Multi-Leader Multi-follower Stackelberg Game in Mobile Blockchain Mining

Suhan Jiang, *Student Member, IEEE,* Xinyi Li, *Student Member, IEEE,* and Jie Wu, *Fellow, IEEE,*

Abstract—The development of Blockchain-based mobile applications are impeded due to the resource limitations of mobile devices. Computation offloading can be a viable solution. In this paper, we consider a two-layer computation offloading paradigm including an edge computing service provider (ESP) and a cloud computing service provider (CSP). We formulate a multi-leader multi-follower Stackelberg game to address the computing resource management problem in such a network, by jointly maximizing the profits of each service provider (SP) and the payoffs of individual miners. We study two practical scenarios: a fixed-miner-number scenario for permissioned blockchains and a dynamic-miner-number scenario for permissionless blockchains. For the fixed-miner-number scenario, we discuss two different edge operation modes, *i.e.*, the ESP is *connected* (to the CSP) or *standalone*, which form different miner subgames based on whether each miner's strategy set is mutually dependent. We propose two models: a hit/miss(H/M) model and a capacity(E_{max}) model to characterize the resource limitation on the ESP side. The existence and uniqueness of Stackelberg equilibrium (SE) in both modes are analyzed, according to which algorithms are proposed to achieve the corresponding SE(s). For the dynamic-miner-number scenario, we focus on the impact of population uncertainty and find that the uncertainty inflates the aggressiveness in the ESP resource purchasing. Numerical evaluations are presented to verify the proposed models.

Index Terms—Cloud computing, edge computing, game theory, load sharing, mobile blockchain mining, reinforcement learning.

1 Introduction

CURRENTLY blockchain technology has been widely adopted, ranging from cryptocurrency, financial services, Internet of Things (IoT) to public and social services. As a distributed ledger, blockchain records data in the form of linked blocks secured by cryptography. Consensus protocol is the core of blockchain, since it regulates the maintenance for such an append-only public ledger in a distributed fashion. Currently, most blockchain applications are on top of a proof-of-work (PoW) protocol. In a PoW-based blockchain network, miners collect blocks of data, verify their integrity, and append them to the blockchain. In order to add a block to the blockchain, miners are required to solve a computationally challenging PoW puzzle. The security and reliability are thus ensured by this mechanism which requires numerous trials and errors for a valid solution. The blockchain grows with the repetitive block-appending processes, each of which is considered as one mining round; meanwhile, the owner of the on-chain block receives monetary rewards as the mining incentive.

The new trend on blockchain technology is using blockchain in mobile app development. However, the energy consumption and the computing power required to perform PoW computation are prohibitively high for mobile devices, thus hindering the practical usage of blockchain in mobile environments. Offloading PoW computation to the external machines has been proven effective in overcoming the aforementioned limitations and promoting mobile blockchain applications. Specifically, both an *edge computing service provider* (ESP) and a *cloud computing service provider* (CSP) can provide computing resources for mobile devices. While a CSP can guarantee a good isolation among multiple computation offloading requests (*i.e.*, there is no competition for cloud computing resources) with a relatively cheap price, significant network delays hamper the performance of cloud computing. Due to the delay-sensitive nature of mining, an ESP is considered as an efficient proximity alternative with the capability of providing low-latency service. However, mobile miners may have to compete against each other for the limited and expensive edge computing resources.

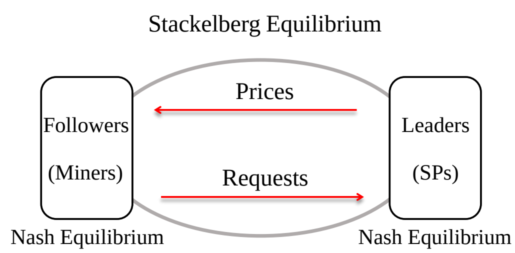
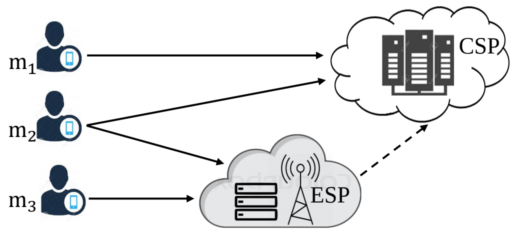


Fig. 1: Mobile blockchain mining network: a multi-leader multi-follower Stackelberg game among SPs and miners .

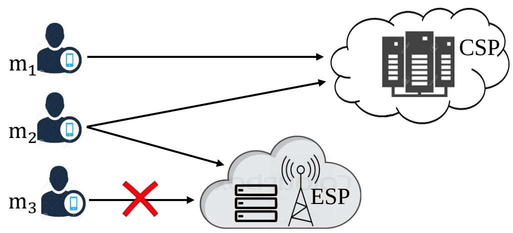
In this paper, we present a hierarchical computation offloading paradigm consisting of two service providers (SPs), *i.e.*, a nearby ESP and a remote CSP, and a set of miners in a mobile blockchain mining network. As depicted in Fig. 1, each miner is willing to offload its PoW computation to either of these two SPs or both

• *S. Jiang, X. Li and J. Wu are with the Department of Computer and Information Sciences, Temple University, Philadelphia, PA, 19122. E-mail:{Suhan.Jiang, Xinyi.Li, and jiewu}@temple.edu*

This research was supported in part by NSF grants CNS 1824440, CNS 1828363, CNS 1757533, CNS 1629746, CNS 1651947, and CNS 1564128. Manuscript received November 9, 2020



(a) ESP is connected to CSP.



(b) ESP is standalone.

Fig. 2: Different operation modes of the ESP.

of them. Once the ESP is overloaded with requests, it responds differently according to its operation mode. Specifically, two edge computing operation modes, *i.e.*, the ESP *connected* to the CSP (Fig. 2(a)) and *standalone* (Fig. 2(b)), have been implemented in practice. Consequently, for an edge computing request which fails to be satisfied by the ESP, it will be sent to the backup CSP in the connected mode (characterized by the dotted line in Fig. 2(a)), or will be rejected in the standalone mode characterized by the dash line in Fig. 2(b)). In the standalone mode, miners can resend those requests rejected by the ESP to the CSP. However, the communication delay will be considerably longer than that in the connected mode where the ESP executes automatic transfers. In the standalone mode, miners' requests are mutually affected and should be dedicated to avoid overloading the ESP.

We exploit game theory to analyze the complex interactions among SPs and mobile miners. To solve the price-based resource management problem, we leverage a multi-leader multi-follower Stackelberg game, which includes two subgames for the SPs (as leaders) and the miners (as followers), respectively. In the SP subgame, each SP has a privilege to set unit prices on its computing resources by anticipating the miners' responses. In the miner subgame, the miners decide their requests according to the observed unit prices. Moreover, we investigate how edge operation modes will affect the miner subgame. We consider two possible models to describe the resource limitation on the ESP side. The first one is a hit/miss(H/M) model, which defines a fixed predicting rate with which a miner's edge request will be fully accepted or transferred to the CSP in the connected mode(rejected in the standalone mode). In this model, the miner subgame is formulated as a classical Nash equilibrium problem (NEP). The second one is a capacity model. In this model, the ESP's capacity, defined as E_max, is a common knowledge among all players. In the capacity mode, due to the limited computing units at the ESP side, whether a miner's edge computing request can be satisfied is affected by other miners' requests. Then, the miner subgame becomes a generalized Nash equilibrium problem (GNEP) in the standalone mode. GNEPs differ from NEPs in that, while in an NEP only the players' objective functions depend on the other players' strategies, in a GNEP both the objective functions and the strategy sets depend on the other players' strategies.

All previous studies assume that the miner number is fixed as a common knowledge in the proposed games. In practice, for permissionless blockchains where miners can randomly join or leave, the miner number may change. Thus, we also discuss the impact of population uncertainty on the miners' strategies by modeling the miner number as a random variable. The major contributions of this paper are as follows:

- We propose a Stackelberg game to solve a price-based computing resource management problem in a mobile blockchain mining network with two SPs.
- We study the proposed Stackelberg game in two practical edge operation modes, thereby formulating two different miner subgames: an NEP in the H/M model and a GNEP in the capacity model.
- We analyze the existence and uniqueness of Stackelberg equilibrium (SE) for both edge operation modes, based on which algorithms are proposed to obtain SE solutions.
- We consider a special case of homogeneous miners and derive explicit-form expressions of the most profitable price strategies for each SP and the optimal resource requests for individual miners in each mode.
- We study the impacts of population uncertainty, which incurs more resource requests at the ESP side.
- We extend our single-CSP single-ESP leader model to a single-CSP multiple-ESP model, which is more in line with the reality.
- We conduct testbed experiments to verify the practicality of our proposed model and perform numerical evaluation in a reinforcement learning framework to validate our analysis. The achieved equilibria are consistent with our theoretical results.

2 System Model and Game Formulation

2.1 A Mobile Blockchain Mining Network

This paper focuses on a mobile blockchain mining network. Corresponding notations are listed in Table 1. We consider N end users, which we also call miners, and two service providers. Fig. 1 depicts an overview of this network. The SP side consists of a nearby ESP and a remote CSP that make profits by contributing their computing power sold by unit. One unit from the ESP is functionally equivalent to one from the CSP. In the proposed network, message transmission time is viewed as communication delay. For simplicity, we assume communication delay between the ESP and miners is negligible as 0, while communication delay between the CSP and the ESP/miners is the same as D_{avg}. Besides, the

TABLE 1: Summary of Notations.

Symbol	Description
P_e/P_c	unit price set by the ESP/the CSP
C_e/C_c	unit cost of the ESP/the CSP
V_e/V_c	utility of the ESP/the CSP
$1-h$	ESP's expected transfer rate
E_{max}	total computing capacity of the standalone ESP
D_c	average delay the CSP
N	total number of miners
m_i	the i-th miner
$U_i/W_i/B_i$	m_i's utility/winning probability/budget
e_i/c_i	number of ESP/CSP units requested by m_i
r_i	m_i's request vector to the SPs, in the form of $[e_i, c_i]^\intercal$
$\boldsymbol{r}$	stacked request vectors of all miners
$\boldsymbol{r}_{-i}$	stacked request vectors of all miners excluding m_i's
R	blockchain mining reward
β	discount rate caused by delay

ESP is assumed to have limited computing capability, while the CSP owns unlimited computing power.

The end-user side is a network with N miners using different mobile devices. We differentiate them in terms of available budget which gives an upper bound on the amount of computing units they can afford. Thus, different types of miners have different requests on computing power. We employ a utility function to describe each miner's expected payoff, *i.e.*, the difference between its expected income and expected cost. The SPs and the miners have bidirectional communications for exchanging price and request information. Miners receive prices from the SPs and transmit their requests to them.

We consider two practical edge operation modes, *i.e.*, connected to the CSP or standalone, differing in whether or not the ESP would share loads with the CSP if it is computationally overloaded. Based on these two modes, we characterize the limited computing capability of the ESP in two different ways. In connected mode, the ESP's computing limitation is captured by an expected transfer rate, *i.e.*, $(1-h)$. That is, A request to the ESP may automatically be transferred to the CSP with a probability of $(1–h)$ in expectation. As an empirical value, $(1-h)$ is common knowledge in the game. Instead, if operating in standalone mode without load sharing, the ESP is limited with E_{max} computing units and hence rejects requests once overloaded.

2.2 SP-Miner Interaction: A Stackelberg Game

We focus on interactions between the SPs and the miners. Each miner's income depends all miners' strategies and its cost varies according to the prices set by each SP. In fact, each SP decides its unit price by considering miners' requests as well as the rival SP's price. Game theory provides a natural paradigm to model the interactions between the SPs and the miners in this network. Each SP sets the unit price and announces it to the miners. The miners respond to the price by requesting an optimal amount of computing units to the SPs. Since the SPs act first and then the miners make their decision based on the prices, the two events are sequential. Thus, we model the interactions between the SPs and the miners using a Stackelberg game. In our proposed game, the SPs are the leaders and the consumers are the followers. It is a multi-leader multi-follower Stackelberg game, two stages of which can be described as follows.

In the first stage, the competition between the ESP and the CSP forms a non-cooperative leader subgame, where each SP optimizes its unit price (P_e/P_c) by predicting the miners' reactions as well as considering the other SP's price strategy. In the second stage, each miner, *i.e.*, m_i, responds to the current prices, by sending request(s) to the target SP(s), considering its budget B_i and requests of other miners'. Since requests are generated for individual utility maximization, a non-cooperative follower subgame is also formed.

2.2.1 Miner Side Utility

Let e_i and c_i be m_i's requests on the ESP and the CSP, respectively. Given the mining reward R, we define m_i's optimization problem below.

Problem 1 (MINER SUBGAME: OP_{MINER}).

$$\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \tag{1a}$$

$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0. \tag{1b}$$

where W_i represents m_i's expected winning probability, an accurate definition and detailed explanations of which will be given in Section 3. Each miner m_i aims to maximize its utility and constraint (1b) ensures that m_i is within its budget.

2.2.2 SP Side Utility

The objective of each SP is to optimize its profit by determining the corresponding unit price. The optimization problem (including OP_{ESP} and OP_{CSP}) at SP side is thus defined as in Eq.(2a) and Eq.(2b) for the ESP and the CSP, respectively.

Problem 2 (SP SUBGAME: OP_{SP}).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E \;\; \textit{where}\; E = \sum\nolimits_{i=1}^{N} e_i \tag{2a}$$

$$\textit{maximize} \quad V_c = (P_c - C_c) \cdot C \;\; \textit{where}\; C = \sum\nolimits_{i=1}^{N} c_i \tag{2b}$$

2.2.3 Stackelberg Game

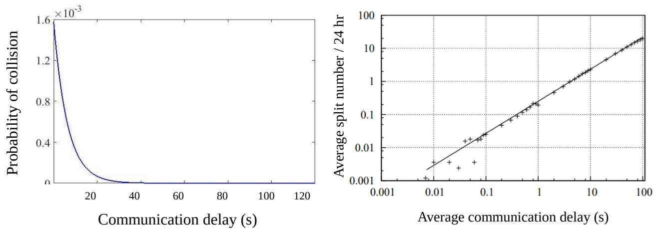
OP_{SP} and OP_{MINER} together form the proposed Stackelberg game. To achieve equilibrium in this game, where neither the leaders (SPs) nor the followers (miners) have incentives to deviate, we need to find its subgame perfect Nash equilibria (NE) in both the leader stage and the follower stage, by applying backward induction. Formally, the SE point(s) is defined as follows.

Definition 1. *Let $[\boldsymbol{E}^*, \boldsymbol{C}^*]$ and $[P_e^*, P_c^*]$ denote the optimal resource request vector of all miners and the optimal computing unit price vector of SPs, respectively. Let $[e_i^*, c_i^*]_{i=1}^N = [\boldsymbol{E}^*, \boldsymbol{C}^*]$, then the point $(\boldsymbol{E}^*, \boldsymbol{C}^*, P_e^*, P_c^*)$ is the Stackelberg equilibrium if the following conditions hold:*

$$V_e(P_e^*, \boldsymbol{E}^*) \ge V_e(P_e, \boldsymbol{E}^*), \forall P_e, \tag{3a}$$

$$V_c(P_c^*, \boldsymbol{C}^*) \ge V_c(P_c, \boldsymbol{C}^*), \forall P_c, \tag{3b}$$

$$U_i(e_i^*, c_i^*, P_e^*, P_c^*) \ge U_i(e_i, c_i, P_e^*, P_c^*), \forall i. \tag{3c}$$



(a) Probability density function of a conflicting block being found while there exists another block being propagated in the network [2].

(b) Average number of blockchain forks per 24 hours as a function of communication delay, averaged over all the nodes in the network [3].

Fig. 3: Communication delay can cause temporary blockchain splits.

3 A Miner's Winning Probability

3.1 Parameter Analysis

As the core part of each miner m_i's utility, W_i is determined by multiple parameters. To win mining rewards, m_i has to be the first to solve its PoW puzzle and propagate its block to reach consensus. The chance for m_i to find a PoW solution is positively correlated to its relative computing power, which is the ratio of m_i's computing power out of all computing power in the network. There is a delay for a mined block to be known by the entire network. During the delay period, another conflicting block may be found and propagates in the network as well. An earlier-mined block can be nullified since its conflicting block may reach consensus faster. Generally, delays may cause the occurrence of conflicting blocks, and then lower the probability of a mined block being accepted by the blockchain. Obviously, W_i is discounted by delays. The relation between the probability of block collision and the delay has been studied in Bitcoin [1], a classic PoW-based blockchain application. Fig. 3(a) provides its block collision probability density function (PDF) with respect to the communication delay, which is subject to an exponential distribution. Thereby, the discount rate, *i.e.*, the block collision cumulative distribution function (CDF), is almost linear to the communication delay, as shown in Fig. 3(b). In this paper, we assume that the proposed network follows the same pattern of collision PDF and CDF as in Bitcoin. For simplicity, we ignore the block propagation time among all miners. Thus, the delay is from the communication time between a miner and an SP. We denote m_i's winning probability will be affected by the a delay discount function, denoting β. Given the closeness of the ESP, we only consider the miner communication delay to the CSP, denoting D_c that incurs a discount rate of β (short for $\beta(D_c)$).

3.2 Expression of Individual Winning Probability

In this part, we will derive an expression of W_i under the assumption that each miner m_i's request, denoted by a vector $r_i = [e_i, c_i]^\intercal$, is fully satisfied at the SP side. Let $\boldsymbol{r} \triangleq \{r_1, r_2, \cdots, r_N\}$ and $\boldsymbol{r}_{-i}$ represent the request profile of all miners and all other miners except m_i, respectively. We denote E in Eq. (2a) and C in Eq. (2b) as the total number of computing units requested on the ESP and the CSP, respectively. $S = E + C$ therefore represents the total requested computing units in the network. The winning probability, in the form of $W_i = W_i^e + W_i^c$, consists of two parts, W_i^e and W_i^c, jointly contributed by the ESP and the CSP, where W_i^e and W_i^c are functions of r_i and $\boldsymbol{r}_{-i}$ given below:

$$W_i^e(r_i, \boldsymbol{r}_{-i}) = e_i/S + e_i \sum\nolimits_{j \neq i} \beta c_j / ES, \tag{4}$$

$$W_i^c(r_i, \boldsymbol{r}_{-i}) = c_i/S - c_i \sum\nolimits_{j \neq i} \beta e_j / ES. \tag{5}$$

We begin with the analysis on W_i^c.

- W_i^c: c_i/S represents the expected chance that m_i mines a cloud-solved block b. Now we discuss the probability that b is discarded before it reaches consensus. With a chance of β, a conflicting block b' would be found during the propagation time D_c. A cloud-solved b' has the same propagation time D_c and thus cannot beat b. However, b will be discarded if b' is found in the edge and hence reaches consensus immediately. W_i^c in Eq. (5) characterizes the fact that, the probability of a successful cloud mining is discounted by the chance that the mined block is discarded due to any conflicting edge-solved block. Here, e_j/E approximates the rate that b' is mined in the edge by another miner m_k. We don't consider the situation, where b' is an edge-solved block for m_i itself, as a discount factor, since m_i still wins.
- W_i^e: m_i's winning probability of edge mining is the addition of (i) the chance that m_i is the first to successfully mine a block using its edge computing power, expressed as e_i/S and (ii) a summed chance that m_i's edge-solved block surpasses a cloud-solved block mined by any other miner m_k. The expression is shown in Eq. (4).

We verify the validity of W_i as a probability mass function.

Theorem 1. *$W_i = W_i^e + W_i^c$ is a valid probability mass function to express the winning probability of individual miners in a mobile blockchain mining network.*

Proof. We present the full verification process by checking that $\sum_{i=1}^N W_i = 1$ holds.

$$\begin{aligned}\sum\nolimits_{i=1}^N W_i &= \sum\nolimits_{i=1}^N (W_i^e + W_i^c) \\ &= \sum\nolimits_{i=1}^N [e_i/S + c_i/S] \\ &\quad + \beta \sum\nolimits_{i=1}^N [e_i(C - c_i)/ES + c_i(E - e_i)/ES] \\ &= 1 + \beta \sum\nolimits_{i=1}^N (e_i C - c_i E)/ES = 1. \end{aligned}$$ □

Thus, we are now ready to conclude that, the winning probability we use is valid, hence our model as well.

3.3 User Requests and SP Responses

All above analysis is based on the assumption that m_i's request r_i is responded to by the ESP and the CSP as what it expects, *i.e.*, if r_i is fully satisfied by the ESP and the CSP as its original form $[e_i, c_i]^\intercal$, the individual winning probability on this condition is denoted by W_i^h and shown in Eq. (6)

$$W_i^h = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES. \tag{6}$$

However, this assumption cannot always hold when we take the ESP's capability into consideration. It is possible that overall requests from the miner side are beyond the ESP's computing capability. We refine the individual winning probability based on whether e_i can be satisfied by the ESP. Now we discuss how r_i will be responded to if e_i is beyond the ESP's capability, in connected mode and in standalone mode, respectively. We denote the corresponding winning probability by W_i^{1-h}.

3.3.1 Failure in connected mode

In this case, e_i would be transferred from the ESP to the CSP, and therefore, r_i is degraded as $[0, e_i+c_i]^\intercal$. The total computing power in the network stays unchanged as S, while $E - e_i$ and $C + e_i$ represent the actual resource allocation by the ESP and the CSP, respectively. Eq. (7) gives the winning probability.

$$W_i^{1-h} = (1-\beta)(e_i + c_i)/S. \tag{7}$$

3.3.2 Failure in standalone mode

Since e_i would be rejected by the ESP, r_i is degraded as $[0, c_i]^\intercal$. Thus, the total computing power of edge computing and that in the network are reduced to $E - e_i$ and $S - e_i$, respectively. Eq. (8) gives the corresponding winning probability.

$$W_i^{1-h} = (1-\beta)c_i/(S - e_i). \tag{8}$$

4 Fixed Miner Number Scenario

In the fixed miner number scenario, we assume that the network contains a fixed set of miners. That is, the number of miners is modeled as a constant, *i.e.*, $N \triangleq n$. We consider two edge computing operation modes: connected and standalone. We apply backward induction to analyze the subgame perfect NE in each stage for both modes. In the connected mode, the uniqueness of the SE is validated by identifying the best response strategies of the miners. In the standalone mode, the existence of the SE is proved by capitalizing on the variational inequality theory. Then, we get the closed-form price and resource allocation solutions to a special homogeneous-miner case for both modes. Besides, we compare the profits at the SP side and the miner side in these two modes.

4.1 Connected Mode

In this mode, the ESP's limited computing capability is characterized by the ESP's expected transfer rate $(1-h)$.

4.1.1 Miner Subgame Equilibrium

Consequently, m_i should consider two possible results: (i) with a probability of h, its request on the ESP is satisfied; (ii) with a probability of $(1-h)$, its request on the ESP is automatically transferred to the CSP with a degraded service. Thus, W_i can reflect these two results by applying the law of total expectation as below.

$$\begin{aligned} W_i &= h \cdot W_i^h + (1-h) \cdot W_i^{1-h} \\ &= h \cdot [(e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/ES] \\ &\quad + (1-h) \cdot (1-\beta)(e_i + c_i)/S \\ &= (1-\beta)(e_i + c_i)/S + \beta h e_i/E, \end{aligned} \tag{9}$$

then the OP_{MINER} problem can be concreted as below.

Problem 1a (MINER SUBGAME: $\text{NEP}_{\text{MINER}}$)**.**

$$\begin{aligned} &\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), && \text{(10a)} \\ &\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0, && \text{(10b)} \end{aligned}$$

where $W_i = (1-\beta)(e_i + c_i)/S + \beta h e_i/E$.

Thus, the existence and uniqueness of an NE of this miner subgame is given by the following theorem.

Theorem 2. *A unique Nash equilibrium exists in* $\text{NEP}_{\text{MINER}}$ *in the connected mode.*

Proof. Denote the equivalent variational inequality (VI) problem $\mathcal{VI}(\mathcal{K}, \mathrm{F}) \equiv \mathcal{NEP}(\mathcal{X}, U)$, where

$$\begin{aligned} &\mathrm{F} := (\nabla_i U_i)_{i=1}^n, \quad \mathcal{X} = ([e_i, c_i]^\intercal)_{i=1}^n, \quad \mathcal{U} = (U_i)_{i=1}^n, \\ &\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_n, \\ &\mathcal{K}_i := \{(e_i, c_i) | P_e \cdot e_i + P_c \cdot c_i \le B_i, e_i \ge 0, c_i \ge 0\}. \end{aligned} \tag{11}$$

Obviously, (i) $\mathcal{K}_i$ is closed and convex, $\forall i$ and (ii) U_i is continuously differentiable and convex w.r.t. $[e_i, c_i]^\intercal$, $\forall i$, then the VI problem has a non-empty solution set. The existence of NE thus follows the sufficient conditions. Further details and the proof of its uniqueness can be found in 4.1.2. □

As a rational player, each miner optimizes its utility by solving the $\text{NEP}_{\text{MINER}}$ problem as follows. Using Lagrange's multipliers λ_1, λ_2, and λ_3 for the constraints in Eq. (1e), the optimization problem is thus converted to the form

$$\begin{aligned} L_i = {} & R \cdot [(1-\beta)(e_i + c_i)/S + \beta h e_i/E] - (P_e \cdot e_i + P_c \cdot c_i) \\ & - \lambda_1 (P_e \cdot e_i + P_c \cdot c_i - B_i) + \lambda_2 e_i + \lambda_3 c_i, \end{aligned} \tag{12}$$

and the complementary slackness conditions are

$$\begin{aligned} &\lambda_1 (P_e \cdot e_i + P_c \cdot c_i - B_i) = 0, \\ &\lambda_2 e_i = 0, \quad \lambda_3 c_i = 0, \quad \lambda_1 > 0, \lambda_2, \lambda_3, e_i, c_i \ge 0. \end{aligned} \tag{13}$$

By the first-order optimality condition $\nabla L_i = 0$, it immediately follows that $\lambda_2 = \lambda_3 = 0$. Thus

$$\begin{aligned} &e_i = \sqrt{\frac{h\beta E_{-i} R}{(1+\lambda_1)(P_e - P_c)}} - E_{-i}, \\ &c_i = \sqrt{\frac{R(1-\beta)(E_{-i} + C_{-i})}{(1+\lambda_1) P_c}} - \sqrt{\frac{h\beta E_{-i} R}{(1+\lambda_1)(P_e - P_c)}} - C_{-i}, \\ &B_i = P_e e_i + P_c c_i, \text{where } E_{-i} = \sum\nolimits_{j \ne i} e_j, C_{-i} = \sum\nolimits_{j \ne i} c_j. \end{aligned} \tag{14}$$

Solving Eq. (14) yields that

$$1 + \lambda_1 = \left[\frac{(P_e - P_c)\sigma_1 \sqrt{E_{-i}} + P_c \sigma_2 \sqrt{E_{-i} + C_{-i}}}{B_i + P_c C_{-i} + P_e E_{-i}} \right]^2, \tag{15}$$

where: $\sigma_1^2 = h\beta R/(P_e - P_c)$ and $\sigma_2^2 = (1-\beta)R/P_c$. Hence substituting Eq. (15) back into Eq. (14) gives the explicit form of the solution to the $\text{NEP}_{\text{MINER}}$ problem, *i.e.*, each miner's best response strategy. This naturally gives a distributed iterative algorithm, allowing each miner to iteratively update its strategy, given the strategies of other miners. When this unique subgame NE is ensured, the algorithm's convergence is also guaranteed. The uniqueness of NE in $\text{NEP}_{\text{MINER}}$ is guaranteed given that F defined in Eq. (17) is strictly monotone.

4.1.2 Proof of Theorem 2

Given the well-formed formulas defined in Problem 1a, we provide full explanations and details for Theorem 2.

First we recap the problem as follows:

Problem 1a (MINER SUB-GAME: $\text{NEP}_{\text{MINER}}$).

$$\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), \tag{16a}$$

$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0, \tag{16b}$$

where $W_i = (1-\beta)(e_i + c_i)/S + \beta h e_i/E$.

W.T.S.: A unique Nash equilibrium exists in $\text{NEP}_{\text{MINER}}$.

Proof. First we show the existence of NE.

Claim 1: There is at least one NE for Problem 1e.

We leverage variational inequality (VI) theory by reformulating the NEP, *i.e.*, NE(s) exist if the equivalent VI problem has a nonempty solution set. Denote $\mathcal{VI}(\mathcal{K}, \text{F}) \equiv \mathcal{NEP}(\mathcal{X}, U)$, where

$$\begin{aligned}
&\text{F} := (\nabla_i U_i)_{i=1}^n, \quad \mathcal{X} = ([e_i, c_i]^\intercal)_{i=1}^n, \quad \mathcal{U} = (U_i)_{i=1}^n, \\
&\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_n, \\
&\mathcal{K}_i := \{(e_i, c_i) | P_e \cdot e_i + P_c \cdot c_i \le B_i, e_i \ge 0, c_i \ge 0\}.
\end{aligned} \tag{17}$$

Since $\mathcal{K}_i$ is closed and bounded, $\forall i$, then the compactness of $\mathcal{K}$ immediately follows. The convexity of $\mathcal{K}$ is trivial by the linearity. Then it suffices to show that U_i is continously differentiable and convex *w.r.t.* $[e_i, c_i]^\intercal \in \mathcal{K}_i, \forall i$. Denote $\mathcal{H}^i$ for the Hessian matrix of U_i as below.

$$\mathcal{H} := \begin{bmatrix} U_{ee}^i & U_{ec}^i \\ U_{ce}^i & U_{cc}^i \end{bmatrix}, \tag{18}$$

where

$$U_{ee}^i = \frac{\partial^2 U_i}{\partial e_i^2}, \quad U_{ec}^i = \frac{\partial^2 U_i}{\partial e_i \partial c_i}, U_{ce}^i = \frac{\partial^2 U_i}{\partial c_i \partial e_i}, \quad U_{cc}^i = \frac{\partial^2 U_i}{\partial c_i^2}.$$

We provide the explicit-form expressions of the Hessian elements as follows:

$$\begin{aligned}
U_{ee}^i =& -(R(1-\beta)/S^2 + \beta h/E^2) \cdot (R(1-\beta)/S^2 + \beta h/E^2) \\
&+ (R(1-\beta)/S + 2\beta h/E - P_e) \cdot (\beta h/E^3 - R(1-\beta)/S^3), \\
U_{ec}^i =& R(1-\beta)/S \cdot (R(1-\beta)/S^2 + \beta h/E^2) \\
&+ 2(R(1-\beta)/S + \beta h/E - P_e) R(1-\beta)/S^3 \\
&- (R(1-\beta)/S^2 - 2R(1-\beta)/S^3 \cdot c_i), \\
U_{ce}^i =& (-R(1-\beta)/S^2 - R(1-\beta)/S^3 \cdot e_i) \\
&+ (-R(1-\beta)/S^3 + 2R(1-\beta)/S^3 \cdot c_i), \\
U_{cc}^i =& 2R(1-\beta)/S^3 \cdot e_i - 2(R(1-\beta)/S^2 - R(1-\beta)/S^3 \cdot c_i).
\end{aligned}$$

Since $det(\mathcal{H}) = U_{ee}^i \cdot U_{cc}^i - U_{ec}^i \cdot U_{ce}^i > 0$, $\forall [e_i, c_i]^\intercal \in \mathcal{K}_i$, and the positive definiteness holds for any i. Therefore $\mathcal{VI}(\mathcal{K}, \text{F})$ is equivalent with $\mathcal{NEP}(\mathcal{X}, U)$ and has a nonempty solution set, we thus prove that *Claim 1* is legitimate. Then we finish the proof for the uniqueness of NE.

Claim 2: There is at most one NE for Problem 1e.

To show the uniqueness of the NE point, we first introduce the matrices $\mathcal{J}_{low}$, defined as

$$[\mathcal{J}_{low}]_{ij} := \inf_{x \in \mathcal{K}} \begin{cases} |\nabla_{ii}^2 U_i|, & \text{if } i = j, \\ -\frac{1}{2}(|\nabla_{ij}^2 U_i| + |\nabla_{ji}^2 U_j|), & \text{else.} \end{cases} \tag{21}$$

We prove the uniqueness of NE solution by showing that $\mathcal{J}_{low}$ is a strictly copositive matrix. We first give the explicit-form expression of $\nabla_{ii}^2 U_i$ and $\nabla_{ij}^2 U_i$ as follows:

$$\begin{aligned}
\nabla_{ii}^2 U_i =& U_{ee}^i + U_{cc}^i \qquad (22a) \\
=& R[-8(1-\beta)(S - e_i - c_i)/S^3] - 2\beta(E - e_i)/E^3, \\
\nabla_{ij}^2 U_i =& \nabla_{ji}^2 U_j \qquad (22b) \\
=& R(1-\beta)[1 - 2(S - e_i - c_i)]/S^2 + h\beta(2e_i - E)/E^3.
\end{aligned}$$

W.L.O.G. we show that the second-order $\mathcal{J}_{low}$ is strictly copositive, the uniqueness of the solution to generalized cases can be simply proved using induction, due to the repetitive pattern of the objective function U_i. Thus, $\mathcal{J}_{low}$ can be written into the form:

$$\mathcal{J}_{low} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}, \tag{23}$$

$$a_{11} = \inf_{(e_1, c_1) \in \mathcal{K}} |\nabla_{11}^2 U_1|, a_{22} = \inf_{(e_2, c_2) \in \mathcal{K}} |\nabla_{22}^2 U_2|, \tag{24}$$

$$a_{12} = (-\frac{1}{2}) \inf_{\substack{(e_2, c_2) \in \mathcal{K} \\ (e_1, c_1) \in \mathcal{K}}} (|\nabla_{12}^2 U_1| + |\nabla_{21}^2 U_2|). \tag{25}$$

Then it suffices to show that $a_{11}, a_{22} \ge 0$ and $a_{12} + \sqrt{a_{11} a_{22}} > 0$, where the non-negativity of the first two terms are trivial.

$$\begin{aligned}
a_{12} + \sqrt{a_{11} a_{22}} =& \inf_{\substack{(e_2, c_2) \in \mathcal{K} \\ (e_1, c_1) \in \mathcal{K}}} R(1-\beta)[1 - 2(S - e_i - c_i)]/S^2 \\
&+ h\beta(2e_i - E)/E^3 \\
&- 8(1-\beta) \sqrt{\prod_{i=1,2} (S - e_i - c_i)}/S^3 > 0.
\end{aligned}$$

Then $\mathcal{J}_{low}$ is strictly copositive as shown above. Since we have shown that F is continuously differentiable with the derivatives bounded on $\mathcal{K}$ (as the derivatives are all linear on the compact solution space $\mathcal{K}$), F is strictly monotone. Therefore NEP has at most one solution.

We conclude our proof since the uniqueness of NE immediately follows combining *Claim 1* and *Claim 2*. □

4.1.3 SP Subgame Equilibrium

With the knowledge of the miners' strategies, each SP makes its decision by solving the NEP_{SP} defined below.

Problem 2a (SP SUBGAME: NEP_{SP}).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E \textit{ where} E = \sum_{i=1}^n e_i, \tag{26a}$$

$$\textit{maximize} \quad V_c = (P_c - C_c) \cdot C \textit{ where} C = \sum_{i=1}^n c_i. \tag{26b}$$

4.1.4 Stackelberg Equilibrium in Connected Mode

We take advantage of a distributed algorithm called Asynchronous Best-response, as is shown in Algorithm 1, to find the unique NE point in OP_{SP} defined in Problem 2. The solution's uniqueness further guarantees the global convergence and SE is achieved, given that NE is found in the leader stage.

4.2 Homogeneous Miners with Identical Budgets

The solutions to the $\text{NEP}_{\text{MINER}}$ are infeasible to express in a symbolic manner. Fortunately, we are able to get the closed-form computation offloading solutions for the $\text{NEP}_{\text{MINER}}$ in a special case. We consider a homogeneous-miner case where each miner is homogeneous with an identical budget B. We are interested in finding an NE where miners decide on a mixed request, buying computing units from both the ESP and the CSP. Thus, constraint (16b) is modified as $e_i > 0$, $c_i > 0$. The corresponding miner side optimization problem can be rewritten as the $\text{NEP}_{\text{HOMOMINER}}$ problem in the following.

Problem 1b (MINER SUBGAME: $\text{NEP}_{\text{HOMOMINER}}$)**.**

$$\begin{aligned} &\textit{maximize} \quad U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i)\,, && (27a)\\ &\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B, \quad e_i > 0, \quad c_i > 0, && (27b) \end{aligned}$$

where $W_i = (e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/(ES)$.

We will provide the explicit-form expression or the pricing strategy for the homogeneous-miner case defined above in Problem (1d).

Theorem 3. *The unique Nash equilibrium for miner m_i in the* $\text{NEP}_{\text{HOMOMINER}}$ *problem is given below*

$$\begin{cases} e_i^* = \frac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}, \\ c_i^* = \frac{B[(1-\beta)(P_e-P_c)-P_c\beta h]}{P_c(1-\beta+h\beta)(P_e-P_c)}, \end{cases} \tag{28}$$

provided that the prices set by the ESP and the CSP satisfy $P_c < \frac{1-\beta}{1-\beta+h\beta}P_e$.

Proof. According to (13), we have $E^2 = \sigma_1^2 \sum_{j\neq i} e_j/(1+\lambda_1)$ and $S^2 = \sigma_2^2 \sum_{j\neq i}(e_j + c_j)/(1+\lambda_1)$ for each miner m_i, which will yield $E^2/S^2 = \sigma_1^2(E - e_i)/[\sigma_2^2(S - e_i - c_i)]$. Then, we calculate the summation of this expression for all the miners: $E/S = \sigma_1^2/\sigma_2^2 = [h\beta/(1-\beta)] \cdot P_c/(P_e - P_c)$. In order to get a mixed strategy, $E/S > 1$ must hold, *i.e.*, Eq.(38) holds. Since all miners are homogeneous, their best response strategies are identical as well, *i.e.*, $E = ne_i$ and $S = n(e_i + c_i)$. By substituting these two equations into Eq. (15), we obtain the NE for miner m_i in Eq.(28). □

Corollary 1. *If the budget B is sufficiently large, the explicit solution to the* $\text{NEP}_{\text{HOMOMINER}}$ *problem is shown in Eq.(41)*

$$\begin{cases} e_i^* = \frac{\beta h R(N-1)}{N^2(P_e-P_c)}, \\ c_i^* = \frac{R(N-1)[(1-\beta)P_e-P_c]}{N^2P_c(P_e-P_c)}. \end{cases} \tag{29}$$

Now, we start to analyze the SP optimization problem, which can be rewritten as follows.

Algorithm 1 Asynchronous Best-Response Algorithm

Output: $j, j \in \{e, c\}$
Input: Initialize k as 1 and randomly pick a feasible $P_j^{(0)}$
for iteration k **do**
  Receive the miners' request vectors $\mathbf{r}^{(k-1)}$
  Predict the strategy of the other SP
  Decide $P_j^{(k)} = P_j^{(k-1)} + \Delta \frac{\partial V_j\left(P_j, P_{-j}^{(k-1)}, \mathbf{r}^{(k-1)}\right)}{\partial P_j}$
  if $P_j^{(k)} = P_j^{(k-1)}$ **then** Stop
  else send $P_j^{(k)}$ to miners and set $k \leftarrow k+1$

Problem 2b (SP SUBGAME: $\text{NEP}_{\text{SPHOMOMINER}}$)**.**

$$\begin{aligned} &\textit{maximize} \quad V_e = (P_e - C_e)\cdot e_i^*, \quad V_c = (P_c - C_c)\cdot c_i^*, && (30a)\\ &\textit{subject to} \quad P_c < \frac{1-\beta}{1-(1-h)\beta}P_e, && (30b) \end{aligned}$$

where $e_i^* = \frac{B\beta h}{(1-\beta+h\beta)(P_e-P_c)}$, $c_i^* = \frac{B[(1-\beta)(P_e-P_c)-P_c\beta h]}{P_c(1-\beta+h\beta)(P_e-P_c)}$.

Theorem 4. *The unique Nash equilibrium for the SPs in the* $\text{NEP}_{\text{SPHOMOMINER}}$ *problem is given below:*

$$\begin{cases} P_e^* = \bar{p}, \\ P_c^* = \frac{C_c\bar{p}(1-\beta) - \bar{p}\sqrt{C_c h\beta(\bar{p}-C_c)(1-\beta)}}{[1-\beta(1-h)]C_c - \beta h P_e}, \end{cases} \tag{31}$$

where $\bar{p}$ is the solution to $\partial V_e/\partial P_e = 0$.

Proof. We start with the optimal P_c^* by analyzing the convexity of V_c. We calculate the first derivative of V_c and find that it is a concave function. Thus, the CSP's optimal price value is the solution to $\partial V_c/\partial P_c = 0$ where $P_c < P_e(1-\beta)/[1-(1-h)\beta]$ and P_c^* is shown in Eq. (31), as is a function dependent on P_e set by the ESP. Given the response strategy of the CSP, the ESP can optimize his payoff by maximizing the re-written V_e as below:

$$V_e = \frac{NB\beta h}{(1-\beta+h\beta)(P_e - P_c^*)} \cdot (P_e - C_e). \tag{32}$$

We calculate the second derivative of V_e and find that $\partial^2 V_e/\partial P_e^2 \le 0$ holds for any valid P_e value. Thus, the ESP has his dominant strategy $P_e^* = \bar{p}$. In this situation, NE is achieved in the leader stage. We analyze P_e^* and P_c^* and find that they only depend on their own operating costs C_e, C_c, and the network delay penalty factor β. □

4.3 Standalone Mode

Similar to the Connected mode, we can model the miner subgame as a Nash equilibrium game using based on the hit/miss rate in the standalone mode. The difference is the winning probability. With a probability of h, miner m_i will be fully satisfied by both the CSP and the ESP. Then, the corresponding winning probability W_i^h is the same as what shows in Eq. (6). However, with a probability of $1-h$, miner m_i's edge request can be rejected. In this case, his winning probability W_i^{1-h} is shown in

Algorithm 2 Price Bargaining

Input: Choose any feasible starting point P_e, P_c

```
for each miner i do
    Receive P_e, P_c
    Predict the optimal requests of other miners
    Decide its computing request [e_i, c_i]^T
    Send e_i to the ESP and send c_i to the CSP
for each operator j, j ∈ {e, c} do
    Receive the optimal requests of miners
    Store the current prices P'_j and P'_{-j},
    Increase decrease the price with a step Δ
    if V_j(P'_j, P'_{-j}) ≤ V_j(P'_j+Δ, P'_{-j}) and
        V_j(P'_j−Δ, P'_{-j}) ≤ V_j(P'_j+Δ, P'_{-j})
    then P_j = P'_j + Δ
    else if V_j(P'_j, P'_{-j}) ≤ V_j(P'_j−Δ, P'_{-j}) and
        V_j(P'_j+Δ, P'_{-j}) ≤ V_j(P'_j − Δ, P'_{-j})
    then P_j = P'_j − Δ
    else P_j = P'_j
    Send P_j to miners
```

Eq. (8). Thus, W_i should reflect these two results by applying the law of total expectation as below.

$$\begin{aligned} W_i &= h \cdot W_i^h + (1-h) \cdot W_i^{1-h} \\ &= h \cdot [(e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/ES] \\ &\quad + (1-h) \cdot (1-\beta) c_i/(S - e_i) \end{aligned} \tag{33}$$

The $\text{NEP}_{\text{MINER}}$ in the standalone mode is the same as Problem 1b except applying the new winning probability using Eq. (33).

Theorem 5. *A unique Nash equilibrium exists in* $\text{NEP}_{\text{MINER}}$ *in the standalone mode.*

The proof of Theorem 5 is quite similar as what we show in section 5.1.2, which we decide to skip here. In this case, there is no explicit expression under the homogeneous miner assumption, given the complex winning probability function.

There also exists another way to model the limitation of the edge computing resource in the standalone mode. It is more likely for miners to know the ESP's capacity in the standalone mode. Thus, we assume the ESP's capacity E_{max} is a common knowledge in this case. Then, it has to reject some requests when overloaded. Thus, the aggregate requests from all miners should be no more than E_{max} to avoid being rejected.

4.3.1 Subgame Equilibrium

In standalone mode, given other miners' requests $\boldsymbol{r}_{-i}$, m_i should ensure that e_i can be satisfied by the ESP. Mathematically, this can be written as $E = \sum_{k=1}^{n} e_k \leq E_{max}$. Under this constraint, its winning probability is expressed in Eq. (34).

$$W_i = (e_i + c_i)/S + \beta(e_i C - c_i E)/ES. \tag{34}$$

Now, we reformulate the OP_{MINER} problem in the below.

Problem 1c (MINER SUBGAME: $\text{GNEP}_{\text{MINER}}$).

$$\begin{aligned} &\textit{maximize} && U_i = R \cdot W_i - (P_e \cdot e_i + P_c \cdot c_i), && \text{(35a)} \\ &\textit{subject to} && E \leq E_{max}, && \text{(35b)} \\ &&& P_e \cdot e_i + P_c \cdot c_i \leq B_i, \;\; e_i, c_i \geq 0, && \text{(35c)} \end{aligned}$$

where $W_i = (e_i + c_i)/S + \beta \cdot (e_i C - c_i E)/ES.$

Constraint (35b) ensures that m_i's request to the ESP can be satisfied. Since all miners' requests are mutually dependent, the $\text{GNEP}_{\text{MINER}}$ problem is a Generalized Nash Equilibrium Problem (GNEP). In $\text{GNEP}_{\text{MINER}}$, the dependence of each miner's strategy set on the other miners' strategies is represented by the (linear) constraint (35b), which includes each miners' request e_i to the ESP. More specifically, since the miners all share a jointly convex shared constraint (JCSC), this subgame is known as a jointly convex game. $\text{GNEP}_{\text{MINER}}$ can be considered as a special case of $\text{NEP}_{\text{MINER}}$, where $h = 1$ and $(1-h) = 0$ due to the given constraint (35b).

4.3.2 Existence of Stackelberg equilibria

Similar with the proof for $\text{NEP}_{\text{MINER}}$ NE in Theorem 2, the existence of $\text{GNEP}_{\text{MINER}}$ NE is easily followed by capitalizing on the variational inequality theory.

Theorem 6. *Given a price set* $\{P_e, P_c\}$ *from the SP side, there exists at least one Nash equilibrium for the non-cooperative subgame at miner side in standalone mode.*

In general, a GNEP could have infinite solutions. Namely, there are multiple NEs in the follower stage, and thus there is no efficient algorithm to obtain the global optimal pricing and computation offloading strategy for the proposed Stackelberg game. Here, we provide a distributed algorithm which first computes a unique variational solution to the $\text{GNEP}_{\text{MINER}}$ problem and then finds the corresponding solution to the SP SUBGAME: GNEP_{SP} problem (defined later) based on the computed miner Nash equilibrium. Note, there is no guarantee that the SE produced by Algorithm 2 is a global optima.

Problem 2c (SP SUBGAME: GNEP_{SP}).

$$\begin{aligned} &\textit{maximize} && V_e = (P_e - C_e) \cdot E, \quad V_c = (P_c - C_c) \cdot C, && \text{(36a)} \\ &\textit{subject to} && E = E_{max}. && \text{(36b)} \end{aligned}$$

4.3.3 Homogeneous Miners with Sufficient Budgets

In standalone mode, solutions to the $\text{GNEP}_{\text{MINER}}$ are infeasible to express in a symbolic manner. Fortunately, we are able to get the closed-form computation offloading solutions for the $\text{GNEP}_{\text{MINER}}$ in a special case. We consider a homogeneous-miner case where each miner is homogeneous with an identical budget B. We assume B is quite large so that each miner's cost under optimal request is affordable. Under this assumption, the constraint (35c) on budget $\text{GNEP}_{\text{MINER}}$ can be removed. We are interested in finding a Nash equilibrium where miners decide a mixed request, buying computing units from the ESP and the CSP. Thus, constraint (35c) is modified as

$x_i > 0, \quad y_i > 0$. The corresponding miner side optimization problem can be rewritten as the GNEP$_{\text{HOMOMINER}}$ problem in the following.

Problem 1d (MINER SUB-GAME: GNEP$_{\text{HOMOMINER}}$).

$$\begin{aligned} &\textit{maximize} && U_i = R \cdot W_i - (P_e \cdot x_i + P_c \cdot y_i) && \text{(37a)} \\ &\textit{where} && W_i = \frac{x_i + y_i}{S} + \beta \cdot \frac{x_i C - y_i E}{SE} && \\ &\textit{subject to} && E \leq E_{max} && \text{(37b)} \\ & && x_i > 0, \quad y_i > 0 && \text{(37c)} \end{aligned}$$

To achieve such an equilibrium in the follower level, the prices set by the ESP and the CSP matters. Then, the Eq. (38) gives the relation between P_e and P_c under which all miners will yield mixed requests.

$$\begin{cases} P_c < (1-\beta)P_e \\ P_c < P_e - \frac{\beta R(N-1)}{N E_{max}} \end{cases} \tag{38}$$

Given P_e and P_c satisfying the Eq. (38), we compute the explicit expression of a miner's request in Nash equilibrium, as is shown in Eq. (39).

$$\begin{cases} x = \frac{\beta R(N-1)}{N^2(P_e-P_c)} \\ y = \frac{R(N-1)[(1-\beta)P_e-P_c]}{N^2 P_c(P_e-P_c)} \end{cases} \quad \text{where} \quad Nx \leq E_{max} \tag{39}$$

There is a special case where all computing units of the ESP are sold out, *i.e.*, $n \cdot x = E_{max}$, by setting dedicate P_e if the following holds:

$$\begin{cases} P_e - \frac{\beta R(N-1)}{E_{max} N} < P_c < \frac{R(N-1)(1-\beta)}{N E_{max}} \\ P_e < \frac{R(N-1)}{N E_{max}}. \end{cases} \tag{40}$$

Then the corresponding equilibrium request of each homogeneous miner is captured by Eq.(41)

$$\begin{cases} x = \frac{E_{max}}{N} \\ y = \frac{R(N-1)(1-\beta)}{N^2 P_c} - \frac{E_{max}}{N} \end{cases} \tag{41}$$

Given the NE point at miner side, utilities of the ESP and the CSP can be rewritten as follows:

$$V_e = \frac{R(N-1)\beta}{N} \cdot \frac{P_e - C_e}{P_e - P_c} \tag{42}$$

$$V_c = \frac{R(N-1)\beta}{N} \cdot \frac{P_c - C_c}{P_c} \cdot \frac{P_e(1-\beta) - P_c}{(P_e - P_c)} \tag{43}$$

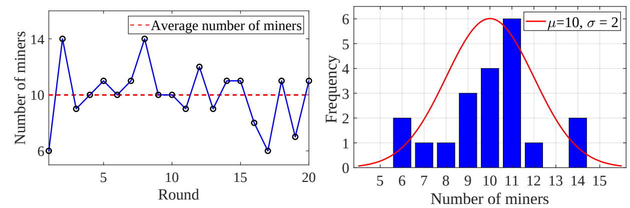
Thus, the optimization problems for the ESP and the CSP are in the below.

Problem 2d (SP SUBGAME: GNEP$_{\text{SPHOMOMINER}}$).

$$\begin{aligned} &\textit{maximize} && V_e = (P_e - C_e) \cdot E, \quad V_c = (P_c - C_c) \cdot C, && \text{(44a)} \\ &\textit{subject to} && E = E_{max}. && \text{(44b)} \end{aligned}$$

The Nash equilibrium in the leader level can be captured by the following equation.

$$\begin{cases} P_e = \frac{\beta R(N-1)h}{N^2[(P_e-P_c)h + P_e(1-h)]} \\ P_c = \frac{C_c P_e(1-\beta) - P_e\sqrt{C_c \beta (P_e - C_c)(1-\beta)}}{C_c - \beta P_e} \end{cases} \tag{45}$$



(a) Statistics on the miner number among 20 mining rounds. (b) Corresponding histogram and distribution $N(\mu, \sigma^2)$.

Fig. 4: A toy example for population dynamics of moblie miners.

TABLE 2: Optimal requests of homogeneous miners with sufficiently large budgets where $\gamma = (N-1)R/N$.

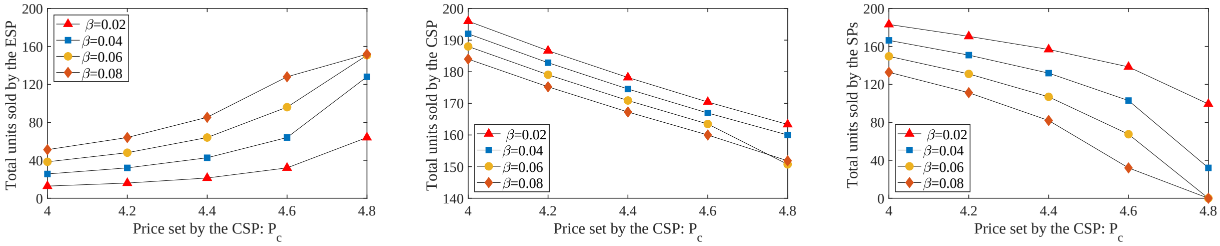
Mode	E^*	C^*	S^*
Connected	$\frac{\gamma\beta}{P_e-P_c}h$	$\gamma\left[\frac{(1-\beta)P_e-P_c}{P_c(P_e-P_c)} + \frac{\beta(1-h)}{P_e-P_c}\right]$	$\frac{\gamma(1-\beta)}{P_c}$
Standalone	$\frac{\gamma\beta}{P_e-P_c}$	$\gamma\frac{(1-\beta)P_e-P_c}{P_c(P_e-P_c)}$	$\frac{\gamma(1-\beta)}{P_c}$

4.4 Comparison of Two Modes

We sum up the main results by comparing these two modes in a homogeneous-miner case. The explicit expressions of all miners' requests in equilibrium are summarized in Table 2, where $\gamma = \frac{(N-1)R}{N}$. As can be explicitly seen in Table 2, the amount of all miners' requests is identical in these two modes, given the same pricing of the CSP. Thus, the total requested computing units is only related to P_c. That is, pricing of the CSP decides the upper bound of the P2P network computing power. Since $h \leq 1$, the ESP would sell more units in standalone mode than in connected mode. Thus, connected mode maximizes the profits of the CSP and also lowers the cost at miner side, while standalone mode maximizes the ESP. The numerical results provided in Section 6 also show that the ESP's equilibrium prices in the standalone mode is higher compared to those in the connected mode. Thus, we conclude that the ESP in the standalone mode gains more profits.

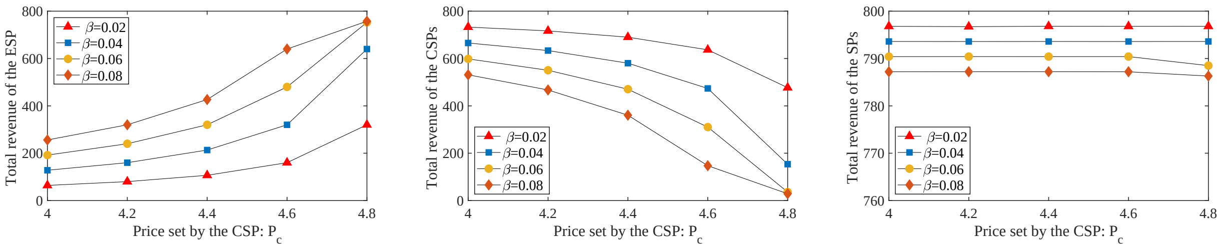
5 DYNAMIC MINER NUMBER SCENARIO

Obviously, in the above analysis, we assume the miner number N is common knowledge in the proposed games. In practice, this scenario is applicable to permissioned blockchains, where miners are pre-selected by a central authority or consortium. However, most blockchains are permissionless, in which anyone can participate in or retreat from the mining process, so the previous scenario may not be suitable. For such situations, we consider a more general scenario by introducing population uncertainty. Games with population uncertainty relax the assumption that the exact number of players is common knowledge. Thus, we model the miner number, N, as a random variable. In particular, N follows a Gaussian distribution with mean μ and variance σ^2. We have $N \sim \mathcal{N}(\mu, \sigma^2)$ where $N = k$ with probability $P(k) = \Phi(k) - \Phi(k-1)$. Fig. 4 gives a toy example where the number of miner can be fit to a Gaussian distribution with $\mu = 10$ and $\sigma^2 = 4$.



(a) Miners' requests to the ESP. (b) Miners' requests to the CSP. (c) Miners' requests to the SPs

Fig. 5: Homogeneous miners with identical budgets and $P_e = 5$.



(a) The ESP's revenue. (b) The CSP's revenue. (c) Total revenue at SP side.

Fig. 6: Homogeneous miners with identical budgets and $P_e = 5$.

In this scenario, we only consider standalone mode and derive the miner utility function U_i as below.

$$U_i(\mu, \sigma^2) = 0.5 \cdot U_i^h + 0.5 \cdot U_i^{1-h} \tag{46}$$

$$U_i^h = P_e \cdot e_i + P_c \cdot c_i - R \cdot W_i^h$$
$$U_i^{1-h} = P_e \cdot e_i + P_c \cdot c_i - R \cdot W_i^{1-h}$$
$$W_i^h = \sum_{k=l}^{u} P(k) \left[(e_i + c_i)/S_k + \beta(e_i C_k - c_i E_k)/(S_k E_k) \right]$$
$$W_i^{1-h} = (1-\beta)(e_i + c_i)/S_\mu$$
$$S_k = E_k + C_k, E_k = \sum_{j=1}^{k} e_j,$$
$$C_n = \sum_{j=1}^{k} c_j, \forall k \in [l, u]$$

Thus, the OP$_{\text{MINER}}$ problem in this scenario can be reformulated as in Eq. (47).

Problem 1e (MINER SUBGAME: OP$_{\text{DYNAMICMINER}}$).

$$\textit{maximize} \quad U_i(\mu, \sigma^2) \tag{47a}$$
$$\textit{subject to} \quad P_e \cdot e_i + P_c \cdot c_i \le B_i, \quad e_i \ge 0, \quad c_i \ge 0 \tag{47b}$$

Problem 2e (SP SUBGAME: OP$_{\text{SP}}$).

$$\textit{maximize} \quad V_e = (P_e - C_e) \cdot E, \ \ V_c = (P_c - C_c) \cdot C \tag{48}$$

The objective function presented in Eq. (47) is so complex that it is infeasible to express its equilibrium expression in a symbolic manner. Therefore, we use numerical analysis to find equilibria in the network. As numerical results will later show in Section 6, we find that with an identical P_e, the uncertainty incurred by the dynamic population renders miners more aggressive to buy computing units from the ESP, even beyond its capability E_{max}. Besides, given the same price P_c from the CSP, we find, in expectation, the total computing units requested by the network are identical with that requested by a network with exactly μ miners.

Miner	x	y	Times of Winning	Probability
m_1	5	10	116	16.1%
m_2	12	5	141	19.6%
m_3	9	9	143	19.9%
m_4	1	20	158	21.9%
m_5	18	1	162	22.5%

TABLE 3: Miner power, actual winning times, and the corresponding winning probability

6 SIMULATION

In this section, we first conduct testbed experiments to verify the practicality of our proposed winning probability function. Then, numerical examples are provided to examine how miners figure out their optimal requests based on the prices of the SPs and how the SPs optimize their unit prices based on their available power and the miners' budgets. We assume the blockchain mining reward R is fixed as 5000. And we assume that, in connected mode, the ESP's expected transfer rate $1-h = 0.1$ is a common knowledge among miners, and in standalone mode, the ESP's resource capacity $E_{max} = 800$ is also known by all miners. The communication delay D_c between the CSP and miners implicitly implies the value of blockchain fork rate β, as β is linear with D_c. When we mention the prices set by the SPs, no matter whether they are optimized or not, $P_e > C_e$ and $P_c > C_c$ always hold.

6.1 Practicality of Winning Probability Function

The most important part is to validate whether our proposed winning probability function is in line with the reality since it is the basis of our paper. To confirm its practicality, we show the show successful PoW-based blockchain mining using our own devices to serving as the CSP and the ESP. We assume there are 5 miners in total and Table 3 shows their mining power (the values of x and y just reflect the ratio rather than the exact assigned computing power). The detailed simulation is described as below. We implement a Bitcoin mining

P_c	4	4.2	4.4	4.6	4.8
Connected	$(3, 38)$	$(5, 33)$	$(7, 32)$	$(10, 26)$	$(16, 20)$
Standalone	$(3, 38)$	$(4, 36)$	$(5, 34)$	$(8, 28)$	$(12, 24)$

TABLE 4: H/M model: Connected Vs Standalone

algorithm in python. We start 10 processes running this algorithm in parallel. Each miner is bound with two processes and the computing power are allocated to each process according to Table 3. To model the CSP, we set a waiting time so that the communication for the value broadcast among all processes will be delayed in 10 seconds if a qualified value is found by a certain CSP process. We run the simulation for 720 times (roughly as Bitcoin mining in 5 days) and show how many times a miner wins in Table 3. We calculate each miner's actual winning probability. For each winning probability, we apply it into Eq. (4) and Eq. (5) and get a value of β. The calculated values of β are quite close to 0.07.

Based on the data provided in the above, we can conclude it is feasible for miners using our proposed function to estimate his winning probability for computing offloading. On this basis, we further conduct experiments to confirm our theoretical analysis.

6.2 Miner Subgame Equilibrium

Our experiments evaluate how the corresponding miner subgame Nash equilibrium is influenced if the parameter values change. We start with a small mobile blockchain mining network with only 5 miners with budgets $B_i,\ \forall i \in [1, 5]$.

6.2.1 Influences from SP side

We first consider the different prices at SP side. Assuming a homogeneous-miner case in the connected mode, where $B_i = 200,\ \forall i \in [1, 5]$ holds, Fig. 5 obviously reflects that, if the CSP's price P_c unilaterally increases, miners tend to buy more units from the ESP, leading to more revenue at the ESP side. Similarly, from Fig. 5, we can also conclude that the blockchain fork rate β caused by the CSP's communication delay also has a negative effect on the number of total units sold by the CSP as well as his total revenue. However, from Fig. 6(c), we find the total revenue at the SP side remains almost unchanged no matter how prices and communication delay change.In the same miner configuration, we analyze the impact of edge operation modes. If the ESP operates in the standalone mode, we see its computing capability is positively related to miners' requests, which can be easily followed in Table 4. From this table, we can conclude that, miners are discouraged from buying units from an ESP working in the standalone mode.

6.2.2 Influences at miner side

Miners also mutually affect each other in this mining network. Fig. 7 shows the changes on all the miners' utilities when their budget of B_i varies from 20 to 200. m_i's requests to the ESP and the CSP keep increasing and its utility follows a similar trend. However, we can see that m_1's total requests to both SPs are similar even with different communication delays at the CSP side.

6.3 SP Subgame

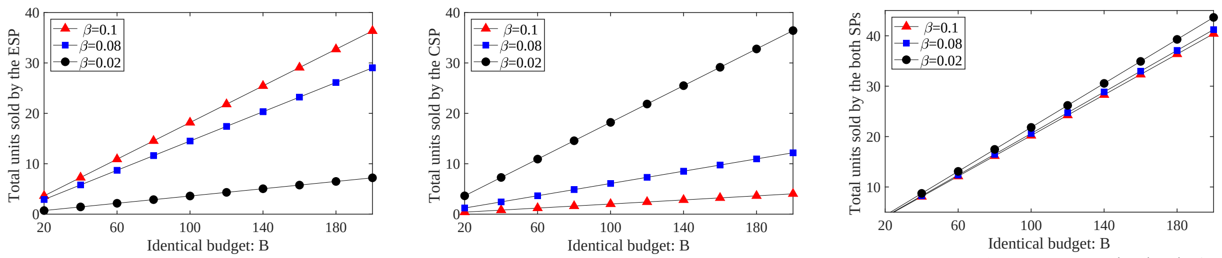
We also study how communication delay and edge operation modes as well as the SP's operating costs affect their equilibrium prices. Fig. 8 depicts the equilibrium prices of the SPs. The ESP's prices increase linearly as its unit operating cost increases. In both modes, the ESP charges a higher price, because it has less power available and its communication delay is 0 in the proposed network. However, its advantage will be shaded if the communication delay at the CSP side decreases. Besides, the ESP's computation limitation also poses an upper bound on its profits. We also discover that the standalone mode allows the ESP a higher price while it decreases the CSP's price and its profits.

According to numerous experiments, we find that the total amounts of sold-out computing units are always approximately equal, if we allow a sufficiently large budget and a fixed number of miners. Besides, we can see that the SP-side welfare is bounded by the total miner budgets in the beginning. However, as the budgets increase to a certain degree, the total welfare of these two SPs are positively related to the mining reward.

6.4 Population Uncertainty

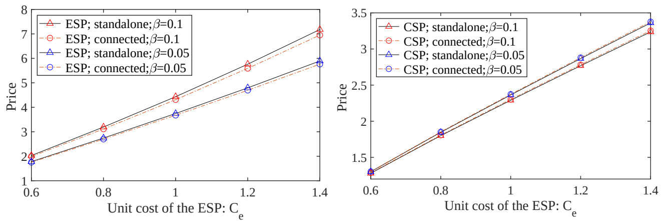
In Section 5,we consider the miner number as a variable subject to a specific Gaussian distribution. To capture the dynamics of the miner number, we use a reinforcement learning (RL) framework to allow miners to learn the population uncertainty and hence improve their strategies. We conduct our simulation within a small mining network of 5 homogeneous miners. We define a time period T as adding 50 blocks. During T, prices from these two SPs are fixed and the miner number changes subject to $N(\mu, \sigma^2)$. The reason why we choose $T = 50$ in our all experiments is that miners' strategies converge after at most 50 blocks added even in such an unstable-population mining network. Once the miners' behavior converges, both the ESP and the CSP update their pricing strategies adaptively. These two steps repeat until a fixed point for both sides is reached. We also apply such a process to a fixed number scenario where $N = \mu$.

In Fig. 9, all unfilled points are the results produced by the RL framework, while all lines are computed using our proposed model. The results of our model are anastomotic with the learned strategies. In Fig. 9(a), we conclude that the uncertainty caused by the miner number renders each miner to buy more units from the ESP, making the total requests sometimes can exceed the ESP's capability. Besides, we also find the variance also affects a miner's request to the ESP, *i.e.*, a larger variance leads to a more ESP-prone miner, according to Fig. 9(b), where $N(5,\ 0.25)$ represents a normal distribution of which the mean is 5 and the variance is 0.25.



(a) A miner's request to the ESP. (b) A miner's request to the CSP. (c) A miner's request to the both SPs.

Fig. 7: m_i's budget B_1 varies from 20 to 200, with 5 miners in total.



(a) $\beta = 0.02$. (b) $\beta = 0.06$.

Fig. 8: The CSP's unit cost is 0.5, while the ESP's unit cost changes.

7 Extended Leader Stage with Single CSP and Multiple ESPs

Our previous discussion focuses on a simplified leader setting with a single ESP. In reality, instead of being under control in a certain place, edge resources should be deployed dispersedly and pervasively in order to provide mobile users with low-latency services. In this section, we extend our base model by considering multiple ESPs independently deploying their own edge computing data centers. There are M ESPs in total and each is denoted as ESP_p. Each ESP_p has its own resource capacity E^k_{max} and unit price P^p_e. We assume that each miner m_i has a preferred ESP_p to which m_i always sends requests. When ESP_p is overloaded, it may have two choices. If there exists some ESP_q that has idle resources and is willing to help ESP_p by offering an assisting price, denoting P^{qp}_e, which is lower than P^p_e, then ESP_p can send m_i's request to ESP_q by paying ESP_q P^{qp}_e for each unit, so that m_i still enjoys zero-delay service while ESP_p also earns money with the unit profit of $P^p_e - P^{qp}_e$. Otherwise, ESP_p will transfer m_i's request to the CSP in the connected mode, or reject m_i's request in the standalone mode. (It is possible that ESP_p may turn to several ESPs for help.) In this case, edge computing resources can be fully utilized and miners have high chance to access to high-speed services.

We can consider that ESPs pool their resources together to jointly serve N miners. Thus, those M ESPs forms a coalition. To keep the coalition stable, the assisting price between any mutual-assisting pair is quite important. Instead of calculating $P^p_e - P^{qp}_e$ for each mutual-assisting pair ESP_q and ESP_p, we apply cooperative game theory to distribute total revenues among all ESPs. On the premise that the ESP set is not partitioned, the Shapley value is popularly used as a fair distribution of the grand coalition's worth to individual ESPs.

8 Related Work

8.1 Mobile Blockchain Applications

There exist two different categories of research in the field of blockchain applications in wireless networks. The first category focuses on blockchain protocols [4–9] to eliminate overhead while maintaining most of blockchain's security and privacy. These research works are beneficial for secure and decentralized data communication in wireless networks. Instead of designing and implementing light-weight blockchain-based protocols, the second category [10–16] investigates pricing and resource management schemes for supporting blockchain applications in a mobile environment. The focus here is on the mining under the PoW consensus [1], which results in the competition among miners to receive a mining reward. Due to limited computing resources of their mobile terminals, miners offload the PoW computations to local edge servers [10, 11]. In this paper, we also study the problem of pricing and computation offloading in mobile blockchain mining under the PoW consensus. However, we consider a more complicated assumption in which miners can perform a two-layer computation offloading to either/both of the ESP and the CSP.

8.2 Cloud Computing and Edge Computing

Cloud computing is becoming the platform of choice for a number of applications due to the advantages of high computing power, low service cost, high scalability, accessibility, and availability. Meanwhile, the success of the Internet of Things and rich cloud services have helped create the need for edge computing, in which data processing occurs in part at the network edge, rather than completely in the cloud. Edge computing could address concerns such as latency, mobile devices' limited battery life, bandwidth costs, security, and privacy. Computation offloading happens in both CC [17, 18] and EC [19, 20], which concerns what/when/how to offload users' workload from their devices to the edge servers or the cloud. One common use case on the EC exploitation is for IoT purposes [21–23].

8.3 Stackelberg Game in Offloading Mechanism

Stackelberg Game is a widely-used model in the field of offloading mechanisms. A large body of existing literature [24–31] focuses on minimizing offloading users' computation overhead in terms of energy and latency.

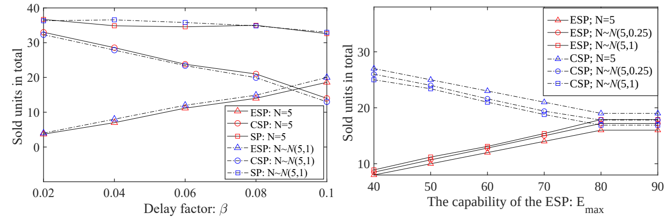
To this end, researchers have developed distributed decision making methodologies. In the field of mobile blockchain mining offloading [10, 11, 32], there are few works and most of them are in the single-leader scenario where mobile miners only offload their computation to an SP, *e.g.* fog. In our paper, we consider a multi-leader multi-follower Stackelberg game to jointly maximize the profit of the SPs and the individual utilities of mobile miners. We assume a resource-limited edge layer working in either stand-alone or connected operation mode with the cloud layer. In this paper, we study the miner subgame as an N-player Nash game. In reality, the number of miners is large, and modeling interactions between SPs and individual miners is difficult. Meanwhile, the miner set is also not fixed in the real-world, indicating that the value of N as well as the mining power in the entire network changes over time. To efficiently find the optimal prices for SPs, we can apply the mean field game theory and reduce a large number of miners to a single mean-field miner. We consider this extension as one of our future works.

8.4 Reinforcement Learning in Incomplete Information Game

Although analysis in game theory always assumes the observable strategies of other players [33,34], in reality, it is more realistic that a player's action is the private information which is unobservable or partially observable by others. Meanwhile, each player's utility function combined with constraints also cannot be fully observed by others. Given this incomplete information setting, analyzing and finding the equilibrium in a game becomes more difficult. Reinforcement learning [35-40] is a technique that allows a player to learn behavior through trial-and-error interactions with other players. During the learning process, a player builds his own belief on the actions of other players' and refines his strategies simultaneously. In our proposed game, leaders (the CSP and the ESP) can estimate the total budgets of all miners, and followers (miners) can probe other miners' strategies as well as the ESP's capacity through the learning properties of their interactions. In addition to applying game-theoretical analysis on the proposed game, we also develop a reinforcement learning framework in our evaluation, allowing all players to select their best response strategies and update their beliefs about unobservable actions of others through repeated interactions with each other in a stochastic environment. This framework confirms our proposed model.

9 Conclusion

In this paper, we have proposed a Stackelberg game between the SPs for optimal price strategies and among the mobile miners for optimal computation offloading requests. We propose two models: a hit/miss model and a capacity(E_{max}) model to characterize the resource limitation on the ESP side. Two practical edge computing operation modes are investigated, *i.e.*, the ESP is connected to the CSP or standalone. First, we characterize the miner number as a constant in both modes. We discuss the existence and the uniqueness of Stackelberg equilibrium in the proposed games and provide algorithms to achieve SE point(s). Our analysis indicates that the connected mode discourages miners from buying computing resources from the ESP. Then, we study the impact of a dynamic miner number. Interestingly, we find that uncertainty incurred by the dynamic population renders miners more aggressive to buy computing resources from the ESP. Numerical experiments based on a reinforcement learning framework have been conducted to further confirm our analysis.



(a) $\beta = 0.02$. (b) $\beta = 0.06$.

Fig. 9: Miner number: fixed vs dynamic.

References

[1] S. Nakamoto, "Bitcoin: A peer-to-peer electronic cash system," 2008.

[2] C. Decker and R. Wattenhofer, "Information propagation in the bitcoin network," in *2013 IEEE Thirteenth International Conference on Peer-to-Peer Computing*.

[3] J. Göbel, H. P. Keeler, A. E. Krzesinski, and P. G. Taylor, "Bitcoin blockchain dynamics: The selfish-mine strategy in the presence of propagation delay," *Performance Evaluation*, vol. 104, pp. 23–41, 2016.

[4] A. Dorri, S. S. Kanhere, R. Jurdak, and P. Gauravaram, "Blockchain for iot security and privacy: The case study of a smart home," in *2017 IEEE international conference on pervasive computing and communications workshops*, 2017, pp. 618–623.

[5] M. A. Khan and K. Salah, "Iot security: Review, blockchain solutions, and open challenges," *Future Generation Computer Systems*, vol. 82, pp. 395–411, 2018.

[6] D. B. Rawat and A. Alshaikhi, "Leveraging distributed blockchain-based scheme for wireless network virtualization with security and qos constraints," in *2018 International Conference on Computing, Networking and Communications*, 2018, pp. 332–336.

[7] Y. Zhang, S. Kasahara, Y. Shen, X. Jiang, and J. Wan, "Smart contract-based access control for the internet of things," *IEEE Internet of Things Journal*, vol. 6, no. 2, pp. 1594–1605, 2018.

[8] H. Yang, Y. Liang, J. Yuan, Q. Yao, A. Yu, and J. Zhang, "Distributed blockchain-based trusted multi-domain collaboration for mobile edge computing in 5g and beyond," *IEEE Transactions on Industrial Informatics*, pp. 1–1, 2020.

[9] H. Yang, J. Yuan, H. Yao, Q. Yao, A. Yu, and J. Zhang, "Blockchain-based hierarchical trust networking for jointcloud," *IEEE Internet of Things Journal*, vol. 7, no. 3, pp. 1667–1677, 2020.

[10] M. Liu, F. R. Yu, Y. Teng, V. C. Leung, and M. Song, "Joint computation offloading and content caching for wireless blockchain networks," in *IEEE INFOCOM 2018-IEEE Conference on Computer Communications Workshops*, 2018, pp. 517–522.

[11] Z. Xiong, S. Feng, D. Niyato, P. Wang, and Z. Han, "Optimal pricing-based edge computing resource management in mobile blockchain," in *2018 IEEE International Conference on Communications*, 2018, pp. 1–6.

[12] Z. Xiong, S. Feng, W. Wang, D. Niyato, P. Wang, and Z. Han, "Cloud/fog computing resource management and pricing for blockchain networks," *IEEE Internet of Things Journal*, vol. 6, no. 3, pp. 4585–4600, 2019.

[13] Z. Xiong, J. Kang, D. Niyato, P. Wang, and H. V. Poor, "Cloud/edge computing service management in blockchain networks: Multi-leader multi-follower game-based admm for pricing," *IEEE Transactions on Services Computing*, vol. 13, no. 2, pp. 356–367, 2020.

[14] N. C. Luong, Z. Xiong, P. Wang, and D. Niyato, "Optimal auction for edge computing resource management in mobile blockchain networks: A deep learning approach," in *2018 IEEE International Conference on Communications*, 2018, pp. 1–6.

[15] Y. Jiao, P. Wang, D. Niyato, and Z. Xiong, "Social welfare maximization auction in edge computing resource allocation for mobile blockchain," in *2018 IEEE international conference on communications*, 2018, pp. 1–6.

[16] M. Yoo and Y. Won, "A study on the transparent price tracing system in supply chain management based on blockchain," *Sustainability*, vol. 10, no. 11, p. 4037, 2018.

[17] N. Fernando, S. W. Loke, and W. Rahayu, "Mobile cloud computing: A survey," *Future generation computer systems*, vol. 29, no. 1, pp. 84–106, 2013.

[18] R. Buyya, C. S. Yeo, S. Venugopal, J. Broberg, and I. Brandic, "Cloud computing and emerging it platforms: Vision, hype, and reality for delivering computing as the 5th utility," *Future Generation computer systems*, vol. 25, no. 6, pp. 599–616, 2009.

[19] Y. Mao, C. You, J. Zhang, K. Huang, and K. B. Letaief, "Mobile edge computing: Survey and research outlook," *arXiv preprint arXiv:1701.01090*, 2017.

[20] W. Shi, J. Cao, Q. Zhang, Y. Li, and L. Xu, "Edge computing: Vision and challenges," *IEEE Internet of Things Journal*, vol. 3, no. 5, pp. 637–646, 2016.

[21] O. Salman, I. Elhajj, A. Kayssi, and A. Chehab, "Edge computing enabling the internet of things," in *2015 IEEE 2nd World Forum on Internet of Things (WF-IoT)*, 2015, pp. 603–608.

[22] S. Abdelwahab, B. Hamdaoui, M. Guizani, and T. Znati, "Replisom: Disciplined tiny memory replication for massive iot devices in lte edge cloud," *IEEE Internet of Things Journal*, vol. 3, no. 3, pp. 327–338, 2015.

[23] X. Sun and N. Ansari, "Edgeiot: Mobile edge computing for the internet of things," *IEEE Communications Magazine*, vol. 54, no. 12, pp. 22–29, 2016.

[24] L. Xiao, C. Xie, T. Chen, H. Dai, and H. V. Poor, "A mobile offloading game against smart attacks," *IEEE Access*, vol. 4, pp. 2281–2291, 2016.

[25] Y. Liu, C. Xu, Y. Zhan, Z. Liu, J. Guan, and H. Zhang, "Incentive mechanism for computation offloading using edge computing: a stackelberg game approach," *Computer Networks*, vol. 129, pp. 399–409, 2017.

[26] X. Wang, X. Chen, W. Wu, N. An, and L. Wang, "Cooperative application execution in mobile cloud computing: A stackelberg game approach," *IEEE Communications Letters*, vol. 20, no. 5, pp. 946–949, 2016.

[27] L. Song, D. Niyato, Z. Han, and E. Hossain, "Game-theoretic resource allocation methods for device-to-device communication," *IEEE Wireless Communications*, vol. 21, no. 3, pp. 136–144, 2014.

[28] H. Zhang, Y. Xiao, S. Bu, D. Niyato, R. Yu, and Z. Han, "Fog computing in multi-tier data center networks: a hierarchical game approach," in *2016 IEEE international conference on communications*, 2016, pp. 1–6.

[29] T. M. Ho, N. H. Tran, C. T. Do, S. A. Kazmi, T. LeAnh, and C. S. Hong, "Data offloading in heterogeneous cellular networks: Stackelberg game based approach," in *2015 Asia-Pacific Network Operations and Management Symposium*, 2015, pp. 168–173.

[30] Y. Sun, H. Shao, X. Liu, J. Zhang, J. Qiu, and Y. Xu, "Traffic offloading in two-tier multi-mode small cell networks over unlicensed bands: A hierarchical learning framework." *TIIS*, vol. 9, no. 11, pp. 4291–4310, 2015.

[31] X. Zhang, L. Guo, M. Li, and Y. Fang, "Social-enabled data offloading via mobile participation-a game-theoretical approach," in *2016 IEEE Global Communications Conference*, 2016, pp. 1–6.

[32] Z. Xiong, Y. Zhang, D. Niyato, P. Wang, and Z. Han, "When mobile blockchain meets edge computing," *IEEE Communications Magazine*, vol. 56, no. 8, pp. 33–39, 2018.