Supplemental Material of "Collaborative Mobile Charging"

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PROOF OF THEOREM 2

Theorem 2: (**Coverage**) Given a sufficiently large WSN $(\mathbf{X}, \mathbf{Y}, \mathbf{B}, \mathbf{T})$ and a charging model $(P, c, v, \eta_1, \eta_2)$, which satisfy conditions $K1K2K3$, the maximum coverages of EqualShare, SolelyCharge, CLCharge, and PushWait are $P/2c$, $P/2c$, P/c, and infinity, respectively.

Proof: In EqualShare, each charger contributes b_i/M energy to sensor node s_i. When M approaches infinity, the share b/K approaches 0. However, every charger has to return to the BS, thus the maximum coverage is $P/2c$. For the same reason, the maximum coverage of SolelyCharge is also $P/2c$. In CLCharge, when a charger turns around, it cannot get any energy from the other chargers before reaching the BS. Hence, the maximum coverage is P/c.

In PushWait, based on our previous analysis, we have:

$$\lim_{M\to+\infty} L_1 = \lim_{M\to+\infty} \sum_{i=1}^{M} \frac{Pd}{2cd^2 i + b}$$

$$\geq \lim_{M\to+\infty} \sum_{i=i_0}^{M} \frac{Pd}{2cd^2 i + b} (\text{let } 2cd^2 i_0 \geq b)$$

$$\geq \lim_{M\to+\infty} \sum_{i=i_0}^{M} \frac{Pd}{4cd^2 i} = \frac{P}{4cd} \lim_{M\to+\infty} \sum_{i=i_0}^{M} \frac{1}{i}$$

Therefore, the maximum coverage of PushWait is infinity, as M approaches infinity. □

PROOF OF THEOREM 3

Theorem 3: (**Optimality of SolelyCharge**) Given a WSN $(\mathbf{X}, \mathbf{Y}, \mathbf{B}, \mathbf{T})$ and a charging model $(P, c, v, \eta_1, \eta_2)$, satisfying conditions $K1K2$, if collaboration among chargers is not permitted, SolelyCharge is optimal.

Proof: Since we do not allow collaboration, energy loss can occur only when a charger transfers energy to a sensor node. Given a fixed WSN, the energy loss during energy transfer is also fixed, which is equal to $(\sum_{i=1}^{size(\mathbf{B})} b_i)/\eta_1$. Therefore, maximizing EUE is equivalent to minimizing $Distance(SolelyCharge)$. Suppose that SolelyCharge requires M chargers to replenish the given WSN. We prove the theorem by induction on M.

When $M = 1$ or $M = 2$, we can prove SolelyCharge is optimal using a similar method as in Theorem 1. We assume that SolelyCharge is optimal for any $M < n$.

$M = n$. Note that, any algorithm must have at least one charger, say $\mathcal{C}$, to charge the farthest sensor node, say s, in the WSN. To improve EUE as much as possible, we let $\mathcal{C}$ charge the sensor nodes that are near s, instead of near the BS. Without loss of generality, we assume that $\mathcal{C}$ charges nodes from s' to s. For sensor nodes between the BS to s', SolelyCharge requires $(n-1)$ chargers, which is optimal due to the induction hypothesis. Putting $\mathcal{C}$ and the $(n-1)$ chargers together, we find that they exactly represent SolelyCharge with n chargers. Therefore, SolelyCharge is optimal. □

PROOF OF THEOREM 4

Theorem 4: (**Necessary condition**) Given a node s that is x_s distance away from the BS, the battery capacity of s is b; using PushWait to deliver b energy to s one time achieves a higher EUE than using PushWait twice.

Proof: We denote PushWait being used once by the 1st scheme, and being used twice by the 2nd scheme. Since the payload energy is fixed, which is b, it is sufficient to prove that the total energy consumed by the 1st scheme is less than that by the 2nd scheme. We prove this by induction on the number of chargers required by the 1st scheme, say M.

$M = 1$ or 2. The 2nd scheme requires at least 2 chargers, both of which must travel $2d$ distance. Therefore, the 2nd scheme consumes more overhead energy than the 1st scheme, and the 1st scheme achieves a higher EUE. I.H.: the 1st scheme is better for any $M < n$.

$M = n$. Imagine that a virtual base station BS' is located at L_n, then, the 1st and 2nd schemes require $(n-1)P$ and Q energy, respectively, to deliver b energy to s from L_n. Due to the induction hypothesis, $Q > (n-1)P$. The task of the 2nd scheme then is to use PushWait twice to deliver Q energy from BS to BS', which requires that

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at least n chargers start from the BS. Since $Q > (n-1)P$, the 1st scheme is optimal. $\square$

PROOF OF THEOREM 5

Theorem 5: (Approx. ratio of ClusterCharging(β)) Given a WSN $(\mathbf{X}, \mathbf{Y}, \mathbf{B}, \mathbf{T})$ and a charging model $(P, c, v, \eta_1, \eta_2)$, satisfying conditions $K1\overline{K2}K3$, the approximation ratio of ClusterCharging(β) is

$$\frac{b_{min}(2cx_N + \sum_{i=1}^{N} b_i)}{P\tau_{max}k\sum_{i=1}^{N} b_i}$$

where $b_{min} = min_{i=1}^{N} b_i$, and

$$k = argmin_{(\sum_{i=1}^{k} \frac{1}{i} \geq \frac{2cx_N\tau_{max}+b_{min}}{P\tau_{max}})} k$$

Proof: Denote by $EUE(alg)$ the EUE of a scheduling algorithm alg. Denote by OPT the optimal solution to scenario $K1\overline{K2}K3$. The main line of this proof is to construct two extreme scheduling algorithms alg_1 and alg_2, such that $EUE(alg_1) < EUE(ClusterCharing(\beta))$, and $EUE(OPT) < ratio(alg_2)$.

Construction of alg_1. Consider the following charging round: we employ PushWait to charge only one sensor node, which is at the farthest point of the WSN, *i.e.*, x_N, and only needs the least possible energy, *i.e.*, b_{min}/τ_{max}. Therefore, the number of chargers used in this round should satisfy:

$$\frac{P}{2kc} + \frac{P}{2(k-1)c} + ... + \frac{P}{4c} + \frac{P - b_{min}/\tau_{max}}{2c} = x_N$$

Then, we have $k = argmin_{(\sum_{i=1}^{k} \frac{1}{i} \geq \frac{2cx_N\tau_{max}+b_{min}}{P\tau_{max}})} k$. Obviously, this charging round is the worst case we can imagine. We have:

$$EUE(ClusterCharging(\beta)) > EUE(alg_1) = \frac{b_{min}}{\tau_{max}kP}$$

Construction of alg_2. Suppose that we have a charger with an infinite battery, then we need only one charger in each round. The EUE reaches maximum when this charger transfers $\sum_{i=1}^{N} b_i$ energy to the WSN, while only traveling $2x_N$ distance. Obviously, we have

$$EUE(OPT) < EUE(alg_2) = \frac{\sum_{i=1}^{N} b_i}{2cx_N + \sum_{i=1}^{N} b_i}$$

Combining them together, we have

$$\frac{EUE(ClusterCharing(\beta))}{EUE(OPT)} > \frac{b_{min}(2cx_N + \sum_{i=1}^{N} b_i)}{P\tau_{max}k\sum_{i=1}^{N} b_i}$$

$\square$