Appendix

A. Proof of Theorem 1

Theorem 1: For two overlapped communities, $C_i$, $C'_i$, and an arbitrary relay set $S$ ($S \subseteq C_i \cap C'_i$), we have:

$$B_{l,l'}(S) = \frac{1}{\sum_{v \in S} \lambda_{v,l'}} + \frac{\sum_{v \in S} \lambda_{v,l}}{\sum_{v \in S} \lambda_{v,l'}}$$

Proof: According to Definitions 3 and 4, we have that $B_{l,l'}(S)$ is exactly the expected delivery delay from $l$ to $l'$ via the first encountered node in $S$. Since the time interval that each node $v$ in $S$ encounters follows the exponential distribution with parameter $\lambda_{v,l}$, the probability density function of node $v$ becoming the first node meeting $l$ is $\lambda_{v,l} \prod_{v \in S} e^{-\lambda_{v,l} t}$. The delivery delay from $l$ to $l'$ via node $v$ is $t$ plus $D_v, v = 1/\lambda_{v,l}$. Then, we have:

$$B_{l,l'}(S) = \sum_{v \in S} \int_0^\infty \lambda_{v,l} \prod_{v \in S} e^{-\lambda_{v,l} t} \left( t + 1/\lambda_{v,l} \right) dt$$

$$B_{l,l'}(S) = \frac{1}{\sum_{v \in S} \lambda_{v,l'}} \sum_{v \in S} \lambda_{v,l}$$

(1)

B. Proof of Theorem 2

Theorem 2: Optimal Opportunistic Routing Rule: the message sender always delivers messages to the encountered relay that has a smaller minimum expected delay to the destination than itself. Concretely, a relay $u$ belongs to the optimal relay set $\tilde{R}_i$ for the delivery from $i$ to $d$, if and only if, $D_{u,d} < D_{i,d}$, i.e.:

$$u \in \tilde{R}_i \iff D_{u,d} < D_{i,d} \tag{2}$$

Proof: We first prove $u \in \tilde{R}_i \Rightarrow D_{u,d} < D_{i,d}$ by contradiction. Assume that $u \in \tilde{R}_i$ while $D_{u,d} \geq D_{i,d}$. Then, we construct a new relay set $R^+ = \tilde{R}_i \cup \{u\}$. By computing $D_{i,d}(\tilde{R}_i)$ and $D_{i,d}(R^+)$, we have:

$$D_{i,d}(\tilde{R}_i) = \sum_{v \in R} \int_0^\infty \lambda_{i,v} \prod_{v \in R} e^{-\lambda_{i,v} t} \left( t + D_{v,d} \right) dt$$

$$D_{i,d}(\tilde{R}_i) = \sum_{v \in R} \lambda_{i,v} D_{v,d}$$

(3)

$$D_{i,d}(R^+) = \sum_{v \in R^+} \int_0^\infty \lambda_{i,v} \prod_{v \in R^-} e^{-\lambda_{i,v} t} \left( t + D_{v,d} \right) dt$$

$$D_{i,d}(R^+) = \sum_{v \in R^-} \lambda_{i,v} D_{v,d}$$

(4)

Then, by comparing $D_{i,d}(\tilde{R}_i)$ and $D_{i,d}(R^+)$, we have:

$$D_{i,d}(\tilde{R}_i) - D_{i,d}(R^+) = \sum_{v \in R^-} \lambda_{i,v} \left( D_{u,d} - D_{i,d}(\tilde{R}_i) \right) \tag{5}$$

That is:

$$D_{i,d}(\tilde{R}_i) \geq D_{i,d}(R^-) \iff D_{u,d} \geq D_{i,d}(\tilde{R}_i) \tag{6}$$

On the other hand, we have $D_{u,d} \geq D_{i,d} = D_{i,d}(\tilde{R}_i)$, according to the assumption. Thus, we can get $D_{i,d}(R^-) \leq D_{i,d}(\tilde{R}_i)$ from Eq.(6). This is a contradiction in that $\tilde{R}_i$ is the optimal relay set to minimize $D_{i,d}$ (if there are multiple relay sets to minimize $D_{i,d}$, we always select the one with the smallest set size in this paper). Therefore, the assumption is wrong, and we should have $D_{u,d} < D_{i,d}$.

Likewise, we can get $D_{u,d} < D_{i,d} \Rightarrow u \in \tilde{R}_i$ by the contradiction method. Assume that $D_{u,d} < D_{i,d}$ and meanwhile $u \notin \tilde{R}_i$. Then, we construct a new relay set $R^+ = \tilde{R}_i \cup \{u\}$. By computing $D_{i,d}(R^+)$, we have:

$$D_{i,d}(R^+) = \sum_{v \in R^+} \int_0^\infty \lambda_{i,v} \prod_{v \in R^+} e^{-\lambda_{i,v} t} \left( t + D_{v,d} \right) dt$$

$$D_{i,d}(R^+) = 1 + \sum_{v \in R^+} \lambda_{i,v} D_{v,d} \tag{7}$$

Then, by comparing $D_{i,d}(R^+)$ and $D_{i,d}(\tilde{R}_i)$ in Eq.(3), we have:

$$D_{i,d}(R^+) - D_{i,d}(\tilde{R}_i) = \frac{\lambda_{i,u}}{\sum_{v \in R^+} \lambda_{i,v}} \left( D_{u,d} - D_{i,d}(\tilde{R}_i) \right) \tag{8}$$

That is:

$$D_{i,d}(R^+) < D_{i,d}(\tilde{R}_i) \iff D_{u,d} < D_{i,d}(\tilde{R}_i) \tag{9}$$

On the other hand, we have $D_{u,d} < D_{i,d} = D_{i,d}(\tilde{R}_i)$ according to the assumption. Thus, we can get $D_{i,d}(R^+) < D_{i,d}(\tilde{R}_i)$ from Eq.(9). This is a contradiction in that $\tilde{R}_i$ is the optimal relay set to minimize $D_{i,d}$. Therefore, the assumption is wrong, and we should have $u \in \tilde{R}_i$.

C. Proof of Theorem 3

Theorem 3: Assume that community $C_i$ has $m$ overlapped communities $C_i \cup \ldots \cup C_m$. Then, the optimal relay set $\tilde{R}_i$ of home $i$, and the optimal betweenness sets $\tilde{S}_{i,l}, (1 \leq i \leq m)$ satisfy:

1) if $v \notin \bigcup_{j=1}^m \tilde{S}_{i,l}$, then $v \notin \tilde{R}_i$;
2) $\tilde{S}_{i,l} \subseteq \tilde{R}_i$, otherwise $\tilde{S}_{i,l} \cap \tilde{R}_i = \emptyset$, for $\forall i \in [1,m]$.

Proof: 1. Since $v \notin \bigcup_{j=1}^m \tilde{S}_{i,l}$ means $v \in \bigcup_{j=1}^m (C_i \cap C_j)$, then without loss of generality, we assume $v \in C_i \cap C_{i-1}$ and $v \in \tilde{R}_i$ to prove the first property by contradiction. Firstly, we construct a new relay set $\tilde{R}' = \tilde{R}_i - (C_i \cap C_{i-1})$ and then compare the delay values, $D_{i,d}(\tilde{R}^{'})$ and $D_{i,d}(\tilde{R}_i)$, the delivery delays from $l$ to $d$ via the new relay set $\tilde{R}^{'}$ and the optimal relay set $\tilde{R}_i$. In fact, the two delay values are the expected values of the delays via nodes in the two relay sets. Consider that a node in $R = \tilde{R}_i - (C_i \cap C_{i-1})$ first visits $l$ and is selected as the real relay. Its contributions to $D_{i,d}(\tilde{R}^{'})$ and $D_{i,d}(\tilde{S}_{i,l})$ are the same. Thus, we only need to consider the contributions of the remaining nodes in $R - \tilde{R}_i (= \tilde{S}_{i,l})$ and $\tilde{R}_i - R$ to $D_{i,d}(\tilde{R}^{'})$ and $D_{i,d}(\tilde{R}_i)$, respectively. Since $\tilde{S}_{i,l}$ is the optimal relay set for the direct delivery from $l$ to $l'$, we thus have $D_{l,l'}(\tilde{S}_{i,l}) + D_{l',d}(\tilde{R}_i - R) + D_{l,d}(\tilde{R}_i)$. That is, the expected delay from $l$ to $l'$ via $\tilde{R}$ is even less
than the delay via $\tilde{R}_l$. This is a contradiction in that $\tilde{R}_l$ is the optimal relay set. Therefore, the assumption about $v \in \tilde{R}_l$ is wrong, and we should have $v \notin \tilde{R}_l$.

2. We are still using the contradiction method, and assume that there exists an integer $i \in [1, m]$ that satisfies $S_{l,i} \not\subseteq R_l$ and $S_{l,i} \cap R_l = R \neq \emptyset$. We also construct a new relay set $R' = R_l - R + S_{l,i}$. Based on a similar analysis as in part 1, we have that $D_{l,d}(R')$ is less than $D_{l,d}(R_l)$. This is a contradiction in that $\tilde{R}_l$ is the optimal relay set. Therefore, the assumption about $S_{l,i} \cap R_l \neq \emptyset$ is wrong, and the theorem is correct. □

D. Proof of Corollary 2

Corollary 1: CAOR can achieve the minimum expected delivery delay.

Proof: A straightforward result in Section 4.3. □

Corollary 2: Assume that $\lambda_{v_1,l'} \geq \lambda_{v_2,l'} \geq \cdots \geq \lambda_{v_n,l'}$, then the optimal betweenness set $S_{l,l'}$ satisfies:

1) $v_i \in S_{l,l'}$;
2) if $v_{i+1} \in S_{l,l'}$, then $v_i \in S_{l,l'}$. That is, $\exists k \in [1, n]$ s.t. $S_{l,l'} = \{v_1, \cdots, v_k\}$;
3) if $S_{l,l'} = \{v_1, \cdots, v_k\}$, then $B_{l,l'}(\{v_1, \cdots, v_i\}) > B_{l,l'}(\{v_1, \cdots, v_i, v_{i+1}\})$ for any $i \in [1, k-1]$.

Proof: At first, we directly prove the second result, which also implies the first result. We consider the optimal opportunistic routing between $l$ and $l'$ via $\{v_1, \cdots, v_n\}$. if $v_{i+1} \in S_{l,l'}$, then we have $D_{v_{i+1},l'} < D_{l,l'}$ according to Theorem 2. Since $D_{v_i,l'} = \frac{1}{\lambda_{v_i,l'}} < D_{v_{i+1},l'} = \frac{1}{\lambda_{v_{i+1},l'}}$, we can get $D_{v_i,l'} < D_{l,l'}$. Using Theorem 2 again, we have $v_i \in S_{l,l'}$. Without loss of generality, let the node in $S_{l,l'}$ with the largest expected delay to community home $l'$ be $v_{k,1}$, i.e., $v_k \in S_{l,l'}$. Then, $v_{k-1}, v_{k-2}, \cdots, v_1 \in S_{l,l'}$, i.e., $S_{l,l'} = \{v_1, \cdots, v_k\}$.

Now we prove the third result. Compare $D_{l,l'}(\{v_1, \cdots, v_i\})$ and $D_{l,l'}(\{v_1, \cdots, v_i, v_{i+1}\})$, we have:

$$D_{l,l'}(\{v_1, \cdots, v_i\}) < D_{l,l'}(\{v_1, \cdots, v_i, v_{i+1}\}) \iff D_{v_{i+1},l'} < D_{l,l'}(\{v_1, \cdots, v_i\}). \quad (10)$$

On the other hand, $v_{i+1} \in S_{l,l'}$, then we can get $D_{v_{i+1},l'} < D_{l,l'}(\{v_1, \cdots, v_i\})$ according to Theorem 2. Thus, $D_{l,l'}(\{v_1, \cdots, v_i+1\}) < D_{l,l'}(\{v_1, \cdots, v_i\})$. □