Approximation Algorithms for Dependency-Aware Rule-Caching in Software-Defined Networks

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Abstract—Software-defined networks (SDNs) can support fine-grained forwarding policies in the underlying switches. The new content addressable memory, Ternary Content Addressable Memory (TCAM), enables fast lookups for matching rules in message forwarding, represented as binary strings with wildcards. However, the cost and power limit the number of matching rules a TCAM can support. Therefore, rule caching is needed to place high-weight (high-hit) rules in the TCAM hardware, while large, but slow, software switches handle cache-miss traffic. We assume that matching these rules form a forest of trees. A rule \( R' \) is a descendant of another rule \( R \) if \( R' \) is a special case of \( R \). Dependent rules are evaluated in a particular matching order: when a rule is included in the cache, all its descendants in the rule set have to be included as well. Our objective is to maximize the number of rule hits, while limiting the number of cached rules. Three greedy rule-caching algorithms are proposed, including two with approximation ratios of 2 and \( \frac{11}{6} \), respectively. In addition, we propose a dynamic programming solution that is optimal but slow. The efficiency of the proposed approaches are evaluated through real data-driven simulations.

Index Terms—Approximation ratio, caching, dependency constraint, greedy algorithms, software-defined networks.

I. INTRODUCTION

In software-defined networks (SDNs), the central controller installs packet-processing rules in switches to manage traffic [1]. There are two types of switches: hardware-based content address memory switches, like Ternary Content Addressable Memory (TCAM) [2], which are fast but small in capacity [3], and software-based switches that are large in capacity but are relatively slow. These two types of switches form a classic cache and main memory pair that stores and processes rules. In this paper, we study an efficient caching scheme for rules subject to a unique constraint on rule placement.

Caching IP address is well known in a general setting [4] as well as in TCAM [5]. The state-of-the-art caching in TCAM uses descendant matching rules [6]. Rules in SDNs are binary string matching rules with wildcards, using * to match any single character. A rule processing order corresponds to a matching between a message header and the binary string of a rule [7]. A string, \( R' \), is a descendant of another string, \( R \), if \( R' \) is a special case of \( R \). \( R' \) and \( R \) are dependent from each other. A rule together with all its descendants form a branch of a tree. Two rules are independent if they do not share a common ancestor in a tree. Rule \( R_1 \) is independent. Together, the rule set forms a forest of trees [8, 9]. For any two dependent rules, there is a matching order. The descendant rule is evaluated first. This order is used to minimize the number of rules used in the matching process.

The matching order complicates the caching policy because when a rule is placed in a cache, all its descendants have to be included in the cache to ensure correctness. In Table 1, if \( R_4; 10** \) is placed in the cache, both its prefixes \( R_2; 1000 \) and \( R_3; 100* \) have to be in the cache. This is because if \( R_2 \) and \( R_3 \) were not placed in the cache, messages with the headers 1000 and 1001 would have matched with \( R_4 \), rather than \( R_2 \) and \( R_3 \), respectively. It has been shown in [6] that an optimal caching, in terms of maximizing total weight (hit ratio), is NP-hard under the above dependency constraint. Several greedy solutions have been proposed, but without no approximation bounds. In this paper, we discuss three greedy solutions, including two with a constant bound in terms of total weight. The first one respects the dependency constraint; the second one breaks the constraint by inserting a dummy rule (called deny rule) in the cache; and the third one combines the first and second greedy solutions. In addition, we use dynamic programming (DP) to solve problem optimally (in a setting without deny rule), but with a high complexity (i.e., heavy-weight). The contributions of this paper can be summarized as follows, where \( n \) is the number of rules and \( k \) is the cache size:

1) Our greedy algorithm with the dependency constraint has a complexity of \( O(n + k \log n + k^2) \) and an approximation ratio of 2.
2) The complexity of the greedy algorithm that breaks the dependency constraint by introducing deny rule is \( O(kn) \) using two-level heaps.
3) The combined greedy algorithm using the above two greedy approaches has an approximation ratio of 24/5.
4) The DP solution has complexity of \( O(k^2n) \).
5) Extensive simulations are conducted together with an optimal solution based on dynamic programming.

TABLE I: Rules with binary strings and weights.

<table>
<thead>
<tr>
<th>Rule</th>
<th>Code</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0***</td>
<td>5</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>1000</td>
<td>7</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>100*</td>
<td>13</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>10**</td>
<td>43</td>
</tr>
<tr>
<td>( R_5 )</td>
<td>1110</td>
<td>20</td>
</tr>
<tr>
<td>( R_6 )</td>
<td>1***</td>
<td>12</td>
</tr>
</tbody>
</table>

Fig. 1: A forest of rules.
The remainder of the paper is organized as follows. Sections 2 and 3 discuss two greedy solutions, one for the basic model using a branch for caching and the other one for an enhanced model that cuts a branch with a dummy rule, together with complexity and performance analysis. Section 4 reviews an optimal solution based on dynamic programming. Section 5 conducts a simulation study. Section 6 concludes the paper.

II. GREEDY SOLUTION ONE

Given a rule set in a forest of trees (see Fig. 1), a tree branch consists of a rule and all its descendants. Due to the space limit of the cache, we can only accommodate a branch of a size up to $k$ (cache size). A max branch is a branch that meets the following conditions: (1) the branch size is $k$, or (2) if its branch size is less than $k$, then it is not a branch of another branch with a size of $k$ or less. In Table 1, when $k = 3$, the max branches are $[R_1]$, $[R_2, R_3, R_4]$, and $[R_5]$; when $k = 5$, the max branches are $[R_1]$ and $[R_2, R_3, R_4, R_5, R_6]$. Note that when $k = 3$, $R_6$ does not belong to any branch, i.e., it cannot be cached. This represents the fact that not all rules can be cacheable due to the dependency condition.

Based on the definitions of the branch and the max branch (see their topological sorting order representations in Fig. 2(a)), we have the following properties: the set of max branches includes all eligible branches that can be placed in the cache. Each rule in a max branch together with its descendants form a branch. For example, in max branch $[R_2, R_3, R_4, R_5, R_6]$, $R_2$ forms branch $[R_2, R_3, R_4]$, $R_3$ forms $[R_2, R_3, R_4]$, $R_4$ forms $[R_2, R_3, R_4]$, $R_5$ forms $[R_5]$, and $R_6$ forms $[R_2, R_3, R_4, R_5, R_6]$.

Now, we calculate the unit benefit for each branch in a max branch. Each rule has a unit cost, $C$, with a weight, $W$. Our goal is to find a branch that maximizes the unit benefit, defined as the ratio of rule weight to rule cost ($\Delta W/\Delta C$). The challenge lies in the fact that once a candidate branch is selected and removed for future consideration, unit benefit values of other branches in the same max branch need to be updated. Therefore, we need to find an appropriate data structure to control the complexity.

The solution is to maintain a special branch that corresponds to the maximum unit benefit for each max branch. Then we build a heap that consists of all these special branches.

**Max branches**

1) Calculate the branch size for each node through a simple traversal of each tree from lowest to highest in the topological sorting order.

![Fig. 2: Topological sorting order for a max branch (a) and a max segment (b) in non-increasing order from left to right.](image)

**Algorithm 1 Greedy Algorithm One**

1: Construct max branches with each max branch arranged in a topological sorting order.
2: Calculate unit benefit values for all branches in the max branches.
3: Find a special branch, called the max unit benefit branch, that has the maximum unit benefit in each max branch.
4: Maintain a heap for all max unit benefit branches.
5: **repeat**
   6: Determine the branch that has the maximum unit benefit in the heap.
   7: Remove the branch and update the corresponding max branch, including its new max unit benefit branch.
   8: Update the heap accordingly based on the new max unit benefit branch selected by b).
9: **until** the cache exceeds its capacity
10: Suppose Steps 5 to 9 stop at round $i + 1$, i.e., the first $i$ branches have not exceeded the cache capacity. Then, we select the larger of the two in terms of weight: the first $i$ branches or the $(i + 1)$th branch.

2) Traverse each tree again from lowest to highest and identify each branch that is not a descendant of another branch of size $k$ or less.
3) Extract the branch as a new max branch before continuing the traversal process.

Fig. 1 shows the branch size for each rule shown in Table 1. When $k = 3$, $[R_1]$, $[R_2, R_3, R_4]$, and $[R_5]$ are three max branches identified through the tree traversal process starting from the root of each tree. It is clear that the complexity for extracting the max branches is $O(n)$. Now we can present the greedy algorithm.

**Theorem 1:** The complexity of Greedy Algorithm One is $O(n + k \log n + k^2)$.

**Proof:** The cost of Step 1 is $O(n)$ for constructing max branches and maintaining a topological sorting order for each max branch. Steps 2 and 3 together cost $O(n)$ because one scan of each max branch is sufficient. Step 4 costs $O(n)$ for heap construction. For the loop in Steps 5 to 9, the max heap in Step 6 costs $O(\log n)$; one max branch update in Step 7 costs $O(k)$, and the heap update in Step 8 costs $\log n$. As there are at most $k$ iterations in Step 5, the overall cost for Step 5 is $O(k(\log n + k))$. Adding all the costs together, we obtain the result. □

Note that $k$ in general is relatively small compared to $n$. If $k = O(\sqrt{n})$ as in many practical cases, the complexity becomes linear $O(n)$.

**Theorem 2:** Greedy Algorithm One has an approximation ratio of 2 compared to the optimal result on the total weight.

**Proof:** By way of algorithm construction, it is easy to verify that all combinations of rules as cache units are considered in the branch selection algorithm, subject to the dependency and cache capacity constraints. The branch selection strictly follows the unit benefit value of each unit. Also, it is clear...
that the total weight of the optimal solution is less than the total weight of the first \(i + 1\) selected branches. As the final result is at least half of that weight, which is either the first \(i\) selected branches or the \((i + 1)\)th branch, whichever is larger, the approximation ratio is bounded by 2.

In Table 1, when \(k = 3\), branch \([R_2, R_3, R_4]\) is selected first with a unit benefit of 21. Then branch \([R_5]\) is selected. When \(k = 5\), branch \([R_2, R_3, R_4, R_5, R_6]\) is selected with a unit benefit of 19.

III. GREEDY ALGORITHM TWO AND COMBINED GREEDY

The dependency constraint causes inefficiencies in cache placement, because when a rule is placed in cache, all its dependents need to be in the cache as well. This problem is more prominent when there are many low-weight rules in the dependents of a high-weight rule. \[6\] introduced a deny rule needed if \(R\) is a leaf rule without any descendants. If \(R\) is an independent single rule (such as \(R_1\) in Table 1), the value for max \(s(R)\) is \(W/C\), as no deny rule is needed.

Fig. 4 shows the construction of a global heap (\(g\)-heap) from the max heaps of all local heaps (\(l\)-heaps). When we remove a rule segment, we do not remove the rule that is currently held by a deny rule. The reason for this is that if that rule is selected, the deny rule can be removed (by “merging” two rule chains into one). In Table 1, if \(R_3\) \(100^*\) is eventually selected, the deny rule \(R_3^*\) \(100^*\) (SS) for \(R_4\) will be removed.

**Theorem 3:** The complexity of Greedy Algorithm Two is \(O(kn)\).

**Proof:** Step 1 costs \(O(n)\) for one scan. Step 2 can use the same scan as Step 1, but for each rule, up to \(k\) updates to its descendants are needed. Therefore, the cost is \(O(kn)\). The cost of building local heaps is bounded by \(O(n)\), as there are at most \(n\) units for all heaps. The size of a global heap is bounded by \(n\) in Step 4. Step 6 costs \(k\) (the number of rounds). Step 7 may affect values for up to \(k\) max \(s(R)\) in all rounds in Steps 5 to 9. Therefore, the total cost is \(O(k \log n)\) for all local heaps’ updates. Step 8 costs \(O(k \log n)\) for all global updates. Therefore, the overall cost is \(O(kn)\). □

We can combine Greedy Algorithm One and Greedy Algorithm Two to create a Combined Greedy Algorithm as follows: we use the same criterion based on the maximum unit benefits of branches and segments for caching. The updates are conducted on the two separate sets of data structures used in these two algorithms. The complexity will remain \(O(kn)\). The reason is that for each selection of a branch in Greedy Algorithm One, one and only one max segment will be affected. Likewise, each selection of a segment in Greedy Algorithm Two will affect one branch.

To derive the approximation ratio for the Combined Greedy Algorithm, we first prove the following bound on unit benefit

### Algorithm 2 Greedy Algorithm Two

1. Construct max segments through traversing the tree once.
2. Calculate \(\max s(R)\) for each rule in the max segments.
3. Maintain a local heap (\(l\)-heap) of \(\max s(R)\) for each max segment. The max heap of this local heap is the max unit benefit segment.
4. Maintain a global heap (\(g\)-heap) for the max heaps of all local heaps.
5. repeat
6. Determine the max heap of the global heap.
7. Remove the segment of the global max heap and update the corresponding max segment, including the new max heap in its local heap.
8. Update global heap based on the new local max heap.
9. until the cache exceeds its capacity
10. Same as Step 10 of Greedy Algorithm One

(b), \(\max s(R)\) is a segment in \(s(R)\) with the maximum unit benefit: \(\Delta W/(\Delta C + 1)\), where \(\Delta C\) is the chain length and the extra 1 is the slot used for the deny rule. No deny rule is needed if \(R\) is a leaf rule without any descendants. If \(R\) is an independent single rule (such as \(R_1\) in Table 1), the value for \(\max s(R)\) is \(W/C\), as no deny rule is needed.
loss when a fork is converted into multiple segments (see one example in Fig. 3 (b)).

**Theorem 4:** The unit benefit loss in terms of the ratio of unit benefit of the fork and the unit benefit of the corresponding multiple segments is 6/5.

**Proof:** Consider a fork with \( l \) branches with weight \( \Delta W \) and cost \( \Delta C \). The unit benefit of the fork is \( \Delta W / (\Delta C + l) \), where \( l \) deny rules are used. The unit benefit of the segments is \( \Delta W / (\Delta C + l + 1) \). Clearly, the fork to segment ratio is \( 1 + 1 / (\Delta C + l) \). Since each branch needs at least one unit, the minimum \( \Delta C \) is \( l + 1 \). The minimum \( l \) is 2. Therefore, the maximum ratio is 6/5.

**Theorem 5:** The Combined Greedy Algorithm has an approximation ratio of 24/5 compared to the optimal scheme in terms of total weight.

**Proof:** We prove that for a fork with a segment rule, its total weight \( \Delta W \) will be at least half the weight of the optimal scheme when the greedy method is used based on the unit benefit. We only need to calculate the “waste” introduced by the slot used for the deny rule. Suppose the weight of the rule used by the deny rule is more than \( \Delta W \), then, the slot for the deny rule would have been selected in an early round because this slot plus one deny rule will generate a segment with a unit benefit of over \( \Delta W / 2 \). That is, there is no waste of “space” used for the deny rule. If the weight of the rule used by the deny rule is no more than \( \Delta W \), then, the optimal solution with the weight of the rule used by the deny rule included has a weight of no more than \( 2\Delta W \). Combining the results for Theorem 2 and Theorem 4, we have an approximation ratio of \( 2 \times 2 \times 6 / 5 = 24 / 5 \). □

In the example shown in Tab. 1, \( \max s(R_1), \max s(R_2), \max s(R_3), \max s(R_4), \max s(R_5), \) and \( \max s(R_6) \) are \([R_1], [R_2, R_3, R_4], [R_3, R_4], [R_3], [R_3], \) and \([R_6] \), respectively. Clearly, \( \max s(R_4) \) has a maximum unit benefit of 21.5 with deny rule \( R_4^0 : 100 \times (SS) \). \( \max s(R_2) \) and \( \max s(R_3) \) are updated to \([R_2, R_3]\) and \([R_3]\) after the removal of \( R_4 \), respectively. \( \max s(R_3) \) is the next candidate with a maximum unit benefit of 20. \( \max s(R_1) \) does not have the deny rule as \( R_3 \) is the head of a rule chain. The next candidate is \( \max s(R_3) \) with a unit benefit of 13 because no new deny rule is used as its immediate predecessor, \( R_4 \), is already in the cache. In this case, deny rule \( R_3^0 : 100 \times (SS) \) is replaced by deny rule \( R_2^0 : 1000 \). If \( k = 5 \), \( \max S(R_6) : [R_6] \) is selected with a unit benefit of 12. Again, no new deny rule is needed as its immediate successors, \( R_4 \) and \( R_5 \), are already in the cache. Therefore, for the Combined Greedy Algorithm, the deny rule is not needed when its immediate predecessor or all its immediate successors of \( \max s(R) \) in the rule set are already in the cache.

**IV. Dynamic Programming Solution**

This section introduces an dynamic programming (DP) solution to maximize the number of rule hits. Since rule dependencies form a forest rather than a tree, a dummy root with a weight of zero is introduced. We start with sorting all rules in a depth-first order. For example, in Fig. 1, such an order can be \( \{R_0, R_1, R_6, R_5, R_4, R_3, R_2\} \) with dummy root \( R_0 \). Let \( T[R, m] \) be the subtree induced by rule \( R \), its first \( m \) children (in terms of the depth-first order), and all descendants of these \( m \) children. For example, in Fig. 1, \( T[R_6, 0] \) only includes \( R_6 \), \( T[R_6, 1] \) includes \( R_5 \) and \( R_6 \), and \( T[R_6, 2] \) includes \( R_2, R_3, R_4, R_5 \), and \( R_6 \). Let \( d(R) \) be the number of children of \( R \).

Let \( O[R, d, m] \) denote the optimal cache-hits in the sense that we cache \( m \) rules out of \( T[R, d] \). These \( m \) rules follow only the dependencies within \( T[R, d] \) and will ignore the other dependencies. By definition, \( O[R, 0, d(0), k] \) is our objective. Let \( W_i \) denote the cache-hit of \( R_i \). The initialization is:

\[
O[R_i, 0, m] = \begin{cases} W_i & \text{if } m \geq 1 \text{ and } i \neq 0 \\ 0 & \text{otherwise} \end{cases}
\]  

(1)

The optimal recurrence pattern is:

\[
O[R_i, d, m] = \max_{0 \leq m' \leq m} \left[ O[R_{id}, d(d_{id}), m'] + O[R_i, d-1, m-m'] \right]
\]  

(2)

In Eq. 2, \( R_{id} \) is the \( d \)-th child of \( R_i \). Note that \( O[R_i, d, m] \) either ignores its descendants under the \( d \)-th child or uses these descendants. For the latter case, we additionally require that the composition of \( O[R_{id}, d(d_{id}), m'] \) and \( O[R_i, d-1, m-m'] \) follow the dependencies within \( T[R_i, d] \). \( R_i \) is cached if and only if all the rules in \( T[R_i, d] \) are cached.

The time complexity of the dynamic programming solution is \( O(k^2n) \). This is because Eq. 2 takes \( O(k) \) to calculate, and we need to compute Eq. 2 for each rule \( R_i \) and each \( m \) (0 \( \leq m \leq k \)). Consequently, the DP solution has a larger time complexity than the greedy solution one, but has a better performance than the greedy solution one. However, the dynamic programming solution may not be able to incorporate deny rules. This is because deny rules lead to an exponential number of combinations, as in Eq. 2.

**V. Implementation**

**A. Real Traffic Traces and Settings**

Simulations are conducted based on the publicly available CAIDA packet trace of 2016 from the Equinix datacenter in Chicago [10] and the real-world Cisco router configuration on a Stanford backbone router [11].

**CAIDA:** The packet traces have a total of 20 million packets sent over 30 minutes on high-speed Internet backbone
C. Performance with the CAIDA traffic trace

Fig. 5 shows results with the CAIDA traffic trace. We find that there are many shallow dependencies in the dependency graph. The depth of the dependency chains varies from 1 to 5. Many leaves’ depths are 2 or 3. These shallow dependencies incur the advantages of our greedy algorithms not so obvious. Fig. 5(a) shows the execution time of the six algorithms with the variability of rules. The dynamic programming method DP, the red line, takes the longest time. The other five take much less time than DP, because they are greedy algorithms that make locally optimal choices without the need to search all possibilities. All of them take at most 30% of Algorithm DP’s execution time. Because of the elaborate heap implementation, Algorithm Branch is faster than Algorithm Dependent by at most 21%, and Algorithm Segment takes only 84% time on average compared to Algorithm Cover. As Algorithm Combined selects the better choice between Branch and Segment, it costs a little more time but the performance improves a lot.

Fig. 5(b) lists the cache-hit traffic ratio over the total traffic on the variability of TCAM size. An advanced Pronto-Pica8 3290 switch has an ASIC that can hold 2,000 OpenFlow rules. We use the logarithm as the x-axis to show the difference more clearly. Algorithm DP does the best among all six algorithms because it sacrifices more time and memory to pursue the optimal solution. Our four algorithms achieve at least a 79.8% hit ratio with 2,000 rules, which is just 1.1% of the total rule table. Algorithm Branch achieves around 77.8%, Algorithm Segment is around 84.2% and Algorithm Combined is around 86.6% against the optimal algorithm DP’s hit ratio.

Fig. 5(c) shows the results of the cache-hit traffic ratio on the variability of the number of packets. We aim at testing our algorithms’ universality. The performance of all four algorithms keeps steady. Algorithm DP still does the best among all six algorithms. It achieves a cache-hit rate around 90.0% when the packet number varies from 0.6 to 20 million.

D. Performance with the Stanford backbone router

The performance with the Stanford backbone router is shown in Fig. 6. The forwarding actions have fewer choices.
Thus, the depth of the dependency chains varies from 1 to 8 and there are fewer shallow dependencies. The performances of our algorithms are better. Fig. 6(a) is the execution time result depending on the forwarding rules that vary from 563 to 18,000. It has more rules than CAIDA. The growth in number makes Algorithm DP try more possibilities while searching for the optimal one. Then, the complexity increases and it needs more time as shown in the red line. Reversely, compared to Fig. 5(a), our three greedy algorithms’ times are only 82%, 79% and 83%, respectively. This illustrates that these algorithms have better performances in the deeper chain.

The cache-hit ratio on the variability of TCAM size is shown in Fig. 6(b). The basic increasing tendency is analogous to Fig. 5(b). With more entries, the cache-hit probability of the hardware switch becomes larger. All six lines stabilize earlier than CAIDA. This is relative to the deeper dependency chain and the less shallow dependencies in this data set. It also illustrates that deeper dependencies need less forwarding entries because applying a deeper dependency rule gains more benefits. Additionally, the optimal solution’s performance is undoubtedly the best and the ratio reaches 92.3% when the TCAM entry number is 2,000. Our three greedy algorithms achieve better ratios than the CAIDA one with the same TCAM size. For example, Algorithm Branch’s ratio is 87.4%, Algorithm Segment’s is 91.3% and Algorithm Combined’s is 93.0% against the Alg. DP when the TCAM size is 2,000.

Fig. 6(c) shows the performance on the variability of the number of packets. It is similar to the CAIDA one in that the more packets there are, the higher the cache-hit ratio is. All our four algorithms’ performances are better than in the CAIDA data set. For 30 million packets, Algorithm DP’s cache-hit ratio reaches 90.2%, Algorithm Combined reaches 89.4%, Algorithm Segment reaches 83.7% and Algorithm Branch reaches 81.9%. Thus, Algorithm DP achieves the best performance in the rule placement at the cost of a longer execution time and more memory space. Algorithm Branch takes the least execution time and achieves the same performance as Algorithm Dependent, the same situation of Algorithm Segment against Algorithm Cover. Algorithm Combined is the best deal because of the trade-off between its complexity and its performance.

VI. CONCLUSION

This paper studies the efficient rule cache problem in SDNs. The hardware switches TCAM enable fast lookups for matching rules, but have limited memory. The software switches have enough memory to store rules, but slow in matching. Consequently, we need to place light-weight (high-hit) rules in TCAM hardware and use software switches to handle the cache-miss traffic. We propose three greedy effective algorithms. The first one with approximation 2 respects the dependency constraint. The second one inserts deny rules that relax the dependency constraint. The third one combines the first and second to have an approximation of $\frac{4}{3}$. We also apply dynamic programming to generate an optimal solution without deny rules, but slow (i.e., heavy-weight), solution. We evaluate our algorithms through real data-driven simulations.

REFERENCES