Energy Efficiency and Contact Opportunities Trade-off in Opportunistic Mobile Networks

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Abstract

In order to discover neighbor nodes, nodes in Opportunistic Mobile Networks (OppNets) have to probe their environment continuously. This can be an extremely energy-consuming process. If nodes probe very frequently, a lot of energy will be consumed in the contact probing process, and might be inefficient. On the other hand, infrequent contact probing might cause nodes to miss many of their contacts. Therefore, there exists a trade-off between energy efficiency and contact opportunities in OppNets. In order to investigate this trade-off, we first propose a model to investigate the contact probing process based on the Random WayPoint (RWP) model, and obtain the expressions of the single detecting probability and the double detecting probability, respectively. Moreover, we also demonstrate that among all contact probing strategies with the same average probing interval, the strategy which probes at a constant interval is optimal. Then, extensive simulations are conducted to validate the correctness of our proposed model. Finally, based on the proposed model, we analyze the trade-off between energy efficiency and the total number of effective contacts in the single and double contact probing processes. Our results show that the total number of effective contacts in the single and double contact probing processes have a lower-bound and an upper-bound, and the good trade-off points are obviously different when the speed of nodes is different.

Index Terms

Opportunistic Mobile Networks, Energy efficiency, Contact probing, Random WayPoint model.

I. Introduction

Recently, with the rapid proliferation of wireless portable devices (e.g., ipad, PDAs, smartphones), a new peer-to-peer (P2P) application scenario – Opportunistic Mobile Networks (OppNets) – has begun to emerge [1], [2], [3], [4]. In OppNets, it is hard to guarantee an end-to-end path due to the time-varying network topology, and thus nodes with data to be transmitted have to exchange data with relay nodes within their communication range. This data exchange process is referred to as the store-carry-forward mechanism, which works as a basic strategy of data transmission in OppNets [5], [6].

In order to enable such data exchange, nodes in the network have to continuously probe the environment to discover other nodes in the vicinity. Not surprisingly, this contact probing is an extremely energy-consuming process [7], [8], [9], [10]. Authors in [8] made measurements on a Nokia 6600 mobile phone to test the energy consumption in the contact probing process, and their results show that the contact

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probing process is as energy-intensive as making a phone call! Moreover, in OppNets, the inter-contact time is generally much larger than the contact duration, due to node sparsity in OppNets [11]; this indicates that nodes in the network will waste a lot of energy in the contact probing process if they probe the environment too frequently. Therefore, it is pressing to investigate saving energy during the contact probing process in OppNets.

One strategy for saving energy is to increase the time between subsequent contact probing. The consequence of this is that nodes in the network may miss many chances to contact others in the contact probing process, and thus opportunities to exchange data are lost. Moreover, if nodes probe the environment too frequently, a lot of energy will be consumed in the contact probing process, and might be inefficient. This points to a trade-off between energy efficiency and contact opportunities in the contact probing process[1]. For strategies which use a constant contact probing interval, the larger the contact probing interval, the greater the number of missed contact opportunities, and vice-versa. In order to investigate the trade-off between energy efficiency and the contact opportunities in OppNets, we first propose a model to investigate the contact probing process based on the RWP model, and obtain the optimal contact probing strategy among all contact probing strategies with the same average contact probing interval. Then, based on the proposed model, we analyze the trade-off between energy efficiency and the total number of effective contacts under different scenarios. Specifically, our contributions in this paper are three-folds:

1) We propose a model to investigate the contact probing process in OppNets, based on the RWP model. Given the distribution of the contact duration in the RWP model, we analytically obtain the expression of the single detecting probability and the double detecting probability, respectively. Moreover, we also obtain the optimal contact probing strategy among all contact probing strategies with the same average contact probing interval.
2) We conduct several simulations to validate the correctness of our proposed model, and our results show that the simulation results are quite close to the theoretical results under different scenarios, which validate the correctness of our proposed model. Furthermore, our results also show that our proposed model can be applied to a more general scenario.
3) Based on the proposed model, we obtain the number of effective contacts detected by a certain node over a certain period, denoted as the total number of effective contacts, and then the trade-off

[1]Since only the effective contacts can be used for data exchange in OppNets, we define contact opportunities as the total number of effective contacts.

between energy efficiency and the total number of effective contacts in OppNets is analyzed under different scenarios.

The remainder of this paper is organized as follows. We present the related work in Section II, and introduce the network model in Section III. Section IV proposes a model to investigate the contact probing process based on the RWP model, and derives the expression of the single detecting probability and the double detecting probability, respectively. Furthermore, Section IV also obtains the optimal contact probing strategy among all contact probing strategies with the same average contact probing interval. Extensive simulations are conducted to validate the correctness of the proposed model in Section V. Then, based on the proposed model, trade-offs between energy efficiency and the total number of effective contacts under different scenarios are analyzed in Section VI. At last, we conclude the paper in Section VIII.

II. RELATED WORK

The stochastic event capturing process in wireless mobile sensor networks is similar to the contact probing process in OppNets. Since sensors with limited energy consume a lot of energy in the stochastic event capturing process, some recent studies have designed energy-efficient schemes for stochastic event capturing in wireless sensor networks [12], [13]. The trade-off between energy efficiency and the quality of monitoring (QoM), in the wireless mobile sensor networks was investigated in [12]. The authors propose a utility function: expected information captured per unit of energy consumption (IPE), to evaluate the overall event capturing performance of a mobile sensor, and systematically analyze the optimal event capturing scheduling under different scenarios. In [13], energy-aware optimization of the periodic schedule for static sensors to capture events was investigated, and four design points: (i) synchronous periodic coverage without coordinated sleep, (ii) synchronous periodic coverage with coordinated sleep, (iii) asynchronous periodic coverage without coordinated sleep, and (iv) asynchronous periodic coverage with coordinated sleep, were all considered. In our study, we focus on investigating the contact probing process in OppNets, which is similar to the stochastic event capturing process, but is more complicated than the memoryless event arrival and departure process of the stochastic event capturing in wireless mobile sensor networks.

Note that nodes consume a lot of energy in the contact probing process, and a high probing frequency means a large amount of energy consumption. Therefore, some studies have investigated the contact probing process to save energy in OppNets [8], [14], [15], [16]. In [8], [14], the impact of contact probing on the probability of missing a contact and the trade-off between the missing probability and energy consumption in bluetooth devices were investigated. Furthermore, though characterizing real world contact patterns in real mobility trace, an adaptive contact probing mechanism, STAR, was proposed. Via real trace-driven simulations, authors show that their proposed mechanism, STAR, consumed three times less energy when compared to a constant contact probing interval scheme. In [15], two novel adaptive schemes for dynamically selecting the parameters of the contact probing process were introduced and evaluated. The proposed schemes enable nodes to adaptively switch between low-power, slow discovery modes and high-power, fast discovery modes, depending on the mobility context. In [16], the impact of contact probing on link duration and the trade-off between the energy consumption and throughput were investigated. In addition, this paper also provides a framework for computing the optimal contact probing frequency under energy limitations.

Different from all the existing studies above, our paper focuses on investigating the contact probing process in OppNets, based on the RWP model, and proposes a model to analyze the trade-off between energy efficiency and the contact opportunities under different scenarios.

III. Network Model

This section introduces the network model related to the contact probing process in OppNets. There have been many mobility models available for evaluating the contact probing process in OppNets, including the Random WayPoint (RWP) model [17], [18], random walk [19], and realistic mobility trace [20]. In this paper, we focus on investigating the contact probing process in OppNets based on the RWP model. In the RWP Model, we consider a two-dimensional system space $\mathcal{S}$ of size S as a square area of width s. With this mobility model, each node selects a target location to move at a speed V selected from a uniformly distributed interval $[V_{min}, V_{max}]$. Once the target location is reached, the node pauses for a random time and then selects another target location with another speed to move again. This process repeats in this manner. For simplicity, we assume that there are N nodes in the network, which move at the same speed V, and with the same pausing time equal to 0.

In OppNets, nodes are in contact with each other only if they are within communication range of each other, and the time when nodes are in contact with each other continuously is called the contact duration, while the time between subsequent contacts is defined as the inter-contact time. We assume that the contact duration T_d is i.i.d. (independent and identically distributed) stationary random variables with CDF (Cumulative Distribution Function) of $F_{T_d}(t)$. Fig. 1 gives an example of the contact duration T_d

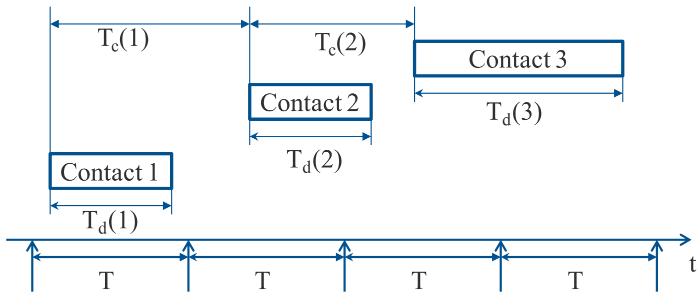


Fig. 1. Illustrating the contacts between two nodes at a constant probing interval T. The upper arrow denotes the probing action of the node.

and the inter-contact time T_c between two nodes at a constant probing interval T. We further assume that each probe consumes equal energy, so that the energy consumption rate of the node can be converted to the average contact probing frequency.

In order to enable data exchanges, nodes in the network have to continuously probe the environment to discover others in the vicinity. We assume that there are N nodes (e.g., portable devices with bluetooth) in the network, and they have the same communication range of r. Since the normal communication range of portable devices with bluetooth is less than $10m$ [21], we assume that $r \leq 10m$. We define two nodes, to be in contact, if they are within communication range of each other. However, if neither node probes its vicinity during their contact with each other, then we have a missed contact. Therefore, we divide the contact in the contact probing process into two kinds: the effective contact and the missed contact. An effective contact happens when either node probes its environment while in contact with another. Since this kind of contact between two nodes can be discovered by one of the two nodes, or both of them, we regard this kind of contact as the effective contact, which can be used for different applications in OppNets. The missed contact happens when neither of the two nodes probes its environment during their contact with each other. Since this kind of contact between two nodes cannot be discovered by the other, we refer to this kind of contact as the missed contact. Note that the contact in OppNets is infrequent, and the contact probing process has a significant effect on the performance of different applications in OppNets. Therefore, in the next section we will propose a model to investigate the contact probing process in OppNets.

IV. Modeling the Contact Probing Process

In OppNets, unlike traditional connected networks (e.g., P2P networks and Internet-accessible networks), nodes are intermittently connected [22], [23]. Nodes in the network can communicate with each other only when they move into the transmission range of each other. Due to frequent link disconnections and dynamic topology in OppNets, contact schedules among nodes are not known in advance. Therefore, nodes in the network have to probe the environment continuously, so as to find the contact which can be used for different applications in OppNets. In this section, we will propose a model to investigate the contact probing process in OppNets based on the RWP model.

A. The Single Detecting Probability

In this part, we investigate the contact probing process in which a contact between two nodes is detected by a certain node only if it is detected by its own probes, i.e. the single contact probing process. Let us define P_{sd} (single detecting probability) as the probability that a contact between two nodes can be detected by a certain node in OppNets. For the following analysis, we assume that for node A, a contact with node B is detected (an effective contact), only if the contact with B is detected by A's probes, or this contact is a missed contact. As shown in Fig. 1, we suppose that node A probes at a constant probing interval T, then for node A, Contact 2 and Contact 3 are effective contacts, while Contact 1 is a missed contact. We will relax this later, to compute the double detecting probability when either A or B's probes detect the contact with each other. Let us consider the contact probing strategy, where each node probes for contacts at a constant probing interval of T (See Fig. 1), and we will discuss all contact probing strategies with the same average contact probing interval later.

There will be a set of different possibilities for calculating the single detecting probability, P_{sd}, depending on the lengths of the probing interval T and the contact duration T_d. Note that if $T_d \geq T$, the contact will always be detected. Therefore, we have the following theorem:

***Theorem** 1:* For a certain node A, with a constant probing interval of T, the single detecting probability can be expressed as:

$$
\begin{aligned}
P_{sd}(T) &= \frac{1}{T}\int_0^T Pr\{T_d + t \geq T\}dt \\
&= 1 - \frac{1}{T}\int_0^T F_{T_d}(t)dt.
\end{aligned} \tag{1}
$$

Proof: Assume that node A probes its vicinity at time $\{T, 2T, ...\}$; here we consider the interval $[0, T]$ to calculate the single detecting probability. Let t be a random variable indicating the time when a contact with A would begin in the interval $[0, T]$. As shown in Fig. 1, t can be expressed as the beginning

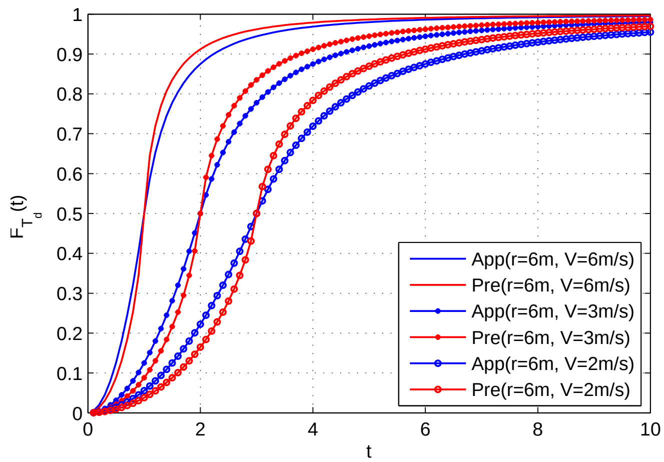


Fig. 2. Comparisons between the approximate value and the precise value of $F_{T_d}(t)$ under different scenarios.

of T_d. Since nodes are randomly moving, we can obtain that t is uniformly distributed over the interval $[0, T]$. Note that a contact will be detected by node A if (a) it happens when A probes its vicinity at time T; (b) it happens during period $[0, T)$, but the contact duration T_d is long enough to be detected by the contact probing time T. Therefore, the single detecting probability $P_{sd}(T)$ is the sum of these two parts, and can be expressed as Eq. (1). ■

It is worth noting that if the contact duration T_d is distributed according to a given distribution, we can analytically obtain the relationship between energy consumption and the single detecting probability $P_{sd}(T)$. As shown in [24], [25], the contact duration T_d in the RWP model is i.i.d. and stationary random variables with CDF of $F_{T_d}(t)$, which can be expressed as:

$$F_{T_d}(t) \ = \ \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\frac{r + Vt}{\sqrt{|r^2 - V^2t^2|}}), \tag{2}$$

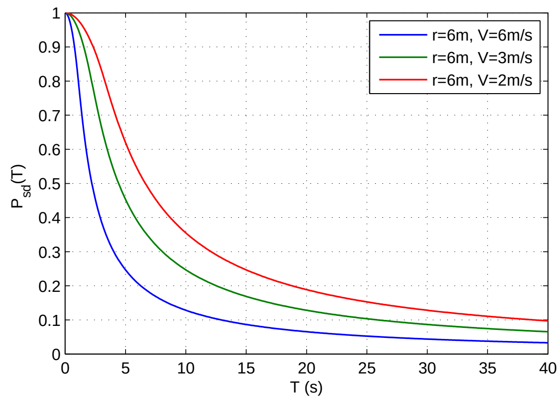
where r is the transmission range of nodes, and V is the moving speed of nodes.

Note that the above equation is hard to integrate. Therefore, in order to facilitate the modeling, we simplify the above expression of $F_{T_d}(t)$ as follows:

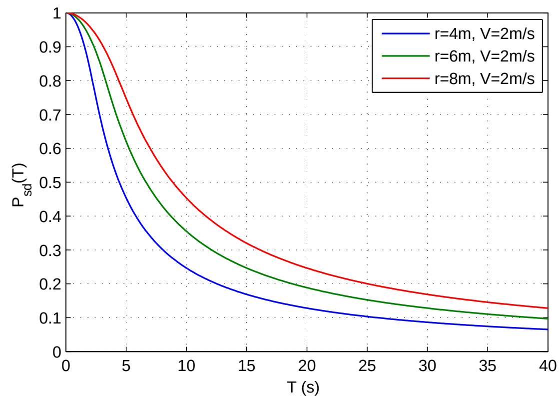
$$F_{T_d}(t) \ = \ \begin{cases} \dfrac{V^2t^2}{2r^2}, & t \leq \dfrac{r}{V}, \\ 1 - \dfrac{r^2}{2V^2t^2}, & t > \dfrac{r}{V}. \end{cases} \tag{3}$$

The appendix describes how to obtain the above expression.

Fig. 2 shows the comparison between the approximate value of $F_{T_d}(t)$ and the precise value of $F_{T_d}(t)$ under different scenarios. It can be found that, as the contact duration T_d increases, the approximate value of $F_{T_d}(t)$ and the precise value of $F_{T_d}(t)$ are very close to each other, especially when $r = 6m$,



(a) When the speed V changes



(b) When the communication range r changes

Fig. 3. The single detecting probability $P_{sd}(T)$ under different scenarios.

$V = 6m/s$. Therefore, in the following, we will simply use the approximate value of $F_{T_d}(t)$ instead of the precise value of $F_{T_d}(t)$ to calculate the detect probability $P_{sd}(T)$ directly.

Substituting Eq. (3) into Eq. (1), we obtain the expression of the single detecting probability $P_{sd}(T)$ as follows:

$$P_{sd}(T) \;=\; \begin{cases} 1 - \dfrac{T^2V^2}{6r^2}, & T \le \dfrac{r}{V}, \\ \dfrac{4r}{3TV} - \dfrac{r^2}{2T^2V^2}, & T > \dfrac{r}{V}, \end{cases} \tag{4}$$

Fig. 3 shows the relationship between the single detecting probability $P_{sd}(T)$ and the contact probing interval T under different scenarios. Fig. 3(a) shows the relationship between the single detecting probability $P_{sd}(T)$ and the contact probing interval T when the speed V changes; meanwhile, Fig. 3(b) shows the relationship between the single detecting probability $P_{sd}(T)$ and the contact probing interval T when the communication range r changes. It can be found that the single detecting probability $P_{sd}(T)$ increases as the contact probing interval T decreases under different scenarios. This is reasonable because if T is smaller, nodes in the network will probe their environments more frequently, resulting in the increase of the single detecting probability $P_{sd}(T)$. It is worth noticing that the upper-bound of $P_{sd}(T)$ is 1 when $T = 0$, and the lower-bound of $P_{sd}(T)$ is 0 when T is close to ∞. It can also be found that the single detecting probability $P_{sd}(T)$ decreases as the speed V increases, and increases as the communication range r increases. The main reason is that the contact duration T_d increases as the communication range r increases or the speed V decreases, while larger contact duration results in the increase of the single detecting probability $P_{sd}(T)$.

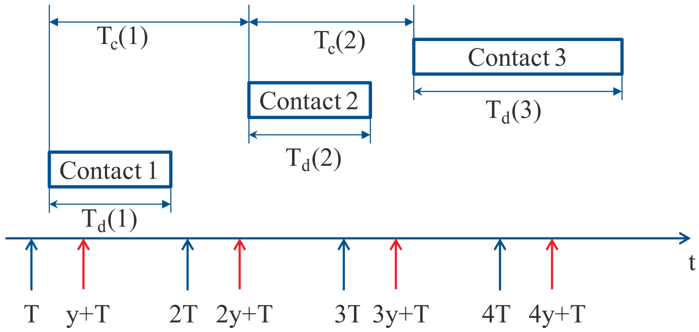


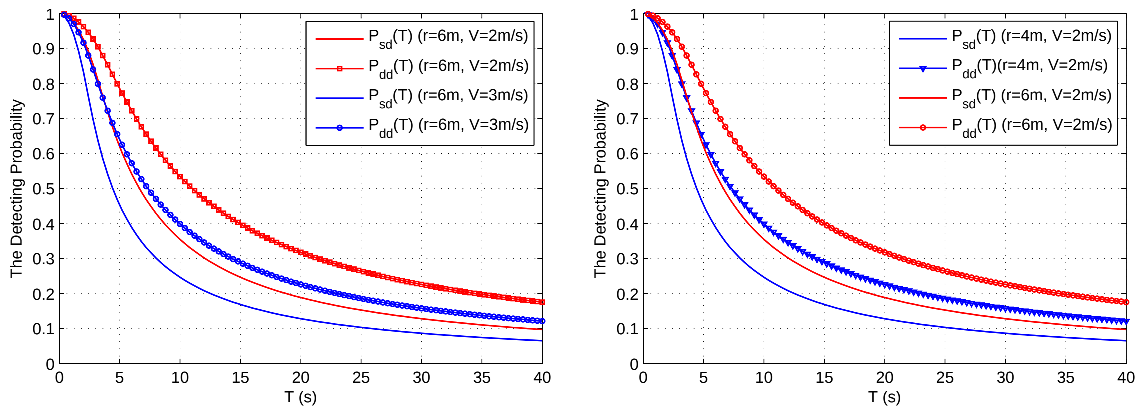
Fig. 4. Illustrating the double contact probing process between two nodes at a constant probing interval T. The upper arrow denotes the probing action of the two nodes.

B. The Double Detecting Probability

In the above, we have given the expression of the single detecting probability, which represents the probability that a contact between two nodes A and B is detected by node A's probes. In this part, we investigate the double contact probing process, which means a contact between nodes A and B is detected (an effective contact) if either node probes the environment during their contact with each other. For example, as shown in Fig. 4, each node probes for contacts at a constant probing interval of T; we suppose that node A probes at times of T, $2T$, . . ., nT, and node B probes at times of y, $y+T$, . . ., $y+(n-1)T$. It can be found that Contact 2 and Contact 3 are detected by node A's probes, while Contact 1 is missed by node A's probes, but Contact 1 is detected by node B's probes. Therefore, Contact 1 is still an effective contact. Consider the case when nodes A and B are independently and periodically probing the environment with a constant probing interval T. Then, the probability that during a contact with each other, either node discovers the other, is given by:

$$\begin{aligned} P_{dd}(T,y) &= \frac{1}{T}[\int_0^y Pr\{T_d+t \geq y\}dt + \int_y^T Pr\{T_d+t \geq T\}dt], \\ &= \frac{1}{T}[T - \int_0^y F_{T_d}(t)dt - \int_0^{T-y} F_{T_d}(t)dt]. \end{aligned} \tag{5}$$

Since the two nodes are probing independently, y is uniformly distributed in $[0,T]$. Then, we obtain



(a) When the speed V changes (b) When the communication range r changes

Fig. 5. Comparison between $P_{sd}(T)$ and $P_{dd}(T)$ under different scenarios.

the double detecting probability $P_{dd}(T)$ as:

$$
\begin{aligned}
P_{dd}(T) &= \frac{1}{T^2}\int_0^T [\int_0^y Pr\{T_d + t \geq y\}dt + \int_y^T Pr\{T_d + t \geq T\}dt]dy, \\
&= \frac{1}{T^2}\int_0^T [T - \int_0^y F_{T_d}(t)dt - \int_0^{T-y} F_{T_d}(t)dt]dy, \\
&= \frac{1}{T^2}\int_0^T [T - 2\int_0^y F_{T_d}(t)dt]dy, \\
&= 1 - \frac{2}{T^2}\int_0^T [\int_0^y F_{T_d}(t)dt]dy.
\end{aligned}
\tag{6}
$$

Substituting Eq. (3) into Eq. (6), we obtain the expression of the double detecting probability $P_{dd}(T)$ as:

$$
\begin{aligned}
P_{dd}(T) &= 1 - \frac{2}{T^2}\int_0^T [\int_0^y F_{T_d}(t)dt]dy, \\
&= \begin{cases} 1 - \frac{2}{T^2}[\int_0^T \frac{V^2y^3}{6r^2}dy], & T \leq \frac{r}{V}, \\ 1 - \frac{2}{T^2}[\int_0^{\frac{r}{v}} \frac{V^2y^3}{6r^2}dy + \int_{\frac{r}{v}}^T y + \frac{r^2}{2V^2y} - \frac{4r}{3V}dy], & T > \frac{r}{V}. \end{cases} \\
&= \begin{cases} 1 - \frac{V^2T^2}{12r^2}, & T \leq \frac{r}{V}, \\ \frac{8r}{3VT} - (7 + 4ln\frac{TV}{r})\frac{r^2}{4V^2T^2}, & T > \frac{r}{V}. \end{cases}
\end{aligned}
\tag{7}
$$

Fig. 5 shows the comparison between the single detecting probability $P_{sd}(T)$ and double detecting probability $P_{dd}(T)$ under different scenarios. Fig. 5(a) shows the comparison between $P_{sd}(T)$ and $P_{dd}(T)$ when the speed V changes, and Fig. 5(b) shows the comparison between $P_{sd}(T)$ and $P_{dd}(T)$ when the communication range r changes. It can be found that, similar to the results in Fig. 3, the double detecting probability $P_{dd}(T)$ also decreases as the contact probing interval T or the speed V increases, and increases as the communication range r increases. It can also be found that the double detecting probability $P_{dd}(T)$ is much larger than the single detecting probability $P_{sd}(T)$, not only when the speed V changes, but also when the communication range r changes. This is reasonable because in the double contact probing process, if either node probes the environment while in contact with another, then this contact can be discovered, or an effective contact. However, in the single contact probing process, if one node misses a contact with another node, then this contact will be missed. Therefore, the double detecting probability $P_{dd}(T)$ is much larger than the single detecting probability $P_{sd}(T)$ under different scenarios.

C. Performance Analysis of the Constant Contact Probing Strategy

In the above parts, we have given the expression of the single detecting probability, and the double detecting probability when the contact probing interval is a constant using the RWP model. In this part, we will analyze the performance of the constant probing strategy in the single contact probing process.

***Theorem** 2:* Consider an environment with N nodes in the network. Note that the distribution of contact duration in the RWP model is i.i.d, and node pairs in the RWP model have identical inter-contact time distributions, with an expected inter-contact-time of $1/\lambda$ [24], [26], [27]. Then, among all contact probing strategies with the same average contact probing interval in the single contact probing process, which do not have pre-knowledge of the contact process, the contact probing strategy which probes at a constant interval performs better than any arbitrary probing strategy in expectation.

Proof: Without loss of generality, we consider that nodes in the networks probe the environment in a large interval of L, and nodes in all strategies probe the environment n times in this interval L. As shown previously, for the strategy which probes at a constant contact probing interval $T = L/n$, the single detecting probability over interval L is $P_{sd}(T) = 1 - \frac{1}{T}\int\limits_0^T F_{T_d}(t)dt$. Assume that an arbitrary strategy probes n times at $t_1, t_2, ..., t_n$, where $t_1 < t_2 < ... < t_n$, and $t_n - t_1 \leq L$. Denote t_0 = 0, then we have n contact probing intervals of $C_1 = t_1 - t_0$, $C_2 = t_2 - t_1$, ..., $C_n = t_n - t_{n-1}$. Since nodes select the contact probing time t_k randomly, and node pairs in the network have identical inter-contact time distributions, with an expected inter-contact-time of $1/\lambda$, the number of expected effective contacts detected by a certain node in the k-th interval $C_k = t_k - t_{k-1}$ is $\lambda(N-1)C_k(1-\frac{1}{C_k}\int\limits_0^{C_k} F_{T_d}(t)dt) = \lambda(N-1)(C_k - \int\limits_0^{C_k} F_{T_d}(t)dt)$. Here, N is the total number of nodes in the the network. Then, the expected single detecting probability over the interval L can be expressed as:

$$\begin{aligned} \bar{P_{sd}} &= \frac{1}{\lambda(N-1)L}[\sum_{k=1}^{n}\lambda(N-1)(C_k - \int\limits_{0}^{C_k} F_{T_d}(t)dt)], \\ &= \frac{1}{L}[\sum_{k=1}^{n}(C_k - \int\limits_{0}^{C_k} F_{T_d}(t)dt)]. \end{aligned} \tag{8}$$

For $C_k \geq T$, we have:

$$\begin{aligned} -\int\limits_{0}^{C_k} F_{T_d}(t)dt &= -[\int\limits_{0}^{T} F_{T_d}(t)dt + \int\limits_{T}^{C_k} F_{T_d}(t)dt], \\ &\leq -\int\limits_{0}^{T} F_{T_d}(t)dt - \int\limits_{T}^{C_k} F_{T_d}(T)dt, \\ &= -\int\limits_{0}^{T} F_{T_d}(t)dt - (C_k - T)F_{T_d}(T). \end{aligned} \tag{9}$$

For $C_k < T$, we have:

$$\begin{aligned} -\int\limits_{0}^{C_k} F_{T_d}(t)dt &= -[\int\limits_{0}^{T} F_{T_d}(t)dt - \int\limits_{C_k}^{T} F_{T_d}(t)dt], \\ &\leq -\int\limits_{0}^{T} F_{T_d}(t)dt + \int\limits_{C_k}^{T} F_{T_d}(T)dt], \\ &= -\int\limits_{0}^{T} F_{T_d}(t)dt + (T - C_k)F_{T_d}(T). \end{aligned} \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (11), then we have:

$$
\begin{aligned}
\bar{P_{sd}} &= \frac{1}{L}\sum_{k=1}^{n}[C_k - \int_0^{C_k} F_{T_d}(t)dt], \\
&\leq \frac{1}{L}\sum_{k=1}^{n}[C_k - \int_0^{T} F_{T_d}(t)dt + (T - C_k)F_{T_d}(T)], \\
&= \frac{1}{L}[\sum_{k=1}^{n} C_k - n\int_0^{T} F_{T_d}(t)dt + (nT - \sum_{k=1}^{n} C_k)F_{T_d}(T)], \\
&\leq \frac{1}{L}[\sum_{k=1}^{n} C_k - n\int_0^{T} F_{T_d}(t)dt + nT - \sum_{k=1}^{n} C_k], \\
&= \frac{1}{nT}[nT - n\int_0^{T} F_{T_d}(t)dt], \\
&= P_{sd}(T)
\end{aligned}
\tag{11}
$$

■

Therefore, according to Theorem 2, we obtain that among all contact probing strategies with the same average contact probing interval, which do not have pre-knowledge of the contact process, the contact probing strategy which probes at a constant interval performs better than any arbitrary probing strategy in expectation.

V. Model Validation

In this section, we conduct several simulations to validate the correctness of our proposed model using MATLAB. In our simulation, we use the network scenario with 10 nodes distributed over $500 \times 500m^2$. Nodes in the scenario move according to the RWP model, and they all communicate using a normal communication range r. According to the assumptions above, we consider that all nodes in the network have the same moving speed V, and we set the pause time to be $0s$.

What's more, since it is not practical to assume that all nodes in the network have the same moving speed V, we also conduct some simulations to test whether our proposed model can be extended to a more general scenario. In this scenario, we consider that the speed of nodes in the network is uniformly distributed in the range of $[V - C, V + C]$, where C is a constant value. Therefore, the average speed of nodes in the network is V, and we can obtain the theoretical results from our proposed model when the average speed of nodes is V. Varying the value of C, we can test whether the simulation results are close to the theoretical results obtained from our proposed model in this scenario.

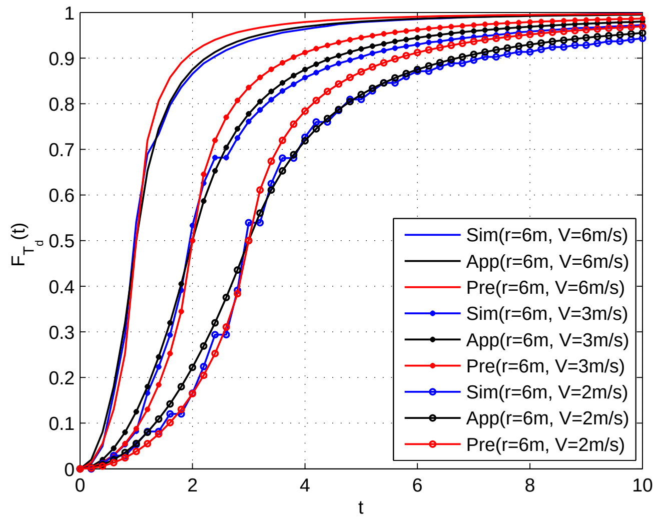
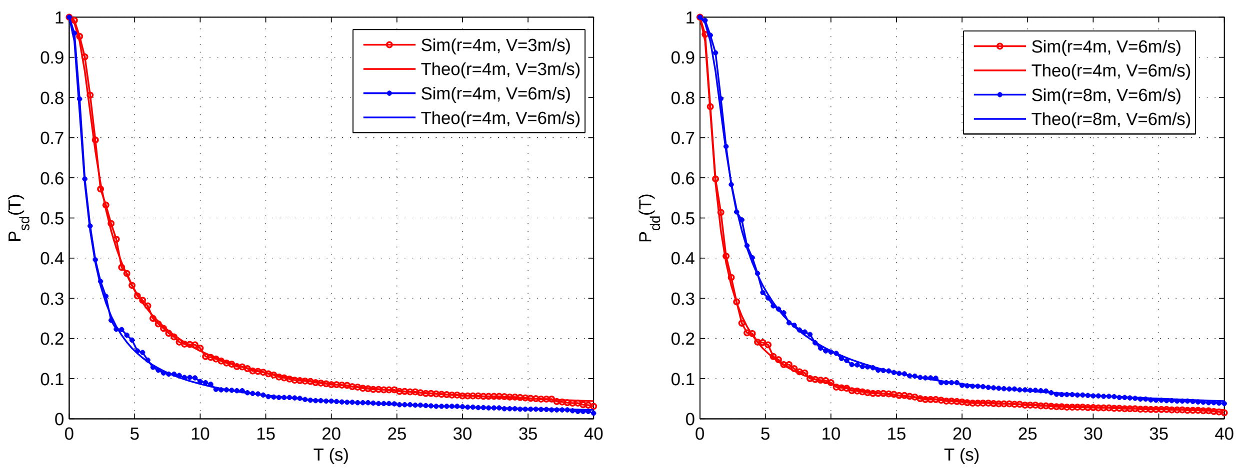


Fig. 6. Comparison between simulation results and theoretical results of $F_{T_d}(t)$ under different scenarios.



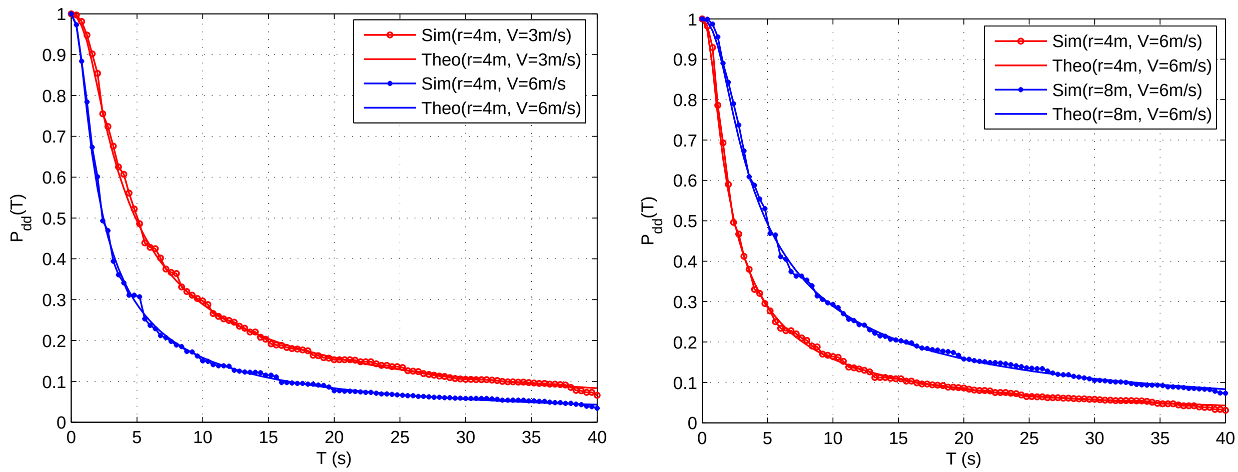
(a) When the speed V changes

(b) When the communication range r changes

Fig. 7. Comparison between simulation results and theoretical results of $P_{sd}(T)$ under different scenarios.

Fig. 6 shows the comparison between simulation results and theoretical results of $F_{T_d}(t)$ under different scenarios. It can be found that with the increase of t, the simulation results of $F_{T_d}(t)$ are very close to the approximate value of $F_{T_d}(t)$ and the precise value of $F_{T_d}(t)$ when $r = 6m$, V = 2, 3 and $6m/s$. It can also be found that with the increase of t, the simulation results of $F_{T_d}(t)$ are much closer to the approximate value of $F_{T_d}(t)$ than the precise value of $F_{T_d}(t)$ when r = $6m$, V = 2, 3, and $6m/s$, except for r = $6m$ and V = $2m/s$ when $t < r/V$. Therefore, in this paper, we simply use Eq. (3) instead of Eq. (2) to calculate the single detecting probability and the double detecting probability directly.

Fig. 7 shows the comparison between simulation results and theoretical results of $P_{sd}(T)$ under different scenarios. Fig. (7)(a) shows the comparison between the simulation results of $P_{sd}(T)$ and the theoretical



(a) When the speed V changes

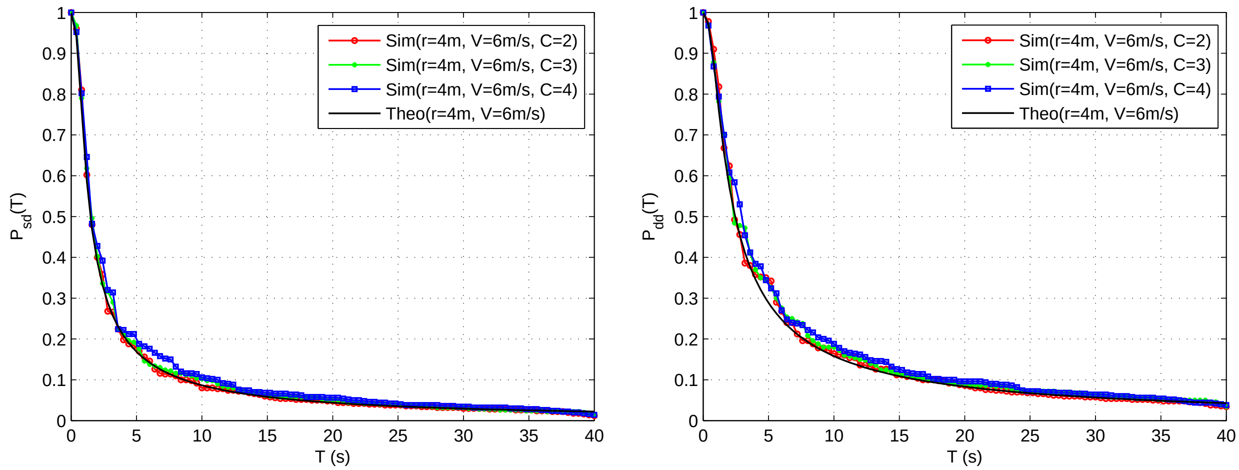
(b) When the communication range r changes

Fig. 8. Comparison between simulation results and theoretical results of $P_{dd}(T)$ under different scenarios.

results of $P_{sd}(T)$ when the speed V changes, and Fig. 7(b) shows the comparison between the simulation results of $P_{sd}(T)$ and the theoretical results of $P_{sd}(T)$ when the communication range r changes. It can be found that with the increase of T, the simulation results of $P_{sd}(T)$ are very close to the theoretical results of $P_{sd}(T)$ not only when the the speed V changes, but also when the communication range r changes.

Fig. 8 shows the comparison between simulation results and theoretical results of $P_{dd}(T)$ under different scenarios. Fig. (8)(a) shows the comparison between the simulation results of $P_{dd}(T)$ and the theoretical results of $P_{dd}(T)$ when the speed V changes, and Fig. 8(b) shows the comparison between the simulation results of $P_{dd}(T)$ and the theoretical results of $P_{dd}(T)$ when the communication range r changes. It can be found that with the increase of T, the simulation results of $P_{dd}(T)$ are also very close to the theoretical results of $P_{dd}(T)$ not only when the the speed V changes, but also when the communication range r changes.

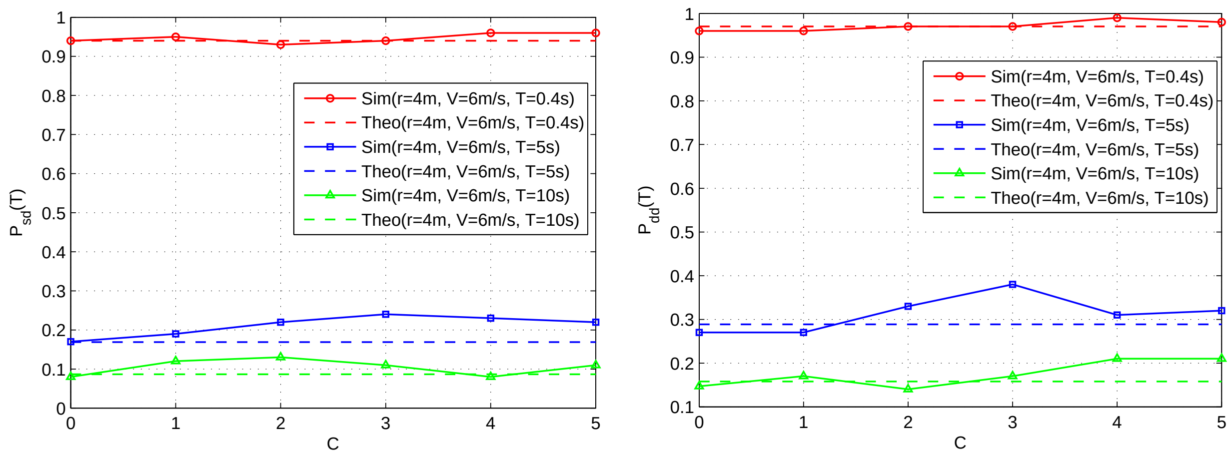
Fig. 9 shows the comparison between simulation results and theoretical results of $P_{sd}(T)$ and $P_{dd}(T)$ when the parameter C changes. Here, C is a constant value. Varying the value of C, we can test whether the simulation results are close to the theoretical results obtained from our proposed model in this scenario. Fig. (9)(a) shows the comparison between the simulation results and the theoretical results of $P_{sd}(T)$ when the parameter C changes. It can be found that with the increase of T, the simulation results $P_{sd}(T)$ when the parameter C changes are very close to the theoretical results of $P_{sd}(T)$, especially when C is small, and T is short or long. Fig. 9(b) shows the comparison between the simulation results and the theoretical results of $P_{dd}(T)$ when the parameter C changes. It can be found that with the increase of T, the simulation



(a) Comparison between simulation results and theoretical results of $P_{sd}(T)$ when the parameter C changes

(b) Comparison between simulation results and theoretical results of $P_{dd}(T)$ when the parameter C changes

Fig. 9. Comparison between simulation results and theoretical results of $P_{sd}(T)$ and $P_{dd}(T)$ under different scenarios.



(a) Comparison between simulation results and theoretical results of $P_{sd}(T)$ when the parameter T changes

(b) Comparison between simulation results and theoretical results of $P_{dd}(T)$ when the parameter T changes

Fig. 10. Comparison between simulation results and theoretical results of $P_{sd}(T)$ and $P_{dd}(T)$ under different scenarios.

results of $P_{dd}(T)$ when the parameter C changes are also very close to the theoretical results of $P_{dd}(T)$, especially when C is small, and T is short or long.. Therefore, our proposed model is also suitable for the more general scenario, in which the speed of nodes in the network is uniformly distributed in the range of $[V - C, V + C]$.

Fig. 10 shows the comparison between simulation results and theoretical results of $P_{sd}(T)$ and $P_{dd}(T)$ when the parameter T changes. Similar to the results in Fig. 9, it can be found that with the increase of C, the simulation results of $P_{sd}(T)$ and $P_{dd}(T)$ are closer to the theoretical results of $P_{dd}(T)$, especially when C is smaller. Furthermore, the simulation results of $P_{sd}(T)$ and $P_{dd}(T)$ are very close to the theoretical results of $P_{sd}(T)$ and $P_{dd}(T)$ when T is short(0.4s), and long(10s), which accord with the results in Fig. 9.

To summarize, we have conducted several simulations to validate the correctness of our proposed model in this section. Via simulations under different scenarios, we show that the simulation results of $F_{T_d}(t)$ are much closer to the approximate value of $F_{T_d}(t)$ than the the precise value of $F_{T_d}(t)$ under different scenarios, except for r = $6m$ and V = $2m/s$ when $t < r/V$; the simulation results of $P_{sd}(T)$ and $P_{dd}(T)$ are also very close to the theoretical results of $P_{sd}(T)$, and $P_{dd}(T)$, respectively, which validate the correctness of our proposed model. Furthermore, we also show that our proposed model can be applied to a more general scenario, in which the speed of nodes in the network is uniformly distributed in the range of $[V - C, V + C]$.

VI. Trade-offs between energy efficiency and the total number of effective contacts

In this section, we introduce the trade-off between energy efficiency and the total number of effective contacts in the single contact probing process and the double contact probing process, while the total number of effective contacts denotes the number of effective contacts detected by a certain node over a certain period. Here, we consider that a certain node, e.g., node A, probes its environment over a certain period L (e.g. node A should probe the environment over a period of 5 hours), then we consider how to decide the probing interval T, so as to make the contact probing process more energy efficient.

According to [26], [27], under the simplifying condition that the pausing time is 0, node pairs in the RWP model have identical inter-contact time distributions, and CDF of the inter-contact time is approximating exponential distribution with contact rate $\lambda = \frac{2rV_{rwp}V}{S}$, where $V_{rwp} \approx 1.754$ is the normalized relative speed for the RWP model, V is moving speed of nodes, r is the transmission range of nodes, and S is the size of the scenario. Then, the number of effective contacts detected by a certain node, e.g., node A, with a certain node, e.g., node B, over period L in the single and double contact probing processes can be expressed as:

$$N_{eff} \;=\; \lambda L P_{sd}(T), \tag{12}$$

and

$$N'_{eff} \;=\; \lambda L P_{dd}(T), \tag{13}$$

where $\lambda = \frac{2rV_{rwp}V}{S}$ is the contact rate between nodes A and B, $P_{sd}(T)$ is the single detecting probability, and $P_{dd}(T)$ is the double detecting probability.

Note that there are N nodes in the network, and node pairs in the RWP model have identical inter-contact time distributions. Therefore, the number of effective contacts detected by node A, over period L in the single and double contact probing processes can be expressed as:

$$N_{eff} = \lambda(N-1)LP_{sd}(T), \tag{14}$$

and

$$N_{eff}^{'} = \lambda(N-1)LP_{dd}(T). \tag{15}$$

Substituting Eq. (4) into Eq. (14), and Eq. (7) into Eq. (15), we obtain the expressions of the total number of effective contacts in the single and double contact probing processes as:

$$N_{eff} = \begin{cases} (1-\frac{T^2V^2}{6r^2})\frac{2r(N-1)V_{rwp}VL}{S}, & T \le \frac{r}{V}, \\ (\frac{4r}{3T}-\frac{r^2}{2T^2V})\frac{2r(N-1)V_{rwp}L}{S}, & T > \frac{r}{V}, \end{cases} \tag{16}$$

and

$$N_{eff}^{'} = \begin{cases} (1-\frac{V^2T^2}{12r^2})\frac{2r(N-1)V_{rwp}VL}{S}, & E \ge \frac{V}{r}, \\ [\frac{8r}{3T}-(7+4ln\frac{TV}{r})\frac{r^2}{4VT^2}]\frac{2r(N-1)V_{rwp}L}{S}, & E < \frac{V}{r}, \end{cases} \tag{17}$$

where r is the transmission range of nodes, V is the moving speed of nodes, and T is the contact probing interval.

In this paper, since we only investigate the energy consumed in the contact probing process, we do not take into account the energy consumed in the data transmission process. We define energy consumption $E = \frac{1}{T}$, which indicates the probing rate of nodes in the network. If the probing rate is larger, nodes in the network will consume more energy in the contact probing process. Then, Eq. (16) and Eq. (17) will be changed to:

$$N_{eff} = \begin{cases} (1-\frac{V^2}{6r^2E^2})\frac{2r(N-1)V_{rwp}VL}{S}, & E \ge \frac{V}{r}, \\ (\frac{4rE}{3}-\frac{r^2E^2}{2V})\frac{2r(N-1)V_{rwp}L}{S}, & E < \frac{V}{r}. \end{cases} \tag{18}$$

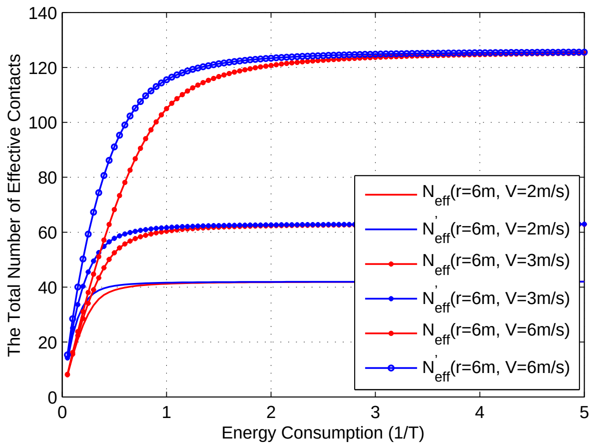
and

$$N_{eff}^{'} = \begin{cases} (1-\frac{V^2}{12r^2E^2})\frac{2r(N-1)V_{rwp}VL}{S}, & E \ge \frac{V}{r}, \\ [\frac{8rE}{3}-(7+4ln\frac{V}{rE})\frac{r^2E^2}{4V}]\frac{2r(N-1)V_{rwp}L}{S}, & E < \frac{V}{r}. \end{cases} \tag{19}$$

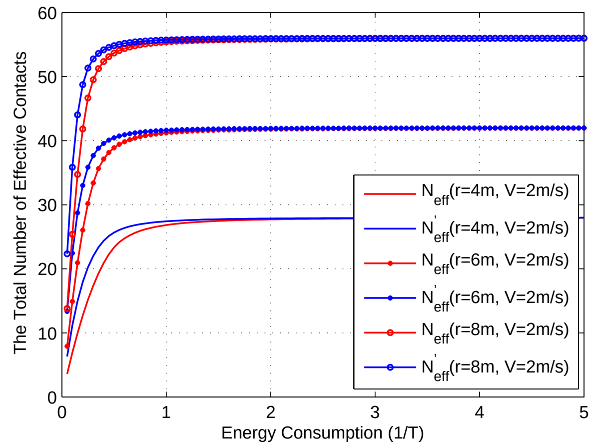
According to Eqs. (18) and (19), when the energy consumption E is close to ∞, we can obtain the total number of effective contacts in the single contact probing process and the double contact probing process as: $N_{eff} = N_{eff}^{'} = \frac{2r(N-1)V_{rwp}VL}{S}$, which is the upper-bound of N_{eff} and $N_{eff}^{'}$. When E equals

0, we can obtain that $N_{eff} = N^{'}_{eff} = 0$, which is the lower-bound of N_{eff} and $N^{'}_{eff}$. Here, for simplicity, we set N=2, $L = 25,000s$ and $S = 500 \times 500m^2$. Therefore, the upper-bound of N_{eff} and $N^{'}_{eff}$ will be changed to $2rV_{rwp}V$.

Fig. 11 shows the trade-off between energy efficiency and the total number of effective contacts in the single and double contact probing processes. Fig. 11(a) shows the trade-off between energy efficiency and the total number of effective contacts in the single and double contact probing processes when the speed V changes, and Fig. 11(b) shows the trade-off between energy efficiency and the total number of effective contacts in the single and double contact probing processes when the communication range r changes. It can be found that the total number of effective contacts in the single and double contact probing processes both increase as the energy consumption increases. This is reasonable because more energy consumption means more frequent contact probing, resulting in the increase of the total number of effective contacts. Furthermore, when the energy consumption increases to a certain value, the increase rate of N_{eff} and $N^{'}_{eff}$ will be very small. Therefore, we define that with the increase of the energy consumption, if N_{eff} and $N^{'}_{eff}$ reach 90% of the upper bound, then this point will be the good trade-off point between energy efficiency and the total number of effective contacts in the single and double contact probing processes. For example, when $r = 6m$, $V = 2m/s$, N_{eff} reaches 90% of the upper-bound when energy consumption is 0.45, and the corresponding value is 1.3 when $r = 6m$, $V = 6m/s$, which are good trade-off points between energy efficiency and the total number of effective contacts in the single contact probing process. $N^{'}_{eff}$ reaches the upper-bound faster than N_{eff} not only when the speed V changes, but also when the communication range r changes. When $r = 6m$, $V = 2m/s$, $N^{'}_{eff}$ reaches 90% of the upper-bound when energy consumption is 0.3, and the corresponding value is 0.9 when $r = 6m$, $V = 6m/s$, which are good trade-off points between energy efficiency and the total number of effective contacts in the double contact probing process. It is worth noticing that good trade-off points in the single and double contact probing processes change as the speed V changes, however, good trade-off points in the single and double contact probing processes are almost the same as the communication range r changes. As shown in Fig. 11(a), when the speed V is smaller, N_{eff} and $N^{'}_{eff}$ reach the upper-bound more quickly, and the good trade-off points in the single and double contact probing processes are obviously different when V = 2, 3, and $6m/s$. Therefore, good trade-off points in the single and double contact probing processes are obviously different as the speed V changes. When the communication range r changes, the good trade-off points



(a) When the speed V changes



(b) When the communication range r changes

Fig. 11. Trade-offs between energy efficiency and the total number of effective contacts in the single and double contact probing processes.

are nearly the same, because N_{eff} nearly reaches the upper-bound at the same point when r = 4, 6, and $8m$, and $N^{'}_{eff}$ also nearly reaches the upper-bound at the same point when r = 4, 6, and $8m$.

Similar to the results in Fig. 3(b), the total number of effective contacts in the single and double contact probing processes also both increase as the communication range r increases. The main reason is that $P_{sd}(T)$ and $P_{dd}(T)$ increase as r increases, resulting in the increase of the total number of effective contacts. It is worth noticing that different from the results in Fig. 3(a), the total number of effective contacts in the single and double contact probing process both increase as the speed V increases. The main reason is that although $P_{sd}(T)$ and $P_{dd}(T)$ decrease as the speed V increases, the contact rate λ increases as V increases, and the contact rate λ increases more quickly, resulting in the increase of the total number of effective contacts.

To summarize, we have obtained the expressions of the total number of effective contacts in the single and double contact probing processes, respectively, and analyzed the trade-off between energy efficiency and the total number of effective contacts under different scenarios. Our results show that the total number of effective contacts in the single and double contact probing processes have a lower-bound and an upper-bound, and the good trade-off points are obviously different when the speed of nodes is different. Our results also show that the single detecting probability and the double detecting probability increase as the speed of nodes decreases, while the total number of effective contacts in the single and double contact probing process increase as the speed of nodes increases. Furthermore, the total number of effective contacts in the double contact probing process reaches the upper-bound much faster than the total number of effective contacts in the single contact probing process, not only when the speed of nodes changes, but also when the communication range changes.

VII. Discussions

For simplicity, we assumed that nodes move at the same speed and the pausing time is equal to 0. Actually, our proposed model can be also extended to the case without this assumption. The pdf of contact duration with pausing time and different speed has been given in [25]. If we substitute the expression into Eq. (1) and Eq. (7), then we can obtain the expression of the single detecting probability and the double detecting probability, respectively. Furthermore, the expected inter-contact time with pausing time and different speed is given as the expected meeting time in [27]. If we substitute the expression into Eq. (14) and Eq. (15), then we can obtain the total number of effective contacts in the single and double contact probing processes, respectively. The only problem is that the pdf of contact duration with pause time and different speed is very complex. It is hard to obtain the exact expression of the single detecting probability and the double detecting probability. In the future work, we will try to solve this problem.

VIII. Conclusions

In this paper, we proposed a model to investigate the contact probing process in OppNets, based on the RWP model. Given the contact duration distribution in the RWP model, we analytically obtain the expression of the single detecting probability and the double detecting probability, respectively, and demonstrate that among all contact probing strategies with the same average contact probing interval, the strategy which probes at a constant interval performs best. Then, we conduct several simulations to validate the correctness of our proposed model. Our results show that the simulation results are quite close to the theoretical results under different scenarios, which validate the correctness of our proposed model. Furthermore, our results also show that our proposed model can be applied to a more general scenario. At last, based on the proposed model, we analyze the trade-off between energy efficiency and the total number of effective contacts under different scenarios. Our results show that the good trade-off points are obviously different when the speed of nodes is different. Moreover, the single detecting probability and the double detecting probability increase as the speed of nodes decreases, while the total number of effective contacts in the single and double contact probing processes increase as the speed of nodes increases, and the total number of effective contacts in the double contact probing process reaches the upper-bound much faster than the total number of effective contacts in the single contact probing process.

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APPENDIX

According to Eq. (2), we have:

$$F_{T_d}(t) = \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\sqrt{\frac{\frac{r}{V} + t}{|\frac{r}{V} - t|}}). \tag{20}$$

If $t \ll \frac{r}{V}$, we have:

$$\begin{aligned} F_{T_d}(t) &= \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\sqrt{\frac{\frac{r}{V}+t}{\frac{r}{V}-t}}) \\ &\approx \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\sqrt{[\frac{r}{V}+t]^2}) \\ &= \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\frac{r}{V}+t) \\ &\approx \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} \frac{Vt}{r} \\ &= \frac{V^2t^2}{2r^2}. \end{aligned} \tag{21}$$

If $t \gg \frac{r}{V}$, we have:

$$\begin{aligned} F_{T_d}(t) &= \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\sqrt{\frac{t+\frac{r}{V}}{t-\frac{r}{V}}}) \\ &\approx \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(\sqrt{[t+\frac{r}{V}]^2}) \\ &= \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} ln(t+\frac{r}{V}) \\ &\approx \frac{1}{2} - \frac{r^2 - V^2t^2}{2rVt} \frac{r}{Vt} \\ &= 1 - \frac{r^2}{2V^2t^2}. \end{aligned} \tag{22}$$

Therefore, we obtain the approximation of Eq. (2) as:

$$F_{T_d}(t) = \begin{cases} \frac{V^2t^2}{2r^2} & t \leq \frac{r}{V} \\ 1 - \frac{r^2}{2V^2t^2} & t > \frac{r}{V}. \end{cases} \tag{23}$$