AoI-aware Incentive Mechanism for Mobile Crowdsensing using Stackelberg Game

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***Abstract*—Mobile CrowdSensing (MCS) is a mobile computing paradigm, through which a platform can coordinate a crowd of workers to accomplish large-scale data collection tasks using their mobile devices. Information freshness has attracted much focus on MCS research worldwide. In this paper, we investigate the incentive mechanism design in MCS systems that take the freshness of collected data and social benefits into concerns. First, we introduce the Age of Information (AoI) metric to measure the freshness of data. Then, we model the incentive mechanism design with AoI guarantees as a novel incomplete information two-stage Stackelberg game with multiple constraints. Next, we derive the optimal strategies of this game so as to determine the optimal reward paid by the platform and the optimal data update frequency for each worker. Moreover, we prove that these optimal strategies form a unique Stackelberg equilibrium. Based on the optimal strategies, we propose an AoI-Aware Incentive (AIAI) mechanism for the MCS system, whereby the platform and all workers can maximize their utilities simultaneously. Meanwhile, the system can ensure that the AoI values of all data uploaded to the platform are not larger than a given threshold to achieve high data freshness. Extensive simulations on real-world traces are conducted to demonstrate the significant performance of AIAI.**

***Index Terms*—Mobile Crowdsensing, Incentive Mechanism, Age of Information, Stackelberg Game**

I. Introduction

With the explosive spread of smart mobile devices, Mobile CrowdSensing (MCS) has become an attractive paradigm of data sensing and collection. A typical MCS system consists of a collection of mobile users (a.k.a., workers) and a cloud platform, through which service applicants can publish their sensing tasks and employ workers to finish these tasks by using their mobile devices (e.g., smartphones, wearables, etc.) [1]–[4]. Due to users' distribution and mobility as well as the diversity of sensors embedded in mobile devices, MCS can accomplish many large-scale sensing tasks that individuals cannot cope with. Thus, it has stimulated extensive applications, such as traffic data collection, air pollution monitoring, seismic amplitude sensing, etc [5]. Moreover, much effort has been devoted to investigating diverse MCS issues, including task allocation [6]–[8], privacy-preserving approaches [9]–[11], incentive mechanism design [12]–[15], and so on.

In this paper, we focus on the incentive mechanism design for MCS with the concerns on the freshness of sensing data and workers' social benefits. Consider such a scenario that the platform incentivizes some workers via social networks to periodically collect the desired data (e.g., traffic data) from a group of Points of Interest (PoIs), so as to provide data services for MCS requesters [4], [5]. Since data valuation and service quality largely depend on the timeliness of data, the platform will try to collect the data with sufficient freshness. More precisely, the platform will ensure that the Age of Information (AoI) values of all collected data are not larger than a certain threshold. Here, AoI, i.e., the elapsed time of data from being collected by the worker to being received and processed by the platform currently, is a widely-used application-independent metric to indicate data freshness [16]–[19]. On the other hand, the workers in the MCS system are social network users so that they can share their collected data with each other to obtain extra social benefits [20]–[23]. For example, when a worker's trajectory of collecting data covers its social neighbors' PoIs, it might piggyback to collect data for the neighbors, which can save their data collection time and costs as well as improve its own social reputation. Then, an important issue is how to design the incentive mechanism to maximize the utilities (i.e., net profits) of the platform and workers simultaneously while taking the above two concerns into consideration.

There are two major challenges in the above-mentioned incentive mechanism design issue. First, the platform wishes to collect fresh data as much as possible, so the system needs to stimulate workers to frequently update their collected data on the platform (i.e., collect and upload the latest data copy to the platform). When workers increase their data update frequencies, the collected data will become fresher and the corresponding AoI values will become smaller, yielding higher data valuation and service quality. However, it will also incur larger data collection costs for workers. Therefore, there must be an optimal trade-off on the data update frequencies of all workers that needs to be addressed. Second, when workers frequently update their data, it might produce a congestion due to the limited communication and processing capabilities of the platform. Thus, there exists a game among workers to compete for data update through queueing, which needs to be addressed in designing the payment strategy of the incentive mechanism. Meanwhile, since workers can bring extra social benefits to the whole system by sharing their data with each other, their social relationships need also to be considered. However, the social relationships are generally unknown or incomplete to the platform in many real-world MCS applications. Therefore, the incentive mechanism involves an incomplete information game, making the design much more challenging.

So far, a wide spectrum of incentive mechanisms has been designed for MCS systems by leveraging different technologies, including auction theory, Stackelberg game, Bayesian game, etc. [12]–[14], [24]–[26]. However, most of them have not discussed the freshness of collected data. Only a few works have studied the pricing issue with AoI concerns [16]–[19], e.g., the authors in [18] proposed a linear AoI-based reward mechanism. However, none of these works investigate the incentive mechanism design with the concerns of AoI and workers' social benefits, which actually involves an incomplete information game with the AoI constraint. Besides, although there have been many researches that devote their efforts to addressing diverse AoI optimization problems [27]–[30], these solutions also cannot be applied to deal with our problem.

To address the above challenges, we model the incentive mechanism design issue as a two-stage Stackelberg game, in which the platform is seen as the leader and workers are treated as the followers. Meanwhile, we model the data update competition among the workers with social benefits as an incomplete information Bayesian sub-game. Besides, we also append the AoI constraint for workers' data collection as well as the constraint on the total data update frequency incurred by the limited resources on the platform side to the Stackelberg game. By means of the Karush-Kuhn-Tucker (KKT) conditions and a backward deduction approach, we derive an optimal solution for this incomplete information Stackelberg game with constraints, based on which we further propose an AoI-Aware Incentive (AIAI) mechanism. More specifically, our major contributions are summarized as follows:

- We propose the AIAI mechanism for MCS systems, which not only can ensure the platform and all workers to obtain their maximum utilities, but also can make the AoI value of each worker's data uploaded to the platform no larger than a given threshold. To the best of our knowledge, this is the first MCS incentive mechanism that takes the AoI values of data and workers' social benefits into account simultaneously.
- We introduce the AoI metric into the incentive mechanism design of MCS systems, and derive the closed-form expression for the AoI of the data that each worker uploads to the platform where workers' social influences with each other are also taken into consideration.
- We model the incentive mechanism design problem as a novel incomplete information two-stage Stackelberg game with constraints. We derive the optimal strategies for this Stackelberg game to determine the reward and the data update frequency for each worker. Moreover, we prove that the optimal strategy forms a unique Stackelberg equilibrium.
- We conduct extensive simulations on real-world traces to demonstrate the significant performance of AIAI.

II. System Model & Problem

A. System Model

We consider a typical MCS system, which consists of a platform on the cloud and a crowd of workers, as illustrated

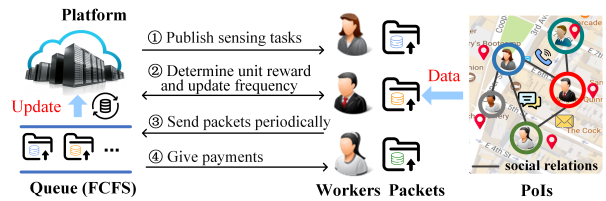


Fig. 1. System Overview

in Fig. 1. The platform has a long-term sensing task, e.g., collecting the latest traffic data from some PoIs. The workers, denoted by $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$, are some social network users who are willing to share data with each other to obtain extra social benefits, e.g., save data collection time and cost by piggyback, improve social reputation, and so on [20]–[23]. In the beginning, the platform publicizes the task and the corresponding requirements to all workers. Next, the platform and workers will jointly determine an incentive mechanism. More specifically, they will determine the data update frequency for each worker and the payment strategy. After that, each worker will perform the sensing tasks and will continuously collect data from some specified PoIs. Note that, the data is packed into packets of fixed-size. Meanwhile, the worker will collect and upload the latest data to the platform with the update frequency determined in advance. The platform will repeatedly receive the data uploaded from each worker, conduct a data cleaning process, and use the cleaned data to update the last version of this data (or directly store this cleaned data if it is the first version). Finally, the platform will periodically pay the reward to each worker according to a pre-determined strategy, until the whole sensing task is completed.

In the above MCS system, the platform will store the latest data copy uploaded by each worker in its cache. Meanwhile, to avoid conflicts, the platform will maintain a queue for cleaning the data from different workers. The queue of data cleaning adopts the First-Come-First-Service (FCFS) strategy. In order to avoid congestion in the queue, the platform needs to ensure that the total data updating frequency is not larger than a specified threshold. In addition, the platform wishes the collected data as fresh as possible. Therefore, it will record the AoI value of data uploaded by each worker and also try to keep the AoI values within a given threshold. For clarification, we define some important concepts and notations as follows.

Definition 1 (***Data Update Frequency and Total Frequency***)**.** The data update frequency of worker i refers to the frequency that the worker collects and uploads the data to the platform, denoted by p_i. We let $P \triangleq (p_1, p_2, \ldots, p_N)$ and P_{-i} ($i \in \mathcal{N}$) denote the data update frequencies of all workers and all workers except worker i, respectively. Moreover, we assume that the total data update frequency of all workers is not larger than a constant $\hat{p}$, i.e., $\sum_{i=1}^{N} p_i \le \hat{p}$.

Definition 2 (***Unit-Reward***)**.** The reward that the platform pays to each worker i ($i \in \mathcal{N}$) is proportional to its data update frequency. We call the reward per data update frequency paid to worker i as the unit-reward, denoted as R_i. Let $\mathcal{R} = \{R_1, R_2, ..., R_N\}$ be the unit-rewards to all workers.

Definition 3 (***Age of Information, AoI***)**.** AoI of data is the time elapsed since the worker collects this data. More specifically, AoI of the data that a worker i ($i \in \mathcal{N}$) uploads to the platform (also called *worker i's data*, or *data i*, for short) is actually the difference between the current time t and the creation time $U_i(t)$ of this data, defined as follows:

$$\delta_i(t) = t - U_i(t). \tag{1}$$

Note that, each worker may need to collect data from multiple PoIs along a planned trajectory, so the collection time should be considered in $\delta_i(t)$. Moreover, if data i has been updated to the platform multiple times, $U_i(t)$ actually refers to the creation time of the latest version of this data.

Definition 4 (***Average AoI and AoI Threshold***)**.** Since the AoI of data might change over time, the average AoI will be adopted in practice. For a time interval of observation $(0, T)$, we define the average AoI of data i as follows:

$$\bar{\delta}_i = \frac{1}{T}\int_0^T \delta_i(t)dt. \tag{2}$$

Actually, $\bar{\delta}_i, \forall i \in \mathcal{N}$ can be seen as a function of p_i and P_{-i}, so we will occasionally adopt the notation $\delta_i(p_i, P_{-i})$ to replace $\bar{\delta}_i$ for clarify. In addition, we set a threshold ε to restrict the average AoI of each data.

Definition 5 (***Social Benefits*** [20]–[23], [25])**.** The workers are assumed to be social network users, so that they can share data with each other via the social network and gain the social benefits from shared data. More specifically, a worker can receive an extra income caused by the data shared by its social network neighbors (See Eq. (3)). Meanwhile, the worker might also ask neighbors to piggyback its desired data which are exactly close to them so as to save the data collection time (See Theorem 1). Here, the social network can be presented by an adjacency matrix $[\upsilon_{ij}]_{N\times N}$, where υ_{ij} indicates the social network influence of worker j on worker i. Without loss of generality, we exclude isolated workers and assume $\upsilon_{ij} = \upsilon_{ji}$, $\upsilon_{ii} = 0$ due to the reciprocity of social relations.

B. Problem Formalization

In the above MCS system, we need to determine the data updating frequency for each worker and the unit-reward for the platform. For formulating this problem, we first define the utilities of each worker and the platform as follows:

Worker's Utility: The utility of worker i refers to the net profit of this worker, which can be defined as follows:

$$\Omega_i(p_i, P_{-i}) = R(p_i) + \Psi_i(p_i, P_{-i}) - \theta_i(p_i). \tag{3}$$

In Eq. (3), the first term $R(p_i) = R_i p_i$ represents the reward that the platform pays to worker i. The second term $\Psi_i(p_i, P_{-i})$ refers to the social benefits. Like in [20], [22], [23], we adopt $\psi_i = \sum_{j\in\mathcal{N}_i} \upsilon_{ij} p_j$ and $\Psi_i(p_i, P_{-i}) = \sum_{j\in\mathcal{N}_i} \upsilon_{ij} p_i p_j$ to model the social network effects and social benefits of worker i respectively, where $\mathcal{N}_i$ denotes the set of all socially-connected neighbors of worker i. The third term $\theta_i(p_i)$ is the cost function of worker i, which is assumed to be monotonically increasing, differentiable, and strictly convex. In this paper, we adopt a widely-used quadratic cost function with coefficients $a \geq 0, b \geq 0$ like in [31]–[33], i.e., $\theta_i(p_i) = s(ap_i^2 + bp_i)$. The tunable parameter s is used for signifying the equivalent monetary worth.

Platform's Utility: The platform's utility is the income that it can gain from all collected data minus the total rewards paid to workers, which can be defined as follows:

$$\Phi = \eta \sum\nolimits_{i=1}^{N} (cp_i - dp_i^2) - \sum\nolimits_{i=1}^{N} R_i p_i, \tag{4}$$

where η is also a tunable parameter denoting the equivalent monetary worth. The first term is the income of the platform, which can be seen as a function of the data update frequencies of all workers. As in [21], [31], [33], we adopt a linear-quadratic function, i.e., $cp_i - dp_i^2$, in which $c > 0$ and $d > 0$ are coefficients characterizing the concavity extent of the function to capture the property of decreasing marginal returns. The second term is the sum of rewards paid to all workers.

After defining the utility functions of workers and the platform, we model the AoI-aware incentive mechanism problem as a two-stage Stackelberg game, where the platform is the leader and the workers are the followers, defined as follows:

$$\textbf{Stage I (\textit{Leader Game}):} \quad \Phi(p_i^*, R_i^*) \geq \Phi(p_i, R_i) \tag{5}$$

$$\textbf{Stage II (\textit{Follower Game}):} \quad \Omega_i(p_i^*, R_i^*) \geq \Omega_i(p_i, R_i) \tag{6}$$

$$\textit{Subject to}: \quad \delta_i(p_i, P_{-i}) \leq \varepsilon, \quad \forall i \in \mathcal{N} \tag{7}$$

$$\sum\nolimits_{i=1}^{N} p_i \leq \hat{p}. \tag{8}$$

In Eqs. (5) and (6), p_i^* and R_i^* represent the optimal data update frequency of worker i and the optimal unit-reward paid by the platform to worker i, respectively. These optimal strategies form a Stackelberg Equilibrium (SE) together. The SE state shows that no one can improve its own utility by deviating from the optimal strategy during the process. The constraint of Eq. (7) means that the AoI value of each worker's data is not larger than the given threshold. Eq. (8) indicates the constraint of total data update frequency.

In stage II, the worker i needs to derive the optimal strategy p_i^* from Eq. (3). However, the second term $\Psi_i(p_i, P_{-i})$ is unknown owing to the uncertainty of social network effects. As a result, the optimal strategy cannot be derived directly. Faced with the problem, we turn it into a Bayesian game with incomplete information, which can be modeled as follows.

Incomplete Information Bayesian Sub-Game (for Stage II): The follower game can be formulated as an N-player Bayesian sub-game with social network effects, expressed as follows:

- The set of players $\mathcal{N}$ is a set of N workers;
- The action of player i is the data update frequency p_i;
- The type of player i is the social network effects ψ_i;
- The payoff of player i corresponding to its type ψ_i and its action p_i is the utility $\Omega_i(p_i, P_{-i})$;
- The strategy of player i is a function stating the action p_i for each type ψ_i at the unit-reward R_i, denoted by $\mathit{\Gamma}_i$: $\varphi_i \times \mathcal{R} \to \mathcal{P}_i$, where φ_i and $\mathcal{P}_i$ are the type space and action space of the worker i, respectively.

In summary, we model the AoI-aware incentive mechanism design into a two-stage Stackelberg game, in which the second stage is embedded with an incomplete information Bayesian sub-game. Moreover, the Stackelberg game is attached with

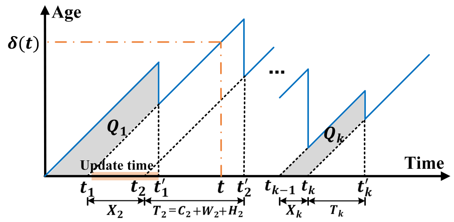


Fig. 2. Example of the AoI

two constraints. Consequently, our problem is transformed into acquiring the optimal strategies p_i^* and R_i^*.

III. Characterizing AoI of Data

In this section, we derive the AoI of data as a premise for solving the two-stage Stackelberg game, in which the impact of the social network on AoI is taken into account. First, we derive the average AoI for the case of a single worker, as a building block. Then, we extend it to the case of multiple workers and calculate the closed-form expression of the AoI of data, which will be applied directly in the next section.

A. Building Block: AoI of Data for A Single Worker

In this subsection, we deduce the closed-form expression of average AoI of data in a queue system with a single source, in which the uploaded data can be seen as coming from a worker. First, we illustrate a sample curve of AoI along with time in Fig. 2. Since AoI is actually the sum of the time the packet collected by the worker, the time it waited in the queue, and the time it spent in the data cleaning process, the curve increases linearly with time when no update completes and is reset to a smaller value upon reception of a new data update.

The average AoI defined in Eq. (2) can be regarded as the area under the curve, which is calculated by the sum of the disjoint geometric part identified by $Q_k, k=\{1,2,...,I(T)\}$. Here, $I(T)$ indicates the number of data packets until the time T. We use $T_k=t'_k-t_k$ to represent the system time of the update, where t_k is the creation time and t'_k is the update time. Through accumulating all areas under the curve, we have

$$\bar{\delta}^T=\frac{Q_1+T_{I(T)}^2/2}{T}+\frac{I(T)-1}{T}\frac{1}{I(T)-1}\sum_{k=2}^{I(T)}Q_k. \qquad (9)$$

From Fig. 2, we observe that the area Q_k can be calculated as the difference between the areas of two isosceles triangles. We define the interarrival time X_k as t_k-t_{k-1}. Then, we have $Q_k=\frac{1}{2}(T_k+X_k)^2-\frac{1}{2}T_k^2$ and further get

$$\bar{\delta}^T=\frac{Q_1+T_{I(T)}^2/2}{T}+\frac{I(T)-1}{T}\frac{1}{I(T)-1}\sum_{k=2}^{I(T)}\left[X_kT_k+\frac{X_k^2}{2}\right].$$

We notice that $Q_1+T_{I(T)}^2/2$ represents a boundary effect that is finite with probability 1, and thus the first term in the above equality will vanish along with the growth of T. Let $p=\lim_{T\to\infty}I(T)/T$ indicate the data update frequency in the steady state. What's more, the remaining term will converge to the corresponding expected value when $I(T)$ approaches to infinity. Hence, we obtain the average AoI of data:

$$\bar{\delta}=lim_{T\to\infty}\bar{\delta}^T=p(\mathbb{E}[XT]+\mathbb{E}[X^2/2]), \qquad (10)$$

where $\mathbb{E}(\cdot)$ is the expectation operator. X and T are the random variables that correspond to the interarrival time and the system time of an update packet, respectively.

Now, we can derive the average AoI of a single worker's data. Consider an $M/M/1$ FCFS queue system with only one source, which contains three critical parameters, i.e., collection time β, data update frequency (a.k.a., arriving rate) p, and serving rate μ. Moreover, the arriving time and serving time follow the Poisson distributions of $1/p$ and $1/\mu$, respectively, and the offered load is denoted by $\rho=p/\mu$. Inspired by [34], the average AoI of data can be easily derived using the queuing theory, i.e., $\bar{\delta}=[(\rho-1)(\rho^2-\mu\rho\beta)+1]/((1-\rho)\rho\mu)$.

B. AoI of Data for Multiple Workers

We extend the above closed-form expression of average AoI of data to the case of multiple workers, in which the AoI value of each worker may be affected by its neighbors. Consider the FCFS $M/M/1$ system with worker i's data update frequency p_i, collection time β_i, serving rate μ, and offered load $\rho_i=p_i/\mu$. The platform hopes to keep data freshness and stimulates workers to update data frequently. However, when worker i continuously sends its data with a high frequency, the AoI value for each other $N-1$ workers might have a rapid increase since it is a competitive N sources queue system with AoI constraints. Thus, the AoI values of some workers' data may exceed the specified threshold ε. To guarantee that each worker's data can satisfy the constraints in Eqs. (7) and (8), we first need to derive the average AoI of each worker's data under the multi-source MCS system and the social network, as shown in the following theorem.

Theorem 1. ***(AoI for Multiple Workers)*** *N workers compete for the data update through an $M/M/1$ FCFS queue, in which each worker i's data update frequency, collection time, serving rate, and offered loads are p_i, β_i, μ, and ρ_i, respectively. Then, the average AoI $\bar{\delta}_i$ of worker i's data satisfies*

$$\bar{\delta}_i=\frac{\alpha\beta_i}{\sum_{j\in\mathcal{N}_i}\upsilon_{i,j}}+\frac{p_i/\mu^2}{1-\rho_{-i}}\left[\frac{\rho_i\rho_{-i}}{(1-\rho_{-i})^2}+\frac{\rho_i/(1-\rho)}{1-\rho_{-i}}+\frac{\rho_{-i}}{\rho_i}\right]+\frac{1}{\mu}+\frac{1}{p_i},$$

where $\rho=\sum_{i=1}^{N}\rho_i$, $\rho_i=p_i/\mu$, *and* $\rho_{-i}=\sum_{j\neq i}\rho_j$. (11)

Proof. The system time of an update T_k can be expressed by $C_k+W_k+H_k$, where C_k, W_k, and H_k represent the collection time, the whole waiting time in the system, and the handling time for data cleaning, respectively. Thus, we have

$$\begin{aligned}\mathbb{E}(X_kT_k)&=\mathbb{E}(X_kC_k)+\mathbb{E}(X_kW_k)+\mathbb{E}(X_kH_k)\\&=\mathbb{E}(X_k)\mathbb{E}(C_k)+\mathbb{E}(X_kW_k)+\mathbb{E}(X_k)\mathbb{E}(H_k).\end{aligned} \qquad (12)$$

The second equality holds since C_k and H_k is independent of X_k. If worker i collects data by itself from multiple PoIs, it will consume the collection time β_i. Consider that workers can share data via the social network. Worker i might obtain some data from its neighbors, and thus the expected collection time can be expressed as $\mathbb{E}(C_k)=\alpha\beta_i/\sum_{j\in\mathcal{N}_i}\upsilon_{i,j}$. Here, α is a constant coefficient. Meanwhile, we already know that there are $\mathbb{E}(H_k)=1/\mu, \mathbb{E}(X_k)=1/p_i$, and $\mathbb{E}(X_k^2)=2/p_i^2$. Then, we substitute these equations and Eq. (12) into Eq. (10) to rewrite the average AoI of worker i's data as follows.

$$\bar{\delta}_i=\alpha\beta_i/\sum_{j\in\mathcal{N}_i}\upsilon_{i,j}+p_i\mathbb{E}(X_kW_k)+1/\mu+1/p_i. \qquad (13)$$

To compute $\mathbb{E}(X_kW_k)$, we consider two cases: $A_k=\{X_k<T_{k-1}\}$ and $G_k=\{T_{k-1}<X_k\}$. Next, we rewrite $\mathbb{E}(X_kW_k)$

as $\mathbb{E}(X_kW_k) = \mathbb{E}(X_kW_k|G_k)P[G_k] + \mathbb{E}(X_kW_k|A_k)P[A_k]$. Inspired by the work [34], we can derive that $\mathbb{E}(X_kW_k|A_k) = \mathbb{E}_1+\mathbb{E}_2$, with $\mathbb{E}_1 = \mathbb{E}[X_k(T_{k-1}-X_k)|A_k] = 1/(\mu^2(1-\rho)(1-\rho_{-i}))$ and $\mathbb{E}_2 = 2\rho_{-i}/(\mu^2(1-\rho_{-i})^2)$. That is, we have

$$\mathbb{E}(X_kW_k|A_k) = (\rho_{-i} - 2\rho\rho_{-i} + 1)/((1-\rho_{-i})^2(1-\rho)\mu^2),$$
$$\mathbb{E}(X_kW_k|G_k) = (1/(\mu - \mu\rho_{-i}) + 1/p_i)(\rho_{-i}/(\mu(1-\rho_{-i}))).$$

Based on the above equalities and the probability $P[A_i] = \rho_i/(1-\rho_{-i})$, Eq. (12) can be deduced as follows.

$$\mathbb{E}(X_kW_k)=\frac{1}{\mu^2(1-\rho_{-i})}\left[\frac{\rho_i\rho_{-i}}{(1-\rho_{-i})^2}+\frac{\rho_i}{(1-\rho)(1-\rho_{-i})}+\frac{\rho_{-i}}{\rho_i}\right]. \quad (14)$$

Finally, we substitute Eq. (14) into Eq. (13) and get the average AoI of worker i's data (i.e., Eq. (11)). □

IV. AoI-Aware Incentive Mechanism

In this section, we propose the AoI-aware incentive mechanism, which exploits the backward induction approach to solve the incomplete information two-stage Stackelberg game with AoI constraints. First, we solve the follower game (i.e., Stage II) including the incomplete social benefit information, through which each worker i can determine its optimal data update frequency p_i^* under a given unit-reward R_i. Then, we turn to the leader game (i.e., Stage I) to derive the optimal unit-reward R_i^* paid by the platform. Finally, we prove that these optimal strategies form a unique Stackelberg equilibrium.

A. Solving the Bayesian Sub-Game

In Stage II, each worker's type (i.e., its social network effects) is uncertain to other workers, forming an incomplete information Bayesian sub-game. Due to the uncertainty of workers' types, we cannot obtain the expected utility for each type of worker directly. Fortunately, many researches have shown that workers' degrees in the social network can indicate their social influences [21]–[23]. Thus, we turn to use each worker's degree in the social network to derive its utility since the distribution of each worker's degree is public and known. Based on this idea, we can derive the expected utility function and then determine the closed-form expression of the optimal data update frequency for each worker.

First, we derive the expected utility of each worker according to Eq. (3). Specifically, we set the social network influence $\upsilon_{ij} = \upsilon$ for all i, j $(i \neq j)$ without loss of generality, where υ is a given social network effect coefficient. Note that, υ can be treated as a variable instead of an exact value. Then, given the reward vector $\mathcal{R}$, the expected utility of worker i is

$$\omega_i(p_i, P_{-i}, \mathcal{R}) = \mathbb{E}[\Omega_i(p_i, P_{-i}), \mathcal{R}]$$
$$= R_ip_i + \upsilon p_i\mathbb{E}[\sum\nolimits_{j\in\mathcal{N}_i} p_j] - (ap_i^2 + bp_i)s. \quad (15)$$

Then, in order to determine the optimal data update frequency p_i^* which can maximize $\omega_i(p_i^*, P_{-i}, \mathcal{R})$, we introduce the graph theory and leverage the degree to describe the type of each worker. Specifically, we model the social network as an undirected graph, whose structure can be described using different degrees. The degree of worker i is denoted as $f \in G$, where $G = \{1, \ldots, f^{max}\}$ and f^{max} is the maximum value of the degree. Let F be the probability distribution of the degree, denoted by $F : G \to [0, 1]$, where $\sum_{f\in G} F(f) = 1$. To a certain extent, the distribution of the degree captures the social network effects from the network interaction patterns. Therefore, we can gain $E[\sum_{j\in\mathcal{N}_i} p_j] = f \times \overline{P_{-i}}$, where f represents the degree of worker i and $\overline{P_{-i}}$ is the average data update frequency of worker i's neighbors. Based on this, each worker's type can be transformed into the degree. Then, the utility of worker i in Eq. (15) can be rewritten as:

$$\omega_i(p_i, P_{-i}, \mathcal{R}) = R_ip_i + \upsilon p_i f\overline{P_{-i}} - (ap_i^2 + bp_i)s. \quad (16)$$

Next, we need to figure out how to express $\overline{P_{-i}}$. Inspired by the work [35], we introduce the concept of "Configuration Model" in network science to model a randomly generated network. We concentrate on the Bayesian game with the symmetric type space, i.e., the workers with the same type f will choose the same data update frequency $p(f)$ and will be awarded the same reward $R(f)$ per data update frequency, which is a widely-adopted assumption in the social network studies [23], [36], [37]. According to the property of the configuration model and the Bayes' rule, for a worker, the degree distribution of its randomly selected neighbor is

$$\overline{F}(f) = F(f)f/(\sum\nolimits_{f'\in G} F(f')f') = F(f)f/\underline{f}, \quad (17)$$

where $\underline{f} = \sum_{f'\in G} F(f')f'$ indicates the mean value of the degrees of the whole social network. We can conclude that a randomly chosen social network neighbor of worker i has the degree distribution $\overline{F}(f)$. Hence, each worker can treat $\overline{F}(f)$ as its neighbors' degree distribution although they might not know the exact values of degrees. Based on Eq. (17), we can compute the average data update frequency of neighbors of a worker with degree f, denoted by $\overline{P_{-f}}$, i.e., $\overline{P_{-f}} = \sum_{f\in G} \overline{F}(f)p(f)$. Substituting it into Eq. (16), we can get that the utility of the worker with degree f satisfies:

$$\omega_f(p(f), P_{-f}) = R(f)p(f) + \upsilon p(f)f\overline{P_{-f}} - (ap^2(f) + bp(f))s. \quad (18)$$

Now, we can solve the Bayesian sub-game to determine the optimal strategies of workers so as to maximize the above utility function and achieve Bayesian Nash Equilibrium (BNE), which is shown in the following theorem.

Theorem 2. ***(Follower's Optimal Strategy).*** *Given any unit-reward $R(f)$, the closed-form expression of the action (i.e., data update frequency) of the follower Game is*

$$p(f) = \frac{1}{2as}R(f) - \frac{b}{2a} + \frac{\upsilon f(\overline{R} - bs)}{2as(2as - \upsilon\overline{f})}, \quad (19)$$

where $\overline{R} = \sum_{f\in G} \overline{F}(f)R(f)$ and $\overline{f} = \sum_{f\in G} \overline{F}(f)f$.

Proof. In order to acquire the closed-form solution of the unique BNE point of the follower game, we first apply the partial derivative of the expected utility in Eq. (18) and get

$$\frac{\partial\omega_f(p(f), P_{-f}, \mathcal{R})}{\partial p(f)} = R(f) + \upsilon f\overline{P_{-f}} - (2ap(f) + b)s. \quad (20)$$

Then, we let $\partial\omega_f(p(f), P_{-f}, \mathcal{R})/\partial p(f) = 0$ and obtain

$$p(f) = \frac{1}{2as}R(f) - \frac{b}{2a} + \frac{\upsilon f}{2as}\overline{P_{-f}}. \quad (21)$$

Since our social network is treated as a configuration model with dense types, we have the following approximation: $\overline{P_{-f}} = \mathbb{E}[p_j||j \in N_i] \approx \mathbb{E}[p(l)||l \in G]$. Thus, we have

$$p(f) = \frac{1}{2as}R(f) - \frac{b}{2a} + \frac{\upsilon f}{2as}\mathbb{E}[p(l)||l \in G]. \quad (22)$$

By plugging Eq. (22) into $\overline{P_{-f}} = \sum_{f\in G} \overline{F}(f)p(f)$, we get

$$\overline{P_{-f}}=\sum_{f\in G}\overline{F}(f)\left[\frac{R(f)}{2as}-\frac{b}{2a}+\frac{\upsilon f}{2as}\mathbb{E}[p(l)||l\in G]\right]$$
$$\Rightarrow \mathbb{E}[p(l)||l\in G]=\frac{1}{2as}\overline{R}-\frac{b}{2a}+\frac{\upsilon\overline{f}}{2as}\mathbb{E}[p(l)||l\in G],$$
$$\Rightarrow \overline{P_{-f}}\approx E[p(l)||l\in G]=(\overline{R}-bs)/(2as-\upsilon\overline{f}), \quad (23)$$

where $\overline{R}=\sum_{f\in G}\overline{F}(f)R(f)$ and $\overline{f}=\sum_{f\in G}\overline{F}(f)f$. By substituting Eq. (23) into Eq. (22), we can get the closed-form expression of data update frequency and finish the proof. □

In Eq. (19), in order to get the value of $p(f)$, a worker only needs to know its own type f and the type distribution of neighbors instead of the exact type values of other workers. As a result, each follower's optimal strategy with incomplete information is solved. It is worth noting that the follower's strategy $p(f)$ relies on the strategy $R(f)$ of the leader (i.e., the platform), so it is necessary to derive the optimal unit-reward for the platform in the next subsection.

B. Solving the Leader Game with Constraints

As the leader, the platform wants to maximize its expected utility $\mathbb{E}[\Phi]$ by finding the optimal unit-reward $R^*(f)$ for each worker. After applying the configuration model, the expected utility of the platform can be expressed as follows.

$$\phi=\mathbb{E}[\Phi]=\mathbb{E}[\eta\sum_{i=1}^{N}(cp_i-dp_i^2)-\sum_{i=1}^{N}R_ip_i]$$
$$=N\sum_{f\in G}F(f)\big[\big(\eta c-R(f)\big)p(f)-\eta dp^2(f)\big], \quad (24)$$

where $F(f)$ is known in advance. When taking the AoI constraint and the total data update frequency constraint into account simultaneously (i.e., Eqs. (7) and (8)), the optimization objective of the platform can be rewritten as follows:

$$\begin{aligned}&\textbf{max} \quad \phi(R(f))\\ &\text{s.t.} \quad g(R(f))=\delta_f(R(f))-\varepsilon\le 0,\\ &\qquad g'(R(f))=N\sum_{f}F(f)p(f)-\hat{p}\le 0. \end{aligned} \quad (25)$$

To find the optimal solution, we construct the Lagrangian function: $\mathcal{L}(R(f),\zeta)=\phi(R(f))+\zeta_1 g(R(f))+\zeta_2 g'(R(f))$, where ζ_1 and ζ_2 represent the Lagrangian multipliers. Since it is a convex optimization problem, the optimal solution must satisfy the Karush-Kuhn-Tucker (KKT) optimality conditions:

$$\partial\mathcal{L}/\partial R(f)|_{R(f)=R^*(f)}=0;\ \zeta_1 g(R(f))=0;\ \zeta_2 g'(R(f))=0;$$
$$g(R(f))\le 0;\ g'(R(f))\le 0;\ \zeta_1\le 0;\ \zeta_2\le 0. \quad (26)$$

To meet the KKT conditions, we consider four cases:

(i) Case 1: $\zeta_1=0,\zeta_2=0$. When the optimal solution of maximizing $\phi(R(f))$ just falls within the feasible region (not including the boundary), the limitation of the feasible region does not work. Therefore, we can solve Eq. (25) by letting the first-order derivative of $\phi(R(f))$ equal to zero directly. For ease of presentation, we define $\Delta=\frac{\upsilon}{2as(2as-\upsilon\overline{f})}$. Due to

$$\frac{\partial\overline{R}}{\partial R(f)}=\overline{F}(f),\ \frac{\partial p(l)}{\partial R(f)}=\frac{\upsilon l\overline{F}(f)}{2as(2as-\upsilon\overline{f})}=\Delta\overline{F}(f)l\ \ (l\neq f),$$
$$\frac{\partial p(f)}{\partial R(f)}=\frac{1}{2as}+\frac{\upsilon f\overline{F}(f)}{2as(2as-\upsilon\overline{f})}=\frac{1}{2as}+\Delta\overline{F}(f)f, \quad (27)$$

we can derive the derivation $\partial\phi/\partial R(f)$ as follows:

$$\frac{\partial\phi}{\partial R(f)}=NF(f)\big[-p(f)+\big(\eta c-R(f)-2\eta dp(f)\big)\big(\frac{1}{2as}+\Delta\overline{F}(f)f\big)\big]+N\sum_{l\neq f}F(l)\big[(\eta c-R(l)-2\eta dp(l))\Delta\overline{F}(f)f\big].$$

Since our social network is regarded as a configuration model with large numbers of workers, we have the approximation: $\sum_{l\neq f}F(l)\big[(\eta c-R(l)-2\eta dp(l))\Delta\overline{F}(f)f\big]=M\overline{F}(f)\sum_{l\in G}F(l)l\big(\eta c-R(l)-2\eta dp(l)\big)$. According to the definition formulas $\overline{R}=\sum_{f\in G}\overline{F}(f)R(f)$, $\overline{f}=\sum_{f\in G}\overline{F}(f)f$, and $\underline{f}=\sum_{f\in G}F(f)f$, we get $\sum_{f\in G}F(f)f^2=\overline{f}\underline{f}$. Moreover, we let $\Lambda=\sum_{f\in G}F(f)fR(f)$ for convenient presentation. Afterwards, we let $\partial\phi/\partial R(f)=0$ and acquire

$$\big[\frac{1}{2as}+(\Delta\overline{F}(f)f+\frac{1}{2as})(1+\frac{\eta d}{as})\big]F(f)R(f)$$
$$=F(f)\big[\frac{b}{2a}-\tilde{\Delta}f+(\Delta\overline{F}(f)f+\frac{1}{2as})\big(\eta c+\frac{b\eta d}{a}-2f\eta d\tilde{\Delta}\big)\big]$$
$$+\Delta\overline{F}(f)\big[-(1+\frac{\eta d}{as})\Lambda+\frac{\eta\underline{f}(ac+bd)}{a}-2\eta d\tilde{\Delta}\underline{f}\overline{f}\big], \quad (28)$$

where $\tilde{\Delta}=\Delta(\overline{R}-bs)$. Based on Eq. (28), we can gain the expression of $R(f)$ easily. However, the expression of $R(f)$ still contains unknown parameters $\overline{R}=\sum_{f\in G}\overline{F}(f)R(f)$ and $\Lambda=\sum_{f\in G}F(f)fR(f)$, which will be worked out below.

Because the degree distribution is known, we can directly calculate an algebraic expression of $\overline{F}(f)$. By substituting $R(f)$ into the definitions of $\overline{R}$ and Λ, we can get two equations which together formulate a system of linear equations with two unknowns, i.e., $\overline{R}$ and Λ. Thus, we can solve the system of simultaneous equations to get the closed expressions of $\overline{R}$ and Λ, defined as $\overline{R^*}$ and Λ^*. Thereafter, we substitute the closed expressions of $\overline{R^*}$ and Λ^* into Eq. (28), and then obtain the closed expression of $R(f)$ as follows:

$$R^*(f)=\frac{2a^2s^2}{(2as\Delta\overline{F}(f)f+1)(as+\eta d)+as}\big[\frac{b}{2a}-\Delta f(\overline{R^*}-bs)$$
$$+(\Delta\overline{F}(f)f+\frac{1}{2as})\big(\eta c+\frac{b\eta d}{a}-2\Delta f\eta d(\overline{R^*}-bs)\big)$$
$$+\Delta f\big(-(1+\frac{\eta d}{as})\frac{\Lambda^*}{\underline{f}}+\frac{\eta(ac+bd)}{a}-2\Delta\eta d(\overline{R^*}-bs)\overline{f}\big)\big]. \quad (29)$$

After the platform has determined the optimal unit-reward $R^*(f)$ according to Eq. (29), each worker i can also determine its own optimal data update frequency $p_i^*(f)$ by substituting $R^*(f)$ and $\overline{R^*}$ into Eq. (19). In short, the closed-form expression of the optimal data update frequency of worker i is

$$p_i^*(f)=\frac{1}{2as}R^*(f)-\frac{b}{2a}+\frac{\upsilon f(\overline{R^*}-bs)}{2as(2as-\upsilon\overline{f})}. \quad (30)$$

By plugging $R^*(f)$ and $p_i^*(f)$ into the utility functions (i.e., ϕ and ω_i), both the platform and each worker can reap their maximum expected utilities, denoted as Φ^* and Ω_i^*, i.e.,

$$\Phi^*=N\sum_{f\in G}F(f)\big[\big(\eta c-R^*(f)\big)p^*(f)-\eta dp^{*2}(f)\big]. \quad (31)$$
$$\Omega_i^*=R^*(f)p^*(f)+\upsilon p^*(f)f\overline{P_{-f}}-(ap^{*2}(f)+bp^*(f))s. \quad (32)$$

(ii) Case 2: $\zeta_1\neq 0,\zeta_2=0$. In this case, $R^*(f)$ and $p_i^*(f)$ derived by Case 1 are not eligible under the AoI constraint. Thus, we attain $R^*(f)$ by letting the $g(R(f))$ equal to 0, i.e.,

$$\frac{\alpha\beta}{f\upsilon}+\frac{p^2(f)}{\mu^2\check{\rho}}\left[\frac{\rho_{-i}(f)}{\mu(\check{\rho})^2}+\frac{1}{\check{\rho}\mu(1-\rho(f))}+\frac{\rho_{-i}(f)\mu}{p^2(f)}\right]+\frac{1}{\mu}+\frac{1}{p(f)}=\varepsilon, \quad (33)$$

where $\check{\rho}=1-\rho_{-i}(f)$, $\rho(f)=\sum_f p(f)/\mu$, $\rho_{-i}(f)=(\sum_f p(f)-p(f))/\mu$, and β is the identical collection time. Combining with $\partial\phi/\partial R(f)+\partial g/\partial R(f)=0$, we can derive $R^*(f)$ through solving Eq. (33) and further gain $p_i^*(f)$ as well.

(iii) Case 3: $\zeta_1=0,\zeta_2\neq 0$. In this case, $R^*(f)$ and $p_i^*(f)$

Algorithm 1: The AIAI mechanism

input : degree distribution $F(f)$, worker i's degree f, and some public parameters a, b, c, d, η, s;
output: $R^*(f)$, $p^*(f)$, Φ^*, and Ω_i^*;

```
1  Platform: Determine its tentative optimal strategy (i.e., the unit-reward R*(f)) according to Eq. (29);
2  for each worker i = 1 ∈ N do
3      Determine its tentative strategy (i.e., the data update frequency p_i*(f)) based on R*(f) and Eq. (30);
4  if δ_i(p_i, P_{-i}) ≤ ε for ∀i then
5      if Σ_{i=1}^N p_i ≤ p̂ then
6          Platform: Obtain Φ* according to Eq. (31);
7          Worker i: Obtain Ω_i* according to Eq. (32);
8      else Solving Eq. (34) and g'(R(f))=0 ⇒ R*(f);
9              Platform: Update its strategy as R*(f);
10             Worker i: Update p_i*(f) based on R*(f);
11             Calculate Φ* and Ω_i* based on Eqs. (31) and (32);
12 else
13     if Σ_{i=1}^N p_i ≤ p̂ then
14         Solving Eq. (33) and ∂L/∂R(f)=0 ⇒ R*(f) ;
15     else Solving Eq. (35) ⇒ R*(f);
16     Platform and Workers: Update p_i*(f), R*(f), Φ*, Ω_i*;
```

derived by Case 1 do not satisfy the total data update frequency constraint. Therefore, we calculate the derivation $\partial\phi/\partial R(f)+\zeta_2\partial g'/\partial R(f)$ and let it equal to zero. Then, we get

$$\frac{\eta cF(f)}{2as}-\frac{F(f)R(f)}{2as}-(1+\frac{\eta d}{as})F(f)p(f)+\Delta\overline{F}(f)(\eta c\underline{f}-\underline{f}\overline{R})$$
$$-2\eta d\Delta\overline{F}(f)\sum_l lF(l)p(l)+\zeta_2\partial g'/\partial R(f)=0. \quad (34)$$

Through solving Eq. (34), we obtain the optimal $R^*(f)$:

$$R^*(f)=\frac{as(\eta c-bs+\zeta_2)}{2as+\eta d}(1-\frac{f}{\underline{f}})+\frac{2asf\hat{p}}{N\underline{f}}+bs-2as\Delta f(\overline{R}-bs),$$

where ζ_2 can be derived by solving $g'(R(f))=0$.

(iv) Case 4: $\zeta_1\neq 0, \zeta_2\neq 0$. When the solutions in the above cases cannot satisfy Eq. (25), we need to solve the equations:

$$\frac{\partial\phi}{\partial R(f)}+\frac{\partial g}{\partial R(f)}+\frac{\partial g'}{\partial R(f)}=0;\ g(R(f))=0;\ g'(R(f))=0. \quad (35)$$

It is perplexing to obtain the closed-form $R^*(f)$ by solving Eq. (35), and we can adopt some mathematical approximating methods (e.g., the bisection, Newton's method, and so on) to acquire an approximation of $R^*(f)$.

C. The Detailed Algorithm Design

Based on the above idea, we propose the AoI-Aware Incentive (AIAI) mechanism, as illustrated in Algorithm 1. First, the leader (i.e., the platform) gives its strategy according to Eq. (29) (Step 1). Then, each follower (i.e., worker) determines its strategy based on the strategy of the platform (Steps 2-3). Next, the AoI of data can be calculated for multiple workers according to Theorem 1, and the platform needs to check whether the AoI of data is not larger than ε (Steps 4-16). In steps 5-7, if there is $\sum_{i=1}^N p_i\le\hat{p}$, we can directly obtain the maximum utilities of the platform and each worker according to Eqs. (31) and (32). Otherwise, the strategy of the platform will be adjusted according to diverse cases and the data update frequency of each worker will be updated accordingly (Steps 8-16). Moreover, the computation complexity is $O(N)$.

D. The Equilibrium Analysis

We analyze the Bayesian sub-game equilibrium and the Stackelberg game equilibrium in this subsection.

Lemma 1. *The follower game exists at least one pure BNE.*

Proof. The work in [38] has pointed out that, if the Bayesian sub-game satisfies the (Milgrom-Shannon) Single Crossing Property of Incremental Returns (SCP-IR), the Bayesian sub-game has at least one pure BNE. Based on Eq. (20), we have

$$\partial^2\omega_i(p_i,P_{-i},\mathcal{R})/\partial p_i\partial\overline{P_{-i}}=\upsilon f>0, \quad (36)$$
$$\partial^2\omega_i(p_i,P_{-i},\mathcal{R})/\partial p_i^2=-2as<0. \quad (37)$$

Therefore, the follower game in Stage II meets the SCP-IR and there exists at least one pure BNE. □

Lemma 2. *[39] For the Bayesian sub-game, there exists at most one equilibrium if the following condition is satisfied:*

$$\left|\frac{\partial^2\omega_i(p_i,P_{-i},\mathcal{R})}{\partial p_i\partial\overline{P_{-i}}}\Big/\frac{\partial^2\omega_i(p_i,P_{-i},\mathcal{R})}{\partial p_i^2}\right|<1,\forall i\in\mathcal{N}. \quad (38)$$

According to Lemmas 1 and 2, when the condition $\upsilon f^{max}-2as<0$ is satisfied, the uniqueness of the BNE of the follower Bayesian sub-game can be guaranteed. Based on this, we prove the existence of the unique Stackelberg equilibrium.

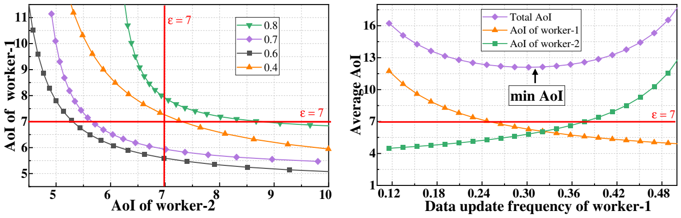
Theorem 3. *The optimal incentive strategy $(R^*(f),p^*(f))$ determined by the AIAI mechanism constitutes the unique Stackelberg equilibrium while satisfying AoI constraints.*

Proof. In the whole two-stage Stackelberg game, each stage can derive its optimal closed-form solution: the unit-reward strategy of the platform and the data update frequency strategies of workers. As the role of the leader in Stage I, the platform can uniquely determine $R^*(f)$ according to Section IV-B. It is worth mentioning that the value of $R^*(f)$ is calculated just by the known distribution and some public parameters. That is, $R^*(f)$ is only associated with the constant input without knowing workers' strategies and social structure information, and the platform cannot gain a larger utility if it uses other strategies. When $R^*(f)$ is determined, workers can pick their optimal strategies based on Eq. (30), and these strategies constitute the unique Bayesian sub-game equilibrium. In a word, each stage has a unique equilibrium under the optimal incentive strategy $(R^*(f),p^*(f))$, and no one can improve its own utility by deviating from the optimal strategy during the process. At last, Eq. (7) guarantees that the AoI values of all workers' data are not larger than the given threshold. Thus, we can conclude that the two-stage game of AIAI has the unique Stackelberg equilibrium while meeting AoI constraints. □

V. Performance Evaluations

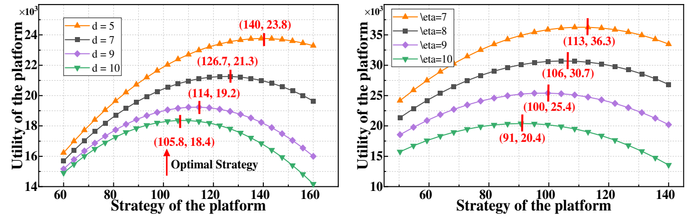
A. Evaluation Methodology

Simulation Settings: We perform our simulations on the real-world data of Chicago Taxi Trips [40]. Each trace records the taxi ID, timestamp, trip seconds, trip miles, pickup/dropoff



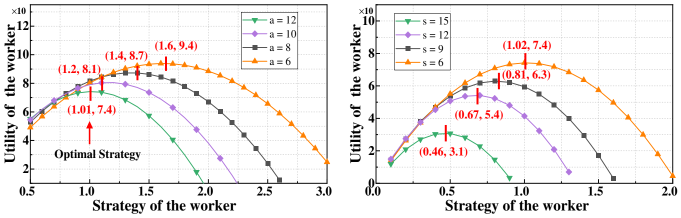
(a) worker-1's AoI vs. worker-2's AoI (b) total AoI and each worker's AoI

Fig. 3. AoI of two competitive workers



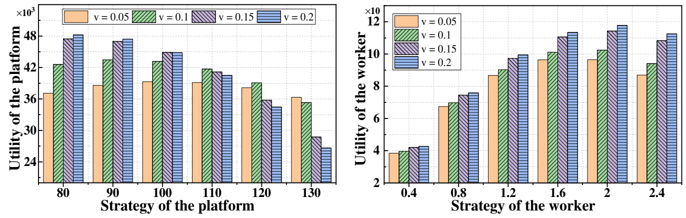
(a) quadratic parameter d (b) conversion parameter η

Fig. 4. Strategy of the platform vs. Utility



(a) quadratic parameter a (b) conversion parameter s

Fig. 5. Strategy of a worker vs. Utility



(a) utility of the platform (b) utility of the worker

Fig. 6. Influence of social network effects

areas, etc. We select a data set of 27055 taxi records. In our simulations, we select some taxi drivers as MCS workers and treat the taxi-hailing requests as sensing tasks. First, we choose 15 PoIs and find 300 taxis from the trace. Then, we choose N taxis as workers, where N ranges from $[50, 300]$. We also simulate the social network based on a real data trace from SNAP (Gowalla) [41], which is a location-based social friendship network built by mobile phone users. We randomly pick N nodes from the network and the social network effect coefficient υ is produced from $[0.01, 0.2]$. The conversion parameters s and η change from $[6, 20]$ and $[7, 10]$, respectively. Meanwhile, the quadratic parameters a and d are set in the range $[5, 15]$ and $[5, 10]$, respectively. Moreover, the default values are $a=5, b=1, c=40, d=5, s=6$, and $\eta=10$.

Compared Algorithms: Since AIAI combines the Stackelberg game and AoI to keep data freshness and solve the incentive problem with incomplete information, we compare AIAI with some existing state-of-the-art studies with incentive mechanism designs [12], [13]. However, the models and problems in these works are different from ours so we cannot compare them directly. Thus, we tailor the basic idea in these algorithms and carefully design three incentive mechanisms for comparison: Auction-based algorithm [13], Contract-based algorithm [12], and AIAI-NS. Here, the auction-based scheme is based on game theory, the contract-based algorithm utilizes the technique of contract theory, and the AIAI-NS mechanism means that we do not consider social network effects.

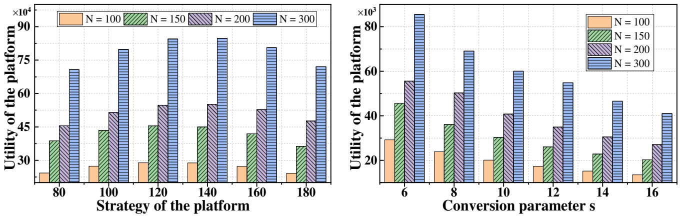
B. Evaluation Results

For the evaluation, we use the following main metrics: AoI, strategy, and utility. To be more precise, we use PU, WU, PS, and WS to denote the platform's utility, a worker's utility, the platform's strategy, and the worker's strategy, respectively.

1) *Evaluation of AoI:* We measure AoI for the $M/M/1$ FCFS queue system with $\mu = 1$. For simplicity, we set that there are only two workers competing for the data update with a fixed total load $\rho_1+\rho_2=\hat{\rho}$ and $\beta_1=\beta_2$. From Fig. 3(a), we can see that the average AoI of worker-1 decreases with the increase of worker-2's AoI. If we set the threshold $\varepsilon = 7$, workers can meet the AoI constraint only when $\hat{\rho} = 0.6$ or 0.7. As shown in Fig. 3(b), the sum of AoI decreases firstly and increases later, and the total AoI can reach a minimum value. The general result for such systems is that the multi-user AoI optimization problem depends on both the total load $\hat{\rho}$ and the allocation of data update frequency among workers.

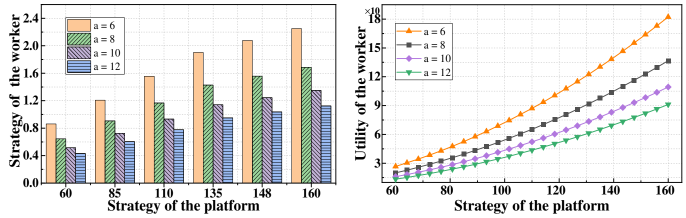
2) *Evaluation of Stackelberg Game:* We first verify the existence of the Stackelberg equilibrium for the platform and workers. In Fig. 4, we change PS and evaluate PU under different parameters (i.e., the quadratic parameter d and conversion parameter η). Similarly, we randomly select a worker and measure its utility by changing the parameters a and s, as illustrated in Fig. 5. We observe that both of PU and WU can find a maximum point, and a larger η will harvest the larger PU. Besides, WU will have an increase when applying a smaller a or s, since the cost of the worker becomes smaller. Then, we evaluate the impact of the social network effects, as shown in Fig. 6. When we enlarge the social network effect coefficient υ, both of the worker and the platform can possess higher utilities. Meanwhile, the optimal strategies of workers become higher along with the increase of υ. This is because workers can obtain more social benefits from their neighbors and are willing to collect data with a high frequency.

Next, we investigate the effect of the number of workers by changing the strategy of the platform from 80 to 180 and adjusting the conversion parameter s in the range $[6, 16]$. In Fig. 7(a), the platform can also determine its optimal strategy under diverse N, which is consistent with Fig. 4. Moreover, Fig. 7(b) shows that increasing N and applying a lower s can improve the profit of the platform. As presented in Fig. 8, we observe the influence of PS on any worker's strategy and utility. When the platform invests more money to incentivize workers, each worker will upload data as frequently as possible so as to acquire more rewards. In addition, a smaller a will result in a high WU which is also reflected in Fig. 5(a).



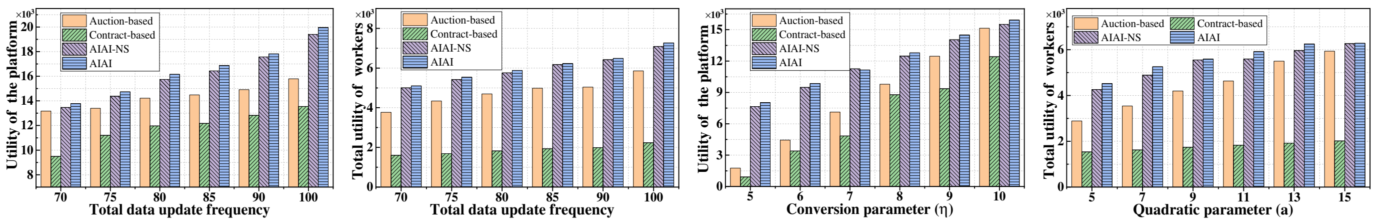
(a) PU under different N (b) PU under different N and s

Fig. 7. Influence of the number of workers



(a) strategy of the worker (b) utility of the worker

Fig. 8. Influence of the strategy of the platform



(a) PU vs. $\hat{p}$ (b) total SU vs. $\hat{p}$ (c) PU vs. η (d) total SU vs. a

Fig. 9. PU and total SU under different incentive mechanisms and varied parameters

Finally, Fig. 9 evaluates the utilities of the platform and all workers under different incentive mechanisms and varied parameters based on the auction theory, contract theory, and game theory. After increasing $\hat{p}$, PU and total WU will have a growth since more workers have chances to participate in the system. Importantly, PU and total WU of AIAI are higher than the compared algorithms. The reason is that the auction-based algorithm and the contract-based algorithm only guarantee the non-negativity of workers' utilities and cannot achieve utility maximization. The WU of the contract-based algorithm grows more slowly than others. This is because the number of contracts is limited and it cannot optimize all workers' utilities compared with AIAI. Besides, we can also note that AIAI performs better than AIAI-NS, which indicates that social network effects can bring in extra benefits for MCS systems.

VI. Related Works

Incentive Mechanism: Many remarkable incentive mechanisms have been designed for various MCS systems [12]–[14], [24]–[26]. Diverse tools have been leveraged in previous studies, such as game theory [25], auction mechanism [13], [24], contract theory [12], and deep learning [14], [32]. For example, [24] proposed a reverse auction-based incentive mechanism to select reliable workers. [32] formulated a multi-leader multi-follower Stackelberg game to deal with the Markov decision process. However, many of them did not take the social network into account, so they cannot be applied to our system. A handful of works integrate the social network effects into incentive mechanism designs [31], [33], [42]–[44]. For instance, the authors in [31] combined the user diversity and social effects into the reward mechanism design. [33] played a Stackelberg game among service providers and users with complete information on the social network effects. Nevertheless, most of these researches ignore the importance of data freshness, especially for time-sensitive MCS applications.

Age of Information: AoI has attracted increasing attention as an information freshness performance metric. There have been plenty of works that focus on addressing various AoI optimization problems [19], [27]–[30]. For example, [27] developed a model-based search structure to maximize collected data while minimizing users' AoI. Only a few researchers have studied the AoI demand with the pricing issue [16]–[19]. For instance, [16] built a dynamic task pricing model by harnessing the AoI timeliness metric in modeling requesters' waiting time costs. [19] derived a long-term decomposition mechanism to maximize the social welfare and ensure the platform freshness conditions. Nevertheless, none of the existing works take the AoI constraint and workers' social benefits into account together, which involves a complex incomplete information game due to the uncertainty of social network effects.

VII. Conclusion

In this paper, we investigate the MCS incentive mechanism design issue with AoI guarantee and social benefits. We first model it as a two-stage Stackelberg game, embedded with an incomplete information Bayesian sub-game. Moreover, we derive the optimal strategies of this game, including the optimal reward paid by the platform and the optimal data update frequency for each worker. We also prove that these optimal strategies form a unique Stackelberg equilibrium. Based on the optimal strategies, we propose the AIAI mechanism, whereby all of the platform and workers can obtain their optimal utilities. Meanwhile, the system can ensure that the AoI values of all data are not larger than a given threshold. Extensive simulations on real-world traces validate its great performance.

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