Hierarchical Edge-Cloud Computing for Mobile Blockchain Mining Game

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1. Blockchain

- PoW-based blockchain mining
  - Mining a block is a puzzle solving race on miners’ computing power

- Mining incentive
  - Each block will be rewarded with $R$
  - Prob. of winning a puzzle solving race
    - $W_i = \text{computing rate} = \frac{\text{individual computing power}}{\text{total computing power}}$

\[
\cdots \xleftarrow{\text{block } (i-1)} \xrightarrow{\text{block } i} \xleftarrow{\text{block } (i+1)} \cdots
\]
Motivation: Apply in Mobile Devices

- Few blockchain applications in mobile environments
  - Mobile devices cannot satisfy mining requirements
    - Limited computing power and energy
  - Solution: computation offloading
    - Offloading incurs delay \(d\) and cost \(C\) from service provider
      - A miner’s utility: \(U_i = R \cdot W_i - C\)
      - \(W_i = (1 - \beta(d)) \times \text{computing rate}\)
        - specific function of delay proportional to computing power
A Two-layer Offloading Paradigm

- Two service providers
  - A remote cloud computing service provider (CSP)
    - Rich resource capacity, low price, long delay
  - A nearby edge computing service provider (ESP)
    - Limited resource capacity ($E_{\text{max}}$), high price, short delay

- Different operation modes
  - ESP is connected to CSP
    - Auto-transfer requests to CSP if overloaded
  - ESP is standalone from CSP
    - Reject requests if overloaded
2. Problem Formulation

1. Nash subgame of N miners to maximize utility $U_i$
   - Decide on resource share from ESP ($e_i$) and CSP ($c_i$)

2. Nash subgame of ESP/CSP to maximize revenue $V_e(V_c)$
   - Decide on the resource unit price $P_e(P_c)$

3. Stackelberg game between miners and ESP/CSP
   - Interplay between leaders (ESP/CSP) and followers (miners).
Miners’ Subgame

- Formulation of strategy and objective
  - Determine $e_i$ and $c_i$ under budget limitation $B_i$ to
    \[
    \text{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i)
    \]

- Winning probability $W_i$ and delay $d$
  - $d$ discounts $W_i$ by $1 - \beta(d)$
    \[
    \beta(d) = 1 - e^{-\lambda d}
    \]
    represent mining difficulty
  - Tradeoff on delay and price
    - CSP lowers cost while decreasing $W_i$
    - ESP increases $W_i$ while adding cost

PDF of a conflicting block being found given another block is being propagated
Validation of Winning Probability

- \( W_i \) combines winning either in edge or cloud

  \[ W_i = W_i^e + W_i^c \]

  \[ W_i^e = \frac{e_i}{E + C} \cdot \left(1 + \frac{\beta C}{E}\right) \quad \text{and} \quad W_i^c = \frac{c_i}{E + C} \cdot (1 - \beta) \]

  where \( E = \sum_{i=1}^{N} e_i \) and \( C = \sum_{i=1}^{N} c_i \)

- **Theorem 1.** \( W_i \) is valid to express winning probability of individual miners in a mobile blockchain mining network

  **Proof:** We present the full verification process by checking that \( \sum_{i=1}^{N} W_i = 1 \) always holds.
Service Providers' Subgame

- Formulation of strategy and objective
  - ESP determines a unit price $P_e$ to
    \[
    \text{maximize } V_e = (P_e - C_e) \cdot E \quad \text{where } E = \sum_{i=1}^{N} e_i
    \]
    ESP unit cost ESP sold-out units
  - CSP determines a unit price $P_c$ to
    \[
    \text{maximize } V_c = (P_c - C_c) \cdot C \quad \text{where } C = \sum_{i=1}^{N} c_i
    \]
    CSP unit cost CSP sold-out units
Stackelberg Game

- A two-stage game
  - Stage 1: ESP/CSP subgame
    - ESP(CSP) optimizes its unit price $P_e(P_c)$ by predicting the miners' reactions as well as considering the rival's price strategy.
  - Stage 2: miner subgame
    - Each miner responds to the current prices, by sending requests to ESP/CSP, considering its budget and other miners' requests.

- Stackelberg equilibrium (SE)
  - Formed by subgame perfect Nash equilibria (NE) in both the leader stage and the follower stage
Game Analysis in Connected Mode

- **Theorem 2.** A unique NE exists in miner subgame
- **Theorem 3.** Stackelberg game has a unique SE
  
  A best response algorithm to find the unique SE point in Stackelberg game.

- **Theorem 4.** If all miners have identical budgets $B$, each miner’s request in NE can be expressed as

\[
\begin{align*}
    e_i^* &= \frac{B\beta h}{(1 - \beta + h\beta)(P_e - P_c)}, \\
    c_i^* &= \frac{B[(1 - \beta)(P_e - P_c) - P_c\beta h]}{P_c(1 - \beta + h\beta)(P_e - P_c)}
\end{align*}
\]
Game Analysis in Standalone Mode

- **Theorem 5.** Given a price set \((P_e, P_c)\), there exists at least one NE in miner subgame.

- **Theorem 6.** SE exists in the Stackelberg game.
  - Note: there may exist more than one SE point.

- A distributed price bargaining algorithm with guaranteed convergence to find one SE point.
The number of miners changes in each round
- Modeled as a random variable $N \sim \mathcal{N}(\mu, \sigma^2)$
- Where $N = k$ with probability $P(k) = \Phi(k) - \Phi(k - 1)$.
4. Experiment

- **Setting**
  - A small network of 5 miners with identical budgets $B=200$
  - Each experiment is averaged over 50 rounds

- **Miner subgame equilibrium**
  - influences of communication delay
    - Delay decreases the number of resources sold by CSP and his revenue.

![Graphs](attachment:image.png)

(a) The ESP’s revenue.  
(b) The CSP’s revenue.
Miner Subgame Equilibrium

- Influences of operation modes
  - Miners are discouraged from buying units from an ESP working in the connected mode.
  - Crosses in (b) the CSP's optimal prices under different communication delays.

(a) $\beta = 0.02$.

(b) $\beta = 0.06$. 
Miner Subgame Equilibrium

- Influences of miners’ budgets
  - Higher budgets, more requests as well as more revenues

(a) A miner’s request to the ESP.

(b) A miner’s request to the CSP.
ESP/CSP Subgame Equilibrium

- Influences of service providers' costs
  - prices increase linearly as unit costs increases
  - ESP charges a higher price

![Graphs showing price increases](image)
Population Uncertainty

- Render miners more aggressive to buy computing resources from the ESP.

![Graphs showing the relationship between delay factor and sold units in total for different scenarios.](image)

(a) $\beta = 0.02$.

(b) $\beta = 0.06$. 
5. Conclusion

- A Stackelberg game with two subgames
  - Consider delay and cost tradeoff in mobile mining environment
  - Model the relation between winning probability and delay
  - Solve a price-based resource management problem

- Two ESP operation modes:
  - Connected vs standalone

- Impacts of population uncertainty

- Experiments to confirm theoretical analysis
Thank you

Q & A