AoI-aware Incentive Mechanism for Mobile Crowdsensing using Stackelberg Game

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Outline

- Background & Motivation
- Related Work & Problem Formulation
- Basic Idea & Solution
- Evaluation & Conclusion
Background

Applications for Mobile CrowdSensing (MCS) systems
**Definition of Age of Information (AoI):** how old the freshest received update is, the elapsed time of data from being collected by the worker to being received and processed by the platform currently.

Motivation

Example

Social Network

- Data freshness \(\rightarrow\) Service quality
- Social benefits: workers can share their collected data with their social neighbors

Key Challenges

- **Strategy Determination** :
  - Data Update Frequency
  - Data valuation
  - Service quality
  - Fresher data \(\rightarrow\) Larger cost
  - Trade-offs: the data update frequencies of all workers

- **Incomplete Game** :
  - Platform: the limited communication and processing capabilities
  - Workers: compete for data update
  - Social relationships \(\rightarrow\)
  - Unknown or incomplete
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Related Work

Goal: Incentive mechanism design while considering data freshness and social networks

AoI optimization

Design with social network effects

Pricing issue with AoI concerns
Auction, Game, Contract, DRL...

Incomplete game
[3] Q. Xu, et al., Game theory and reinforcement learning based secure edge caching in mobile social networks, TIFS2020
Problem Formulation

System Model

- **Workers** \( \{1,2,\ldots,N\} \): collect and upload the data to the platform with data update frequency \( p_i \)
- **Unit-Reward** \( R_i \): the reward per data update frequency paid to worker \( i \rightarrow R = \{R_1, R_2, \ldots, R_N\} \)
- **Average AoI**: the time elapsed since the worker collects this data, \( \delta_i(t) = t - U_i(t) \rightarrow \bar{\delta}_i \)
- **Social networks**: an adjacency matrix \([v_{ij}]_{N \times N}\); \( v_{ij} \) is the social influence of worker \( j \) on worker \( i \).

Procedure

1. The requester \( \rightarrow \) the platform
   - A long-term sensing task

2. Platform: optimal payment
   - Worker: optimal update frequency

3. Worker \( i \): collect data from PoIs
   - Upload data packets with \( p_i \)

4. Pay rewards \( R \) to workers
Problem Formulation

➢ **Worker’s Utility:** the reward + social benefits − cost

\[
\Omega_i(p_i, P_{-i}) = R_ip_i + \Psi_i(p_i, P_{-i}) - s(ap_i^2 + bp_i).
\]

The reward that the platform pays to worker \(i\)

Social benefits caused by the social network effects

\[
\Psi_i(p_i, P_{-i}) = \sum_{j \in N_i} v_{ij} p_ip_j.
\]

➢ **Platform’s Utility:** the income that it can gain from all collected data − the total payments

\[
\Phi = \eta \sum_{i=1}^{N} (cp_i - dp_i^2) - \sum_{i=1}^{N} R_ip_i.
\]

Income: a linear-quadratic function of the data update frequencies of all workers

The sum of rewards paid to all workers
Problem Formulation

➢ Two-stage Stackelberg game: the platform is the leader and the workers are the followers

Stage I [Leader Game]: \[ \Phi(p_i^*, R_i^*) \geq \Phi(p_i, R_i) \]
Stage II [Follower Game i]: \[ \Omega_i(p_i^*, R_i^*) \geq \Omega_i(p_i, R_i) \]
Subject to: \[ \delta_i(p_i, P_{-i}) \leq \varepsilon, \ \forall i \in N \]
\[ \sum_{i=1}^{N} p_i \leq \hat{p} \]

Incomplete Information Bayesian Sub-Game:
- The set of players \( i \) \( \rightarrow \) a set of \( N \) workers;
- The action of player \( i \) \( \rightarrow \) the data update frequency \( p_i \);
- The type of player \( i \) \( \rightarrow \) the social network effects;
- The payoff of player \( i \) with its type and its action \( \rightarrow \) the utility \( \Omega_i \);
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Basic Idea

1. Characterizing AoI of Data
   • Derive AoI for a single worker
   • Derive AoI for Multiple Workers:
     -- neighbors influence
     -- competitive N sources system
     -- the closed-form expression of average AoI of data

2. Solving the Follower Game
   • Graph theory → Degree of a node $f$
     → the type of the worker
   • Rewrite the utility function:
     -- the degree distribution
     -- average strategy of neighbors
   • Optimal data update frequency $p^*(f)$

3. Solving the Leader Game
   • Rewrite the utility function:
     -- substitute $p^*(f)$ to $\Phi$
     -- AoI and total frequency constraint
     -- Karush-Kuhn-Tucker (KKT) conditions
   • Optimal unit-reward $R^*(f)$

4. Algorithm & Analysis
   • Propose the AoI-Aware Incentive (AIAI) mechanism
   • Follower game
     -- Bayesian Nash Equilibrium
   • Unique Stackelberg equilibrium
Characterizing AoI of Data

**AoI for a single worker**

\[
\delta^T = \frac{1}{T} \left( Q_1 + \sum_{k=2}^{I(T)} Q_k + \frac{T^2_{I(T)}/2}{2} \right) \\
= \frac{Q_1 + T^2_{I(T)/2}}{T} + \frac{I(T) - 1}{T} \frac{1}{I(T) - 1} \sum_{k=2}^{I(T)} Q_k.
\]

\[\delta = \lim_{T \to \infty} \delta^T = p(\mathbb{E}[XT] + \mathbb{E}[X^2/2])\]

\[\tilde{\delta} = [\rho - 1] \left( \rho^2 - \mu \rho \beta \right) + 1 / \left( (1 - \rho) \rho \mu \right)\]

**AoI for Multiple Workers**

\[
\mathbb{E}(X_k T_k) = \mathbb{E}(X_k C_k) + \mathbb{E}(X_k W_k) + \mathbb{E}(X_k H_k)
\]

\[
\downarrow \quad = \mathbb{E}(X_k) \mathbb{E}(C_k) + \mathbb{E}(X_k) \mathbb{E}(W_k) + \mathbb{E}(X_k) \mathbb{E}(H_k)
\]

\[
\tilde{\delta}_i = \frac{\alpha \beta_i}{\sum_{j \in N_i} \upsilon_{i,j}} + \frac{p_i / \mu^2}{1 - \rho_i} \left[ \frac{\rho_i \rho_i - 1}{(1 - \rho_i)^2} + \frac{\rho_i / (1 - \rho_i)}{\rho_i} + 1 / \mu + 1 / \rho_i \right]
\]
Solving the Follower Game

**Step 1**
- Derive the expected utility of each worker
  \[
  \omega_i(p_i, P_{-i}, \mathcal{R}) = \mathbb{E}[\Omega_i(p_i, P_{-i}, \mathcal{R})] = R_i p_i + v p_i \mathbb{E}\left[\sum_{j \in \mathcal{N}_i} p_j\right] - (ap_i^2 + bp_i)s.
  \]
- Worker’s type → the degree:
  \[
  \omega_i(p_i, P_{-i}, \mathcal{R}) = R_i p_i + v p_i \overline{P_{-i}} - (ap_i^2 + bp_i)s.
  \]

**Step 2**
- Degree distribution: \( F(f) \)
- Derive the utility of the worker with degree \( f \)
  \[
  \omega_f(p, P_{-f}) = R(f)p + v p(f) \overline{P_{-f}} - (ap^2(f) + bp(f))s.
  \]
- Bayesian Nash Equilibrium, BNE:
  \[
  \Gamma_i(\psi_i) \in \text{argmax}_{p_i \in \mathcal{P}_i} \Omega_i(p_i, \Gamma_{-i}, \psi_i, \psi_{-i}).
  \]

**Step 3**
- Apply the partial derivative:
  \[
  \frac{\partial \omega_f(p(f), P_{-f}, \mathcal{R})}{\partial p(f)} = R(f) + v f \overline{P_{-f}} - (2ap(f) + b)s.
  \]
- Derive the average data update frequency of neighbors \( \overline{P_{-f}} \)
  \[
  \overline{P_{-f}} \approx E[p(l) | l \in \mathcal{G}] = \frac{\overline{R} - bs}{2as - v \overline{f}}
  \]

**Step 4**
- Given any unit-reward \( R(f) \), the closed-form expression of the action:
  \[
  p(f) = \frac{1}{2as} R(f) - \frac{b}{2a} + \frac{vf(\overline{R} - bs)}{2as(2as - v \overline{f})}
  \]
Solving the Leader Game

Follower Game

Optimal data update frequency:

\[ p(f) = \frac{1}{2as} R(f) - \frac{b}{2a} + \frac{vf(R - bs)}{2as(2as - vf)} \]

Substituting to \( \Phi \)

\[ \phi = E[\Phi] = E[\eta \sum_{i=1}^{N} (cp_i - dp_i^2) - \sum_{i=1}^{N} R_i p_i] \]
\[ = N \sum_{f \in G} F(f) [(\eta c - R(f))p(f) - \eta dp^2(f)] \]

Maximize \( \phi(R(f)) \)

Subject to

\[ g(R(f)) = \delta_f(R(f)) - \varepsilon \leq 0, \]
\[ g'(R(f)) = N \sum_f F(f)p(f) - \hat{p} \leq 0 \]

Lagrangian function

\[ L(R(f), \zeta) = \phi(R(f)) + \zeta_1 g(R(f)) + \zeta_2 g'(R(f)) \]
Algorithm 1: The AIAI mechanism

**Input:** degree distribution $F(f)$, worker $i$'s degree $f$, and some public parameters $a, b, c, d, \eta, s$.

**Output:** $R^*(f), p^*(f), \Phi^*$, and $\Omega_i^*$.

1. **Platform:** Determine its tentative optimal strategy (i.e., the unit-reward $R^*(f)$) according to Eq. (31);

2. **for each worker** $i = 1 \in N$ **do**
   - Determine its tentative strategy (i.e., the data update frequency $p_i^*(f)$) based on $R^*(f)$ and Eq. (32);

3. **if** $\delta_i(p_i, P_{-i}) \leq \varepsilon$ **for** $\forall i$ **then**
   - **if** $\sum_{i=1}^{N} p_i < \hat{p}$ **then**
     - **Platform:** Obtain $\Phi^*$ according to Eq. (33);
     - **Worker** $i$: Obtain $\Omega_i^* $ according to Eq. (34);
     - **else** Solving Eq. (36) and $g'(R(f))=0 \Rightarrow R^*(f)$;
       - **Platform:** Update its strategy as $R^*(f)$;
       - **Worker** $i$: Update $p_i^*(f)$ based on $R^*(f)$;
       - Calculate $\Phi^*$ and $\Omega_i^*$ based on Eqs. (33) and (34);
   - **else**
     - **if** $\sum_{i=1}^{N} p_i \leq \hat{p}$ **then**
       - Solving Eq. (35) and $\partial \mathcal{L} / \partial R(f) = 0 \Rightarrow R^*(f)$;
     - **else** Solving Eq. (37) $\Rightarrow R^*(f)$;

4. **Platform and Workers:** Update $p_i^*(f), R^*(f), \Phi^*, \Omega_i^*$.
Equilibrium Analysis

**Lemma**

✓ The follower game exists at least one pure **Bayesian Nash Equilibrium**.

Proof:
\[
\begin{align*}
\frac{\partial^2 \omega_i(p_i, P_{-i}, R)}{\partial p_i \partial P_{-i}} &= vf > 0, \\
\frac{\partial^2 \omega_i(p_i, P_{-i}, R)}{\partial p_i^2} &= -2as < 0.
\end{align*}
\]

The follower game meets the Single Crossing Property of Incremental Returns.

**Theorem**

✓ The optimal incentive strategy \((R^*(f), p^*(f))\) determined by the AIAI mechanism constitutes the **unique Stackelberg equilibrium** while satisfying AoI constraints.

Proof:

- The platform in Stage I uniquely determines \(R^*(f)\): only associated with the constant input.
- Given \(R^*(f)\), workers can pick their optimal data update strategies.
- No one can improve its own utility by deviating from the optimal strategy during the process.
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Evaluation

Experimental Settings

Real Dataset
- Chicago Taxi Trips
taxi driver → worker
- SNAP (Gowalla)
  mobile users → social friendship

Parameter settings
- \( N \) ranges from \([50, 300]\)
- The conversion parameters \( s \) and \( \eta \) change from \([6, 20]\) and \([7, 10]\)
- \( a=5, b=1, c=40, d=5, s=6 \)

Compared Algorithms
- Auction-based algorithm
- Contract-based algorithm
- AIAI-NS mechanism

Evaluation Metrics
- AoI
- Worker/Platform’s Strategy
- Worker/Platform’s Utility
The existence of the Stackelberg equilibrium for the platform:

(a) quadratic parameter $d$

(b) conversion parameter $\eta$

Influence of the strategy of the platform under different $d$

Influence of the strategy of the platform under $\eta$
The existence of the Stackelberg equilibrium for workers:

Influence of the strategy of the worker under different $a$

Influence of the strategy of the worker under $s$
Evaluation

- Influence of the number of workers and the strategy of the platform:

  Utility of the platform under different $N$
  $\Rightarrow$ Increasing $N$ can improve the profit of the platform

  Utility of the worker under different $PS$
  $\Rightarrow$ Investing more money can incentivize each worker to update data
Evaluation

- Different incentive mechanisms and varied parameters

(a) PU vs. $\hat{p}$

(b) total SU vs. $\hat{p}$

(c) PU vs. $\eta$

(d) total SU vs. $a$
We investigate the MCS incentive mechanism design issue with AoI guarantee and social benefits.

We model the problem as a two-stage Stackelberg game, embedded with an incomplete information Bayesian sub-game.

We derive the optimal strategies of this game and prove that these optimal strategies form a unique Stackelberg equilibrium.

We propose the AIAI mechanism -- the platform and workers can obtain their optimal utilities; -- meet the AoI constraint.

Extensive simulations on real-world traces validate its great performance.
Thank you for your attention!

Question?

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