Bundle Charging: Wireless Charging Energy Minimization in Dense Wireless Sensor Networks

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***Abstract*—Using a mobile charger to wirelessly charge sensors is a promising yet not well-solved technique. Existing trajectory planning schemes for wireless charger either (1) fail to optimize the one-to-many characteristic of wireless charging or (2) fail to jointly optimize the charger movement cost and the charging cost. The objective of this paper is to find the optimal trajectory planning for a mobile charger in terms of energy minimization in the quadratic attenuation charging model. There exists a trade-off between charging efficiency and trajectory distance. If the mobile charger comes close to sensors, the charging efficiency is high, but the entire charging trajectory of the charger will be long and vice versa. To address this trade-off, we propose the idea of charging bundle and optimize the charger's trajectory based on the charging bundle rather than each sensor. The optimal charging bundle generation problem and the bundle trajectory optimization problem are discussed gradually. Both of them are proven to be NP-hard. Then, we first propose a greedy bundle generation algorithm with an approximation ratio of $\ln n$, where n is the number of sensors. After that, we propose a TSP-based solution and further optimize the TSP-trajectory by jointly considering the adjacent charging locations. Theorems are proposed to effectively find the optimal location. Extensive experiments show that our scheme achieves a much better performance than traditional schemes.**

***Index Terms*—wireless energy transfer, wireless sensor networks, trajectory planning, mobile charging.**

I. Introduction

Due to the battery capacity of wireless sensor networks, energy is by far one of the most critical design issues in the deployment of wireless sensor networks. The recent breakthrough in wireless energy transfer technology provides a promising alternative for powering these sensor nodes. Ideally, the lifetime of a Wireless Rechargeable Sensor Network (WRSN) can be extended infinitely for perpetual operations. During wireless charging, charging vehicles, called Mobile Chargers (MCs), can approach sensors in close proximity [1–3]. Many applications can benefit from WRSNs. Without batteries attached to a node, we can design much smaller and more flexible sensor nodes that can be attached to objects that are not traditionally instrumented, like fruit and medical pills.

To ensure efficient operations, one major challenge in this field is to design the charging trajectory of the mobile charger so that all sensors are recharged while the system operating cost is minimized [1, 3–6]. There are two different operating costs: the charger movement cost and the wireless charging cost. The movement cost of a charger can be simply approximated through the overall trajectory length [1, 4, 7]. The wireless charging cost is more complicated. There are two inherent properties of wireless charging: (1) the charging time quadratically increases along with the charging distance, i.e., the distance from the mobile charger to the sensor [2, 3, 8], and (2) wireless charging is one-to-many charging, i.e., all sensors within the charger's charging range can be charged to some degree [1, 3] due to the coupled magnetic resonance characteristic of wireless charging [9].

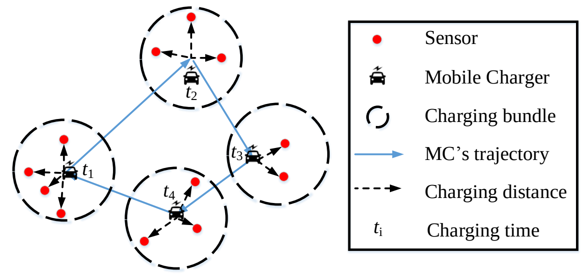
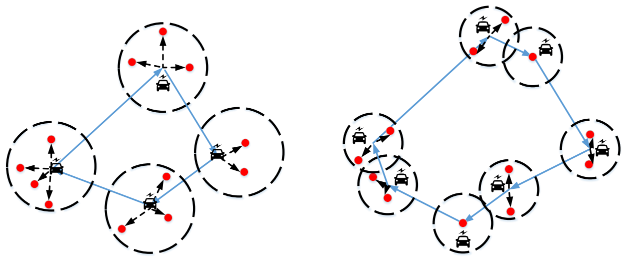


Fig. 1. An illustration of bundle charging.

However, most previous trajectory planning schemes for the mobile charger [1, 4–6, 10] only address some part of the operation cost or use impractical charging models. The authors in [3] does not consider the charger movement cost. The authors in [1, 5] assume the charging time for a specific sensor can always be ignored as long as the sensor is within the transmission range of the mobile charger. In a typical wireless rechargeable sensor such as Intel Research's Wireless Identification and Sensing Platform (WISP) [11], the charging time for voltage to reach $1.8V$ to power a WISP equipped with a $100\mu F$ capacitor can be as long as 155 seconds when the mobile charger is 10.0 meters away [3]. The trajectory optimization approaches of mobile chargers in [4, 6] are based on the one-to-one charging mechanism and thus may not be optimal in dense networks.

Based on the aforementioned situation, we re-think the trajectory planning optimization problem for the mobile charger in terms of minimizing operation cost. A novel idea of *bundle charging*, i.e., a set of nearly sensors, is proposed to optimize the practical one-to-many charging property during the mobile charger trajectory planning. The argument is that the trajectory



(a) Configuration 1 (b) Configuration 2

Fig. 2. An illustration of different charging bundle configurations.

of the mobile charger should not be optimized based on each sensor, which leads to a long trajectory length. Instead, a mobile charger should take advantage of the one-to-many charging property of wireless charging to judiciously cluster location-proximity sensors into a charging bundle. Fig. 1 illustrates the idea of bundle charging in this paper. Sensors in the network are clustered into a set of charging bundles, and the mobile charger will stop at a location of every charging bundle so that all the sensors can be fully charged. In Fig. 1, there are 12 sensors clustered into 4 charging bundles. The mobile charger will stop 4 times rather than 12 times for $\{t_1, t_2, t_3, t_4\}$ seconds in each charging bundle. The time t is determined by the sensor with the farthest charging distance in each charging bundle to meet the charging requirement.

To get the trajectory of the charger, we need to solve two problems gradually: (1) how to generate charging bundles; and (2) how to visit these charging bundles efficiently so that the overall energy consumption is minimized. These two problems are non-trival because of the trade-off between the moving cost and the charging efficiency of mobile charger. In the charging bundle generation, a small bundle radius has a high charging efficiency. However, the mobile charger needs to visit many charging bundles to charge all the sensors, which leads to a long trajectory length. An illustration is shown in Fig. 2, where we generate two different charging bundle configurations for a sensor network. In this example, Fig. 2(a) has four charging bundles and Fig. 2(b) has six charging bundles. In Fig. 2(a), the trajectory length of the mobile charger is shorter; in Fig 2(b), the charging efficiency is higher. The trajectory planning with the given charging bundle configuration is still hard, since selecting a charging position in each charging bundle can be reduced to TSP with neighborhoods [12], which is NP-hard.

In this paper, we first prove that the proposed two problems are NP-hard. Then, we discuss the trade-off between moving cost and charging cost in different scenarios, and we analyze different charging bundle configurations in different scenarios. The greedy bundle generation algorithm is introduced first, which turns out to have a $\ln n$ approximation ratio, where n is the number of sensors. For the charging tour generation, we propose a TSP-based solution to connect charging bundles, and further optimize the TSP-tour through tour lengh reduction. The idea is that we can adaptively adjust the TSP-tour depending on the moving cost and the charging efficiency, i.e., charging a faraway sensor at a low charging rate to trade for the tour length reduction. Theorems are proposed to reduce the optimization time complexity.

The contributions of this paper are summarized as follows:

- To our best knowledge, we are the first to jointly consider the movement cost and charging cost of wireless charging through the trajectory optimization of the mobile charger in a practical setting.
- We propose a novel idea of bundle charging to trade off charging cost and charging efficiency; the optimal bundle radius is analyzed based on different cost scenarios.
- We iteratively optimize the charging tour by carefully considering the two inherent properties of wireless charging. The overall operation cost is further reduced.
- We verify the optimality of the proposed scheme by using extensive simulations and a real testbed.

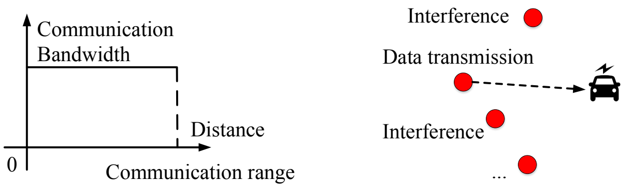
The remainder of the paper is as follows. The related works are in Section II. The problem statement is introduced in Section III. The charging bundle generation algorithm is provided in Section IV. The bundle trajectory optimization is presented in Section V. The simulation results are shown in Section VI. We validate the proposed methods in a real testbed shown in Section VII. The conclusion and acknowledgement are in Section VIII and IX, respectively.

II. Related Works

In this section, we summarize the issues in wireless rechargeable sensor networks [1–4, 7, 10, 11, 13] and the difference with data collection problem by using mobile mules and other related problems [5, 14] in WSNs.

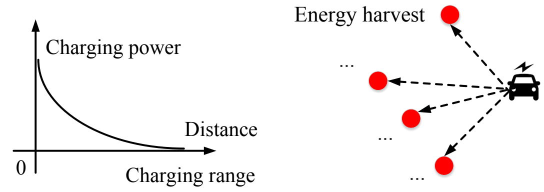
1) Data collection using mobile mules: The initial idea of mobile data collection is to balance the sensor energy consumption, since the sensors near the sink node have more communication and thus run out of energy faster [15–17]. The data delivery path may be multi-hop. Later works [18, 19] proposed to use mobile mules to collect data directly through single-hop communication. In [18], a periodic path was designed for the mobile mule to collect all data. In [19], based on different data collection deadlines, the mobile mule greedily chooses the next sensor to collect data. In [20], they consider the case where there are multiple mobile mules. The collaboration between mobile mules is considered in [21–24].

2) Mobile wireless charging: Early work [1] assumes that charging can be finished as long as the sensor reaches a certain proximity-range. Under this assumption, the mobile charging problem is the same as the traditional data collection problem. In [14], the authors considered a case where there are a set of static wireless chargers. The problem is how to schedule these chargers so that they can charge the sensors in the network in the shortest time by considering the radio interference. In [2, 25], the authors considered a case where the trajectory of the mobile charger is given. In [2], the authors tried to find the maximum constant speed of the mobile charger so that all sensors are fully charged. In [25], the authors discussed the velocity control for a charger so that this mobile charger can maximize the charged energy to sensors with a given time.



(a) Distance vs. efficiency (b) Communication manner

Fig. 3. An illustration of data collection.



(a) Distance vs. efficiency (b) Communication manner

Fig. 4. An illustration of wireless charging.

3) Trajectory planning of mobile charger: In [26, 27], the authors try to minimize the total number of chargers in order to charge all sensors. Traditional approaches [4, 6, 28] for trajectory planning in wireless charging are to reduce the problem into the classic traveling salesman problem with neighborhoods [29], and the mobile charger should reach every sensor at least once. This problem is NP-hard [12, 30], and several approximation solutions [12, 30] are proposed. However, they ignore the fact that only reaching each neighborhood is insufficient. The mobile charge should stay for at least a certain time to fully charge sensors for each location, and improper location leads to large charging cost. The authors in [3] considered a similar problem with this paper. However, their objective is to minimize the overall charging latency. Instead, we focus on the charging energy minimization and the movement latency is also considered. In [31, 32], the authors consider a stochastic charging scheduling based on event dynamics.

III. Model and Problem Formulation

In this section, we first introduce the charging model and the network model followed by the problem formulation.

A. Wireless Charging Model

In [1, 5], the authors assumed that wireless charging procedure is instant. However, due to the weakness of electromagnetic radiation, the power transfer efficiency drops rapidly over distance. Therefore, it may take a while to fully charge sensors, especially for distant sensors. Particularly, we use the WISP-reader charging model, proposed in [2, 3, 8], but our work can externd to other charging models with the minimum modification. We refer to an empirical model of wireless recharging as follows:

$$p_r = \frac{\alpha}{(d+\beta)^2} p_c, \qquad \alpha = \frac{G_s G_r \lambda \eta}{4\pi L_p}, \tag{1}$$

where d is the distance between the MC and the sensor, β is a parameter to adjust the Friis equation for the short distance transmission. G_s is the transmit gain parameter, G_r is the receive gain parameter, L_p is the polarization loss, η is the rectifier efficiency, λ is the average wavelength. p_c is the source power, and p_r is the received power.

We refer to [8] to get the value of the parameters. In [8], the WISP tag has a transmit gain $G_s = 8dBi$. The transmit frequency of readers ranges between $920-925MHz$; therefore, the average wavelength λ is about $0.33m$. The WISP tag has a linearly polarized dipole antenna and has a receive gain $G_r = 2dBi$ according to [10]. Note that wireless energy transfer is immune to the neighboring environment and does not require a line of sight between the energy charging and receiving nodes.

The key difference between the data collection problem and the wireless charging problem is that the locations within the proximity range are the same for data collection problems but are different for wireless charging problems, since the charging efficiency decreases quadratically with the charging distance. Another difference is that data collection procedure is a one-to-one manner but wireless charging is a one-to-many manner. An illustration of their difference is shown in Figs. 3 and 4.

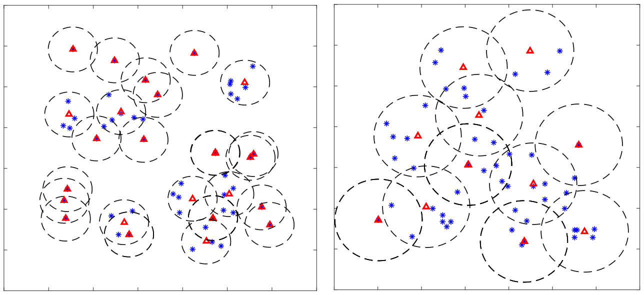
B. Network Model

In this paper, we focus on the environment where sensors are densely deployed. The potential applications are environments monitoring, such as dense jungles (for habitat monitoring applications), battlefields (for enemy troop movement monitoring), e.g., smart dust from DARPA [33], etc. We assume that if n sensors (i.e., nodes) run out of power, the charging procedure is triggered. These sensors and their locations are denoted as $\{s_1, s_2, \cdots, s_n\}$ alternatively in this paper. $\{b_1, b_2, \cdots b_N\}$ charging bundles are generated so that a mobile charger is deployed from the base-station to charging sensors bundle by bundle. Since the monitoring area is usually no man's land or a dangerous area, we assume that no obstacles exist and the mobile charger can move in all possible directions. During the charging tour, each sensor should be charged at least δ, which is the minimum energy requirement. This paper focuses on the energy minimization problem during wireless charging. Specially, we consider both the moving energy cost and the charging energy cost of the mobile charger, the same as [1, 3, 6, 7]. The following are two definitions:

Definition 1. *Charging bundle (CB) is the set of sensor nodes charged by the mobile charger at the same time.*

Definition 2. *Anchor point of a charging bundle is the position from which the mobile charger conducts charging.*

Bundle charging takes advantage of the one-to-many charging manner and the mobile charger stops at the anchor point and operates in charging mode to charge sensors. Note that we do not consider charging during movement. Due to the quadratic charging power attenuation as in Eq. 1, charging sensors at a position which is closest to the sensor is always the best in terms of charging efficiency than any moving charging strategies. In fact, bundle charging is the major contribution that distinguishes our proposed work from existing works. In



(a) CB configuration 1 (b) CB configuration 2

Fig. 5. An illustration of different charging bundle configurations.

the traditional mobile charger trajectory planning problem, the charging tour is optimized in a sensor-based granularity, i.e., each sensor is an anchor point. However, this strategy leads to a long charging tour length, especially in a dense network.

In this paper, we assume that the wireless charging is omni-directional, therefore, the charging time for a charging bundle is determined by the sensor which is the farthest away from the anchor point. To minimize the max distance for all the sensors in a charging bundle, an important observation is that *the optimal anchor point is always in the center of a circle that can cover all the sensor nodes*. This observation can be proved through contradiction. Based on this observation, we define the charging bundle radius r as follows:

Definition 3. *Charging bundle radius is the circle radius centered at the anchor point, where sensors within the circle belong to a charging bundle.*

An illustration of different charging bundle configurations is shown in Fig. 5, where blue stars are sensors' positions, red triangle points are anchor points, and sensors within a dotted circle generate a charging bundle.

Note that there is a slight difference regarding the charging efficiency between one-to-one charging and one-to-many charging [34]. In this paper, we ignore this difference due to two reason: (1) the number of sensors in a bundle is not limited; (2) the charging powering is much larger than sensors' energy receiving power.

C. Problem Formulation

To solve the bundle charging problem, we formulate a two-step approach: (1) we need to find the optimal bundle configuration with a given bundle radius; (2) we need to generate the shortest path with given bundle configuration.

To conduct bundle charging, the first problem that we need to solve is the Optimal Bundle Generation (OBG) Problem, i.e., how to optimally generate charging bundles to conduct the bundle charging. An intuitive idea is to assign all sensors into a few charging bundles, a small number of charging bundles lead to a short charging tour, and vice versa. Therefore, we propose the OBG problem with the given radius, which tries to minimize the number of charging bundles:

$$\begin{aligned} \min \quad & \sum_{b_i \in B} x_i, \\ \text{s.t.} \quad & \sum_{s_i : b_i \in B} x_i \geq 1, \quad x_i \in \{0,1\} \qquad \forall s_i, \end{aligned} \tag{2}$$

TABLE I

SUMMARY OF SYMBOLS

Symbol	Interpretation
n	the total number of sensors
s_i	a sensor i's location in the network
b_i	a charging bundle i
B	a set of all charging bundles
N	the total number of charging bundles
x_i	a boolean decision variable for b_i
p_c	the charging power of the mobile charger
p_r	the collected power of a sensor
E_m	the unit moving energy power of the mobile charger
y_{ij}	a boolean decision variable for path between b_i to b_j
l_i	the charging location of the charger in charging bundle b_i
$d(l_I, l_j)$	the shortest path between two charging location l_i and l_j
δ	the charging threshold of a sensor

where B is the union set of all the possible charging bundles, x_i is a decision variable, where $x_i = 1$ ($x_i = 0$) means that charging bundle b_i is selected (not selected). The objective is that we want to minimize the number of selected charging bundles. The constraint means that for any sensor, at least one charging bundle with it is selected to make sure it can be fully charged.

The second problem is the Bundle Trajectory Optimization (BTO) problem: given N charging bundles, how to optimize the charging tour to conduct charging in terms of energy minimization. It can be formulated as follow:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \sum_{j=1, i \neq j}^{N} d(l_i, l_j) y_{ij} E_m + \sum_{i=1}^{N} p_c t_i, \\ \text{s.t.} \quad & \sum_{i=1, i \neq j}^{N} y_{ij} = 1, \forall j; \sum_{j=1, i \neq j}^{N} y_{ij} = 1, \forall i; y_{ij} \in \{0,1\}, \\ & \sum_{i=1}^{N} p_r(i,j) t_i \geq \delta, \qquad j \in \{1, n\}, \end{aligned} \tag{3}$$

where E_m is the moving power consumption per length and p_c is the charging power per second, respectively, l_i is the anchor point location in charging bundle i, $d(l_i, l_j)$ is the distance between locations l_i and l_j, and t_i is the corresponding charging time in charging bundle i. y_{ij} is a decision variable, where $y_{ij} = 1$ ($y_{ij} = 0$) means that there is (not) a path from charging bundle i to charging bundle j, and $p_r(i,j)$ is the charging power for sensor j in charging bundle i. The first constraint means that each charging bundle should have an arrival from exactly one other charging bundle, and from each charging bundle, there should be a departure to exactly one other charging bundle. The second constraint is the charging energy constraint for each sensor.

IV. Charging Bundle Generation

In this section, we discuss the hardness of OBG problem first, then we propose a greedy bundle generation solution with a $\ln n$ approximation ratio.

A. Hardness Proof

We first prove that the proposed OBG problem is NP-hard.

Algorithm 1 MinDisk

Input: Set of nodes C and corresponding boundary nodes R.
Output: Disk D, disk center c and radius r.

if $\{C = \phi \parallel |R| = 3\}$ **then**
  Calculate the smallest enclosing disk through R.
else
  Choose $s_i \in C$;
  $D\{c, r\}$:= MinDisk($C - \{s_i\}, R$);
  if $\{s_i \notin D\}$ **then**
    $D\{c, r\}$:= MinDisk($C - \{s_i\}, R \cup \{s_i\}$);
Return D.

Algorithm 2 Charging Bundle Generation

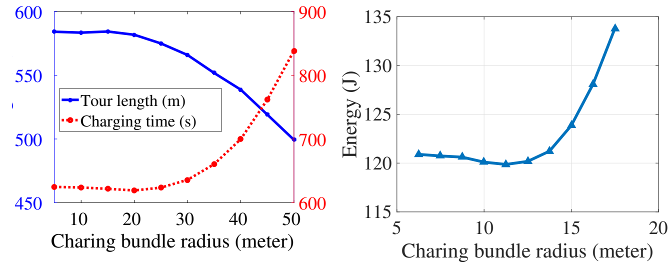
Input: Sensor distribution and bundle radius r.
Output: The charging bundle vector X.

for every node in the network **do**
  Find all its neighbors within radius r and generate all potential charging bundle candidates;
  for {every potential charging bundle candidate} **do**
    Call MinDisk algorithm to find the smallest enclosing disk with radius r';
    if $\{r' < r\}$ **then**
      Add b_i to candidate set;
while $|X| < n$ **do**
  Find the charging bundle b_i with maximal cardinality.
  Add b_i to X;
  for node $n_i \in b_i$ **do**
    Delete all the charging bundle with node n_i.
The charging bundle vector X.

Theorem 1. *The proposed OBG problem is NP-hard.*

Proof. To prove the NP-hardness of the OBG problem (Problem 1), we prove that it can be reduced to the set cover problem in polynomial time. The set cover (Problem 2) is as follows: given a set of nodes $\{e_1, e_2, \cdots, e_n\}$ and a collection of sets $S = \{s_1, s_2, \cdots, s_m\}$, whose union equals the universe, the set cover problem is to identify the smallest sub-collection of S whose union equals the universe.

Given any network on which we need to solve Problem 2, we instantiate Problem 1 in a special case. Let us set each sensor in Problem 1 as a node in Problem 2 and a node set where nodes belong to a charging bundle in Problem in as a set in Problem 2. Therefore, we claim that if there is a polynomial solution which can minimize the total number of selected sets in Problem 2, then it is an optimal charging bundle selection method in Problem 1. On the other hand, the solution of Problem 2 is also the solution of Problem 1, since all nodes in Problem 1 are covered in this case. Since Problem 2 is NP-hard, Problem 1 is also NP-hard. □



(a) Energy consumption (b) Tour length and charging time

Fig. 6. An illustration of the trade-off in bundle charging.

B. Charging Bundle Generation

To check if a finite set of sensors can be assigned to a charging bundle with a given bundle radius r, we revise the MinDisk algorithm [35] into a decisional version. The original MinDisk algorithm computes the Smallest Enclosing Disk (SED), D, and returns the radius and center of the disk. It works recursively; assume we already know a small enclosing disk for $k-1$ points $\{s_1, \cdots, s_{k-1}\}$ and the nodes in boundary are denoted as R. Now, there are two rules for the new D after adding the k^{th} point, which can be proved through contradiction.

- s_k lies inside D. The new D for $\{s_1, \cdots, s_k\}$ is the same for $\{s_1, \cdots, s_{k-1}\}$.
- s_k does not lie inside D. However, s_k must lie on the boundary of the new D.

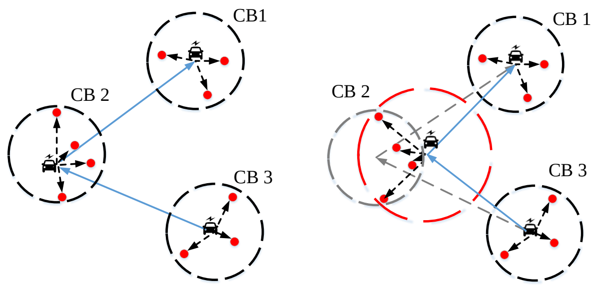
By taking advantage of the two aforementioned properties, we do not need to check unnecessary enclosing disks, and the expected running time is linear.

The proposed charging bundle generation algorithm is shown in Algorithm 2. For every node, we first find all its neighbors with a given bundle radius r. All the feasible charging bundles must be in the combination of its neighbors. The decisional MinDisk algorithm is used to check the feasibility. Therefore, we get all the charging bundle candidates. After that, we greedily select the charging bundle which can cover most uncovered sensors until all sensors are covered.

Theorem 2. *Algorithm 2 is a* $\ln n{+}1$ *approximation algorithm for the proposed OBG problem.*

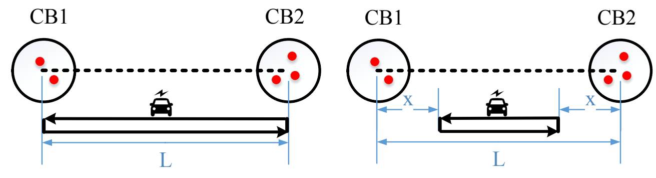
Proof. The proof is based on the greedy property of Algorithm 2. Let us assume that the amount of sensors covered in the round i as x_i. Then, let $z_i = n - \sum_{j=1}^{i} x_j$, which is the amount of remaining uncovered sensors after i rounds of Algorithm 2. Initially, z_0 equals n. Suppose the optimal solution uses k charging bundles to charge n nodes. Therefore, there exists at least one charging bundle that must have n/k sensors. Due to the greedy property of Algorithm 2, Algorithm 2 always selects the charging bundle with the maximal cardinality at each step. Then, we have $x_1 \geq z_0/k$ and the reason is that there exists an optimal solution which uses k charging bundles to charge all sensors. Therefore,

$$z_1 \leq n - n/k = n(1 - 1/k) \tag{4}$$



(a) Center of charging bundle (b) Optimal anchor point

Fig. 7. An illustration of charging tour optimization.



(a) Charging tour 1 (b) Charging tour 2

Fig. 8. An illustration of different charging tour.

Otherwise, there is a contradiction. Note that $z_2 < z_1(1 - 1/(k-1)) \leq z_1(1-1/k) \leq n(1-1/k)^2$. We would like to determine the number of rounds after which our greedy algorithm will cover all sensors. Suppose that it takes m rounds to cover all sensors, the uncovered sensor is less than one. Therefore,

$$z_m = n(1-1/k)^m = n(1-1/k)^{k\cdot\frac{m}{k}} \leq 1. \tag{5}$$

It is equivalent to

$$(1-1/k)^{k\cdot\frac{m}{k}} = e^{-\frac{m}{k}} \leq 1/n. \tag{6}$$

Therefore, $m \geq k \ln n$ and Algorithm 2 is a $\ln n + 1$ approximation algorithm. □

C. Optimal Bundle Radius Discussion

How to determine the optimal charging bundle radius r is challenging. A small r has a higher charging efficiency (i.e., a smaller maximum charging distance that leads to a shorter charging time). However, the mobile charger needs to move more charging bundles to charge all the sensors, which leads to a long trajectory length. We conduct a set of experiments to get the optimal bundle charging radius as shown in Fig. 6. The detailed experimental setting can be found in Section VI. As the charging bundle radius increases, the trajectory length decreases, and the total charging time increases, shown in Fig. 6(a). As for the total energy consumption, it first decreases to an optimal point, then begins to increase as shown in Fig. 6(b). Therefore, in the experiments, it is good to try different charging bundle radii until a best bundle radius r is found.

V. Charging Tour Optimization

In this section, we first prove the NP-hardness of the proposed problem. A TSP-based solution is proposed and then optimized by reducing the influence of distant bundles.

A. NP-hardness Proof

Theorem 3. *The proposed BTO problem is NP-hard.*

Proof. To prove the NP-hardness of the BTO problem (Problem 1), we prove that it can be reduced to the Traveling Salesmen Problem with Neighborhoods (TSPN) in polynomial time. The TSPN (Problem 2) is as follows: the network is partitioned into a certain number of neighborhoods, the TSPN is to find a tour which visits all cities, i.e., reach into the neighborhood, the total length of the tour is minimized.

Given any network on which we need to solve Problem 2, we instantiate Problem 1 as below. Let us set a circle which can cover sensors in a charging bundle as a neighborhood in Problem 2. In this case, visiting a neighborhood in Problem 2 is the same as to visit a charging bundle in Problem 1. Assuming that the charging energy is much lower than the moving energy consumption, we ignore it in the Problem 1, Under this assumption, the overall cost is determined by the path length in both problems. In this instance, the solution of Problem 2 is also the solution of Problem 1 and vice versa. Since Problem 2 is NP-hard. Problem 1 is also NP-hard. □

Note that the BTO problem is different from TSPN. We not only need to find a shortest tour, the maximal distance from the anchor point in each neighborhood should also be minimized in order to ensure the charging efficiency.

B. TSP-Tour Generation

We begin the discussion when there are only two charging bundles in the network to address the drawback of aforementioned TSP-tour optimization, followed by the discussion in the general case. Based on the TSP-tour generation approach, the mobile charger will always reach the center of each charging bundle, shown in Fig. 8(a). However, this solution can be further improved in the case when moving is costly. In Fig. 8(b), the mobile charger does not reach the center of each charging bundle.

Assume the distance between the two charging bundles' centers is L. The mobile charger charges each charging bundle in a position, whose distance is x to the center. In this method, the charger saves some moving distance at the cost of lower charging efficiency; the extra energy can be used to charge sensors over a longer time. Let us denote the original charging time as t, where t is determined by the charging constraint of the sensor,

$$\left(\frac{\alpha}{(\beta+L+r)^2} + \frac{\alpha}{(\beta+r)^2}\right)t \geq \delta. \tag{7}$$

If the charger reduce its movement length, the charger can get a new charging time t' with the same amount of total energy consumption, i.e., $p_c(t'-t) = 2E_m x$, which means that the saved movement energy is used to charge sensors. Hence, the worst charged energy difference for a sensor is

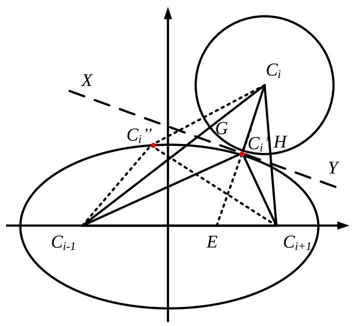


Fig. 9. Proof of Theorems 4 and 5.

$$
\begin{aligned}
f(x) = &(\frac{\alpha}{(\beta+L+r-x)^2} + \frac{\alpha}{(\beta+r+x)^2})t' - \\
&(\frac{\alpha}{(\beta+L+r)^2} + \frac{\alpha}{(\beta+r)^2})t. \\
\approx &\frac{\alpha}{(\beta+r+x)^2}(t+\frac{2E_m x}{p_c}) \\
&- \frac{\alpha}{(\beta+r)^2}t + \frac{2\alpha E_m x}{p_c(\beta+r+L)^2}
\end{aligned} \tag{8}
$$

where the first part of Eq. 8 is the energy charged in the new strategy and the second part of Eq. 8 is the energy charged in the baseline method. If $f(x)$ is a positive number for some positive x, it means that reducing charger movement can further improve the charging efficiency. The optimal x is solvable through the standard numerical method.

If there are multiple charging bundles, we optimize the optimal anchor point iteratively. Assume that there are three contentious charging bundles in the TSP-based solution, denoted as C_{i-1}, C_i, C_{i+1}, the optimal solution might be C_{i-1}, C_i', C_{i+1}, where C_i' is a point inside the triangle $\triangle C_{i-1}, C_i, C_{i+1}$ as shown in Fig. 9.

However, possible positions of C_i' are infinite, which makes an exhaustive search inefficient. Facing this challenge, we prove the following theorems to reduce the search space.

Theorem 4. *The optimal anchor point position is located at the tangency point of the ellipse centered at points C_{i-1} and C_{i+1} and the circle centered at point C_i.*

Proof. Let us assume that the optimal charging distance between the anchor position with the center of the charging bundle is d. Therefore, all the possible anchor points form a cycle. In Fig. 9, all anchor points form a circle centered at C_i. Given a certain moving energy (i.e., given a certain moving distance), based on the definition of ellipse, all of its possible positions form an ellipse, centered at C_{i-1} and C_{i+1} shown in Fig. 9. Then, the tangency point of the ellipse with the radius of the circle centered is denoted as C_i'. Suppose that there exists one point C_i'' which is the optimal anchor position, but C'' is not at the tangency point of the ellipse and circle. An example of C'' is shown in Fig. 9. From Fig. 9, it is clear to find that C_iC_i'' is longer than C_iC_i'. Then, we can replace C_i'' by C_i' to save the overall energy consumption, which is a contradiction that C_i'' is optimal. □

Algorithm 3 Charging Tour Optimization

Input: Graph and charging bundle set and charging radius r.
Output: The charging tour vector V.
Call TSP solver based on the center points generated from Algorithm 2. The output is the visiting order and positions;
for $i = 2 : N-1$ **do**
  Calculate the energy consumption of three adjacent centers C_{i-1}, C_i, and C_{i+1}.
  for $d = 0 : \max$ **do**
    Calculate C_i' based on Theorems 4 and 5.
  Update C_i with best C_i'.

Theorem 5. *The line C_iC_i' is the bisector of the $\angle C_{i-1}C_i'C_{i+1}$.*

Proof. Based on Theorem 4, C_i' is always at the tangency point of the ellipse centered at points C_{i-1} and C_{i+1} and the circle centered at point C_i. Let us denote the coordinate of C' as $(a\cos\theta, b\sin\theta)$, where a and b are the semi-major axis and semi-minor axis of the ellipse centered at points C_{i-1} and C_{i+1} as follows, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Therefore, the tangent line passing point C_i', denoted as XY, shown in Fig. 9, is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$. It is because $XY \perp CC_i'$ due to the definition of tangency point. Similarly, lines $C_{i-1}C_i'$ and $C_{i+1}C_i'$ are

$$y = \frac{b\sin\theta}{a\cos\theta + a}x + \frac{ab\sin\theta}{a\cos\theta + a}, \tag{9}$$

and

$$y = \frac{-b\sin\theta}{a - a\cos\theta}x + \frac{ab\sin\theta}{a - \cos\theta}. \tag{10}$$

If we use e_j to denote the slope of the line j, the slope of these three lines are $e_{XY} = \frac{-b\cos\theta}{a\sin\theta}$, $e_{C_{i-1}C_i'} = \frac{b\sin\theta}{a\cos\theta + a}$, and $e_{C_{i+1}C_i'} = \frac{b\sin\theta}{a\cos\theta - a}$, respectively. Through calculation, we have

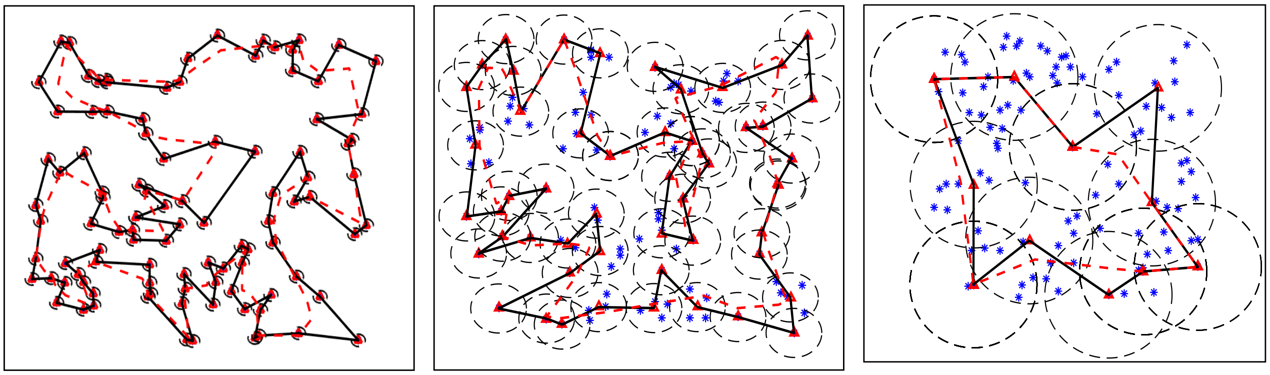
$$\tan\angle C_{i-1}C_i'X = \frac{e_{C_{i-1}C_i'} - e_{XY}}{1 + e_{C_{i-1}C_i'} \times e_{XY}} = \frac{\frac{ab}{\sin\theta}}{a^2 + b^2\frac{\cos\theta}{1-\cos\theta}}, \tag{11}$$

and

$$\tan\angle YC_i'C_{i+1} = \frac{e_{XY} - e_{C_{i+1}C_i'}}{1 + e_{C_{i+1}C_i'} \times e_{XY}} = \frac{\frac{ab}{\sin\theta}}{a^2 + b^2\frac{\cos\theta}{1-\cos\theta}}. \tag{12}$$

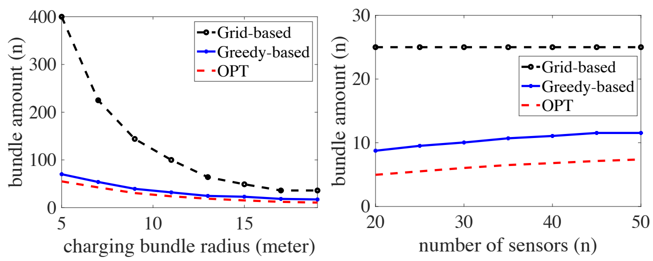
That is, $\angle C_{i-1}C_i'X = \angle YC_{i+1}C_i'$, since $\angle C_{i-1}C_i'X + \angle C_{i-1}C_i'E = \angle C_{i+1}C_i'Y + \angle C_{i+1}C_i'E = 90°$, and therefore, $\angle C_{i+1}C_i'E = \angle C_{i-1}C_i'E$. Theorem 5 is proved. □

With Theorem 5, if we find that $\angle C_{i-1}C_i'X$ is smaller than $\angle C_{i-1}C_i'Y$, the optimal solution is in arc $C_i'G$. Otherwise it is in arc $C_i'H$. Therefore, the position of optimal position with given d can be found effectively through the binary search. The time complexity is reduced from $0(h^2)$ to $O(\log h)$, where h is the discretization level. Then, we can try all different d by discretization to get the optimal anchor point effectively. The tour optimization algorithm is shown in Algorithm 3. It iteratively adjusts the TSP-solution to optimize the tour in terms of energy consumption.



(a) $r = 10m$ (b) $r = 50m$ (c) $r = 100m$

Fig. 10. A network configuration with 50 nodes and three running examples.



(a) Different radius (b) Different sensor number

Fig. 11. Different bundle generation.

VI. Simulation Evaluation

In this section, we demonstrate the effectiveness of the proposed algorithms through extensive experiments.

A. Experimental Setting

We assume wireless rechargeable sensor nodes are randomly deployed over a 2-D square field with the side length $L = 1000m \times 1000m$. The number of nodes in the experimental is 40 to 200. We set $\alpha = 36$ and $\beta = 30$ in Eq. 1, which are obtained through [3]'s experiments. The charging capacity is $2J$ also drawn from [3]. Based on [4], a mobile charger consumes energy at a rate of $5.59J/m$. When charging is operated, it consumes $0.9J/min$ ($5mA \times 3V \times 60$s). During the experiments, we generate different numbers of sensors from 50 to 200 to simulate the different node densities. Each point in the simulation is obtained by an average of 100 runs with different random seeds.

B. Algorithm Comparison

For the charging bundle generation, we compare three algorithms: (1) Grid-based charging bundle generation algorithm [8] partitions 2D area into grids, and each grid is a charging bundle. (2) Greedy-based charging bundle generation algorithm is proposed in this paper. (3) Optimal charging bundle generation algorithm is obtained through the exhaustive search.

For trajectory planning of mobile charger, we compare the following four algorithms.

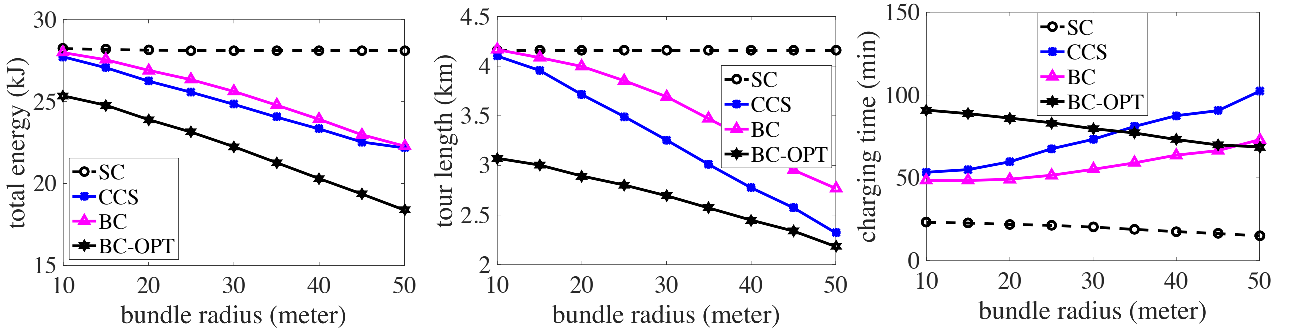
- Single Charging (SC) algorithm [6] does not consider the bundle charging technique. The SC algorithm generates a TSP path to connect all the sensors in the network.
- Combine-Skip-Substitute (CSS) algorithm [36] is optimized for data collection. In the CSS algorithm, a pre-defined radius parameter, communication range, is defined for each sensor, the mobile ferry visits the intersection of the transmission range of multiple sensors to save trajectory length.
- Bundle Charging (BC) algorithm is proposed in this paper; it charges sensors bundle by bundle.
- Bundle Charging Optimization (BC-OPT) algorithm is proposed in this paper based on BC algorithm and the tour optimization in Section V is also considered.

Fig. 10 shows three different configurations with 50 nodes. The black line is the TSP tour based on the charging bundle, i.e., the charging tour generated by BC algorithm. The dotted red line is the optimized tour based on BC algorithm, i.e., the charging tour generated by BC-OPT algorithm. Fig. 10(a) is the configuration when bundle radius is very small. In that case, the proposed BC-OPT algorithm is similar to the SC algorithm, since sensors are visited one by one. With the bundle radius increasing, the number of charging bundle decreases significantly, leading to a short tour length but a low charging efficiency for some sensors. The tour generated by BC-OPT algorithm is more smooth than that of BC algorithm.

C. Experimental Results

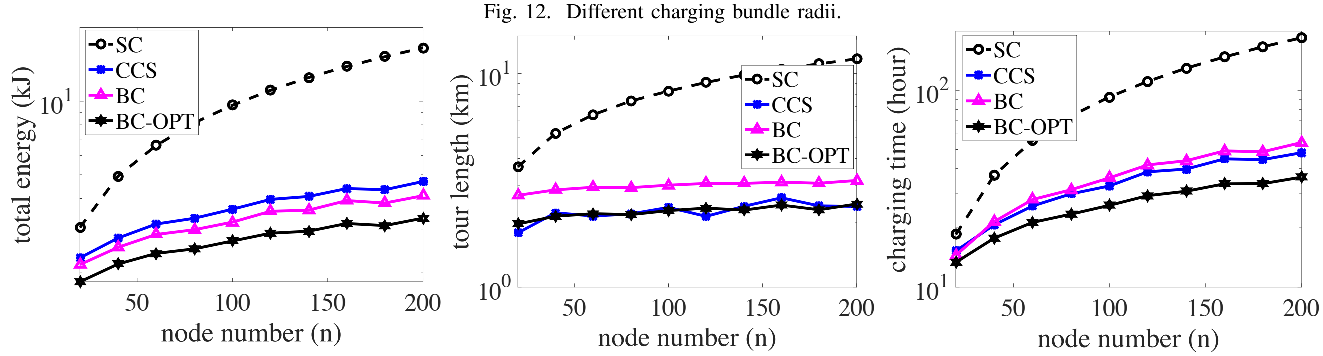
In the experiments, we not only evaluate the total energy of different algorithms but also show the tour length and the total charging time for the charger to fully charge all sensors in the network to provide more insight for proposed algorithms.

1) Bundle generation amount: The performance results of different bundle generation algorithms are shown in Fig. 11. Fig. 11(a) shows that the proposed greedy-based bundle generation algorithm's performance is very close to the optimal solution. In addition, the greedy based algorithm is much better than the grid-based bundle algorithm when the charging bundle radius is small. Fig. 11(a) shows that when the number of sensors is small, the proposed bundle generation algorithm



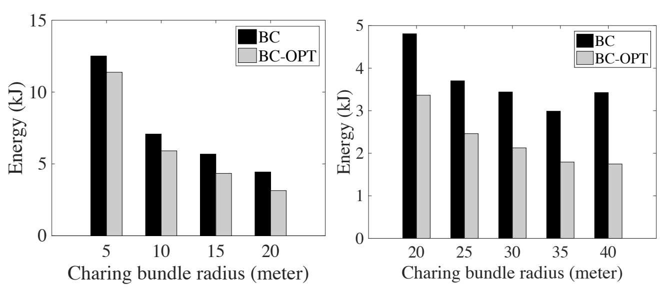
(a) 100 nodes (b) 100 nodes (c) 100 nodes

Fig. 12. Different charging bundle radii.



(a) $r = 100m$ (b) $r = 100m$ (c) $r = 100m$

Fig. 13. Different network densities.



(a) total energy consumption (b) total energy consumption

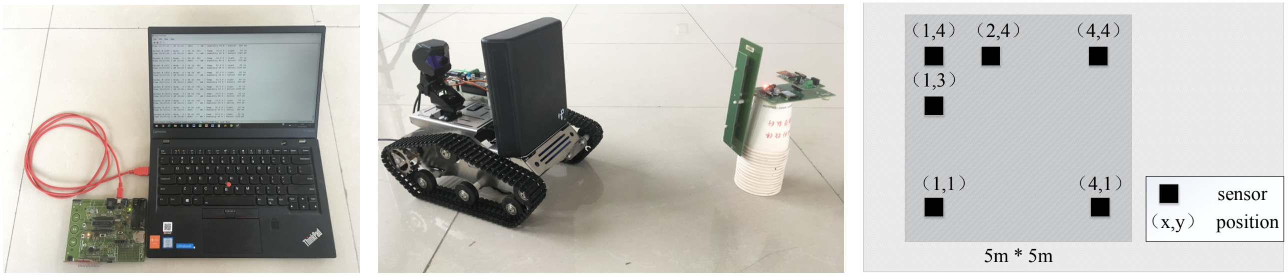
Fig. 14. An illustration of optimal radius, 200 nodes.

performs well. When there are many sensors, the greedy-based algorithm's performance is close to the grid-based solution.

2) Different bundle radii: Fig. 12 shows the result of different bundle radii. The performance of BC-OPT algorithm is the best, followed by the order of BC, CCS, and SC algorithms in terms of energy minimization shown in Fig. 12(a). Fig. 12(a) demonstrates the efficiency of bundle charging, reducing by 38% total energy consumption of the traditional SC algorithm. In this setting, with the bundle radius increasing, the performance of BC-OPT algorithm becomes better than other three algorithms. The reason can be discovered from Figs. 12(b) and 12(c). Fig. 12(b) shows that CCS, BC, and BC-OPT algorithms all reduce the TSP-tour length. The BC-OPT algorithm reduces 25% than the CCS algorithm when bundle radius is $10m$. When it comes to the average charging time for each sensor in Fig. 12(c), the SC algorithm is optimal. However, as the charging bundle radius increases, the average charging time in BC-OPT algorithm reduces. This may be because although the charging time for one specific sensor increases in BC-OPT algorithm, BT-OPT algorithm takes advantage of bundle charging and charges multiple nodes at the same time, so that the average charging time decreases. For the CCS and BC algorithms, the average charging time increases with the increasement in bundle radius.

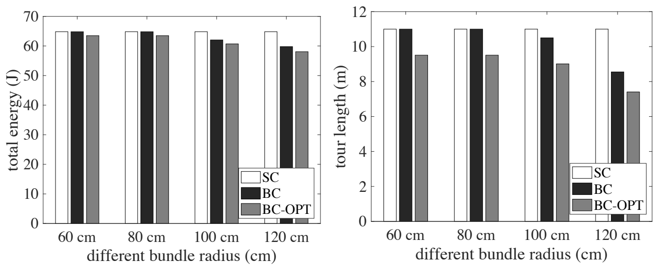
3) Different node numbers: Fig. 13 shows the performance results of proposed algorithms in different sensor numbers. It is clear that the SC algorithm gradually becomes worse as the network density increases. Fig. 13(a) shows the bundle-based charging can significantly reduce the total energy consumption during charging. When the network is dense, $n = 200$, the BC algorithm uses less than half of the energy compared to SC algorithm. The BC-OPT algorithm is the best in terms of energy minimization, followed by BC algorithm and then CCS algorithm. The BC-OPT reduces about 20% total energy consumption. The BC algorithm reduces 5% of the total energy consumption than CCS algorithm, when $n = 40$. However, when $n = 200$, the BC algorithm reduces 10% of total energy consumption than CCS algorithm. An interesting result is shown in Figs. 13(b) and 13(c). That is, the BC-OPT and the CCS algorithms get similar results in terms of the average tour length, as shown in Fig. 13(b). However, the BC-OPT algorithm maintains a lower charging time than the CCS algorithm, as shown in Fig. 13(c). The reason is that the CCS algorithm has the similar concept of charging bundle, but it does not optimize the charging location.

4) Optimal charging bundle radius: Due to the trade-off between bundle radius and charging efficiency, it is important



(a) AP (b) Mobile charger (left) and sensor (right) (c) Sensor distribution

Fig. 15. Testbed.



(a) Total energy consumption (b) Total tour length

Fig. 16. An illustration of the experimental results in the testbed.

to find the optimal bundle radius. The results by using different charging bundle radii from 5 to 40 are shown in Fig. 14. It shows that there exists an optimal charging bundle radius for the BC algorithm as shown in Fig. 14(b). Therefore, to optimize the performance, we should select the charging bundle radius carefully. For the BC-OPT algorithm, its performance improves as the network becomes dense, which demonstrates the effectiveness of tour optimization. Fig. 14(b) shows that when charging bundle radius is large, the BC-OPT algorithm reduces nearly a half more than the BC algorithm.

VII. Testbed Validation

We implement the proposed method in a real testbed as shown in Fig. 15. Our testbed consists of a robot car with a TX91501 power transmitter produced by Powercast [37] and we use this robot car as the mobile charger in the experiments. Each sensor has a P2110 Powerharvester Receiver [37] so that it can receive energy when the mobile charger is nearby and the collected energy can be reported to the labtop through an AP. In this testbed, the TX91501 has a wireless transmitter power of $3W$ and its charging frequency is $915MHz$, which makes the wave length to be $\lambda = 0.33m$. There are 6 sensors in total in the testbed and we set them with coordinates $(1,1), (1,3), (1,4), (2,4), (4,4)$ and $(4,1)$ in a $5m{*}5m$ square area of the office room as shown in Fig. 15(c). Then we can record the charging utility from deployed sensors. The robot car moves at a speed of $0.3m/s$ and we refer the moving energy consumption of $5.59J/m$, the same as simulation and the energy consumption of $4mJ$ [38].

Fig. 16 shows the performance results of the proposed algorithm by choosing different bundle radii. Initially, the bundle radius is small, and therefore, there is only one sensor in each charging bundle. In this case, the proposed BC and BC-OPT algorithms have the same performance as the SC algorithm. Along with the increase of the charging bundle radius, the proposed BC and BC-OPT achieve a better performance than SC algorithm. In Fig. 16(a) , when r equals $1.2m$, BC and $BC-OPT$ reduces the overall charging energy about 8% and 13%, respectively, as shown in Fig. 16(a). It is because the BC and BC-OPT generate charging tours with a much shorter length. In the experiments, the BC-OPT algorithm reduces more than 20% tour length compared with SC algorithm.

Compared with the simulation result in Section VI, the advantages of the proposed BC and BC-OPT algorithms are not fully present in the testbed experiments for the following two reasons: (1) due to limited number of available sensors in our testbed, the opportunity to generate charging bundles is relatively limited; (2) due to the relatively small experiment test, the total movement energy consumption is relatively small. Therefore, we can conclude that the proposed bundle charging scheme can achieve a even better performance, i.e., lower total energy consumption in real applications.

VIII. Conclusion

In this paper, we argue that how to minimize the operation cost of the wireless charger in the wireless sensor network is still not well-solved. Existing trajectory planning schemes either (1) fail to optimize the one-to-many character of wireless charging or (2) fail to jointly optimize the charger movement cost and the charging cost. The main idea is that we need to consider a set of sensors rather than a single sensor during charging optimization. We first propose a greedy-based charging bundle generation algorithm with an approximation performance bound $\ln n$, where n is the number of sensors. For the trajectory planning optimization, we optimize the TSP-based solution based on the adjacent charging locations to further reduce the charging tour length. Extensive simulation and testbed experiments show that our scheme achieves a much better performance than existing schemes.

IX. Acknowledgement

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