# Optimizing Roadside Unit (RSU) Placement in Vehicular CPS 

Jie Wu
Computer and Information Sciences
Temple University

## RSU Placement

- Roadside advertisement

Attracting shoppers
Variation of maximum coverage problem
Traffic flow monitoring
Tracking traffic flow
Variation of set cover problem


## Roadside Advertisement

Passengers, shopkeeper, and roadside unit (RSU)
Shopkeeper disseminates ads to passing vehicles through RSUs
Passengers may go shopping, depending on detour distance


## Roadside Advertisement

RSUs placement optimization
Given a fixed number of RSUs and (traffic) flows, maximally attract passengers to the shop

Tradeoff between traffic density and detour probability


## Graph Model: $G=(V, E)$

V: a set of street intersections (vertices)

- One shop and RSUs located at street intersections

E: streets (directed edges)
Traveling path is the shortest path


## Detour Model

- Shopkeeper disseminates ads to passengers through RSUs
- Passengers in a flow may detour to the shop
- Detour probability depends on detour distance: $d_{1}+d_{2}-d_{3}$



## Property

For a given flow, the first RSU on its path always provides the best detour option (compared to all other RSU locations on the path)

- Insight: first RSU provides the highest traveling flexibility

- Redundant ads do not provide extra attraction


## Detour probability

- For a traffic flow, $f$, with a detour distance, $d$
$p(d)$ : the detour probability, decreasing utility function
An expectation of $|f| p(d)$ passengers detour to the shop



## Related Work: Maximum Coverage

- Use a given \# of sets (s) to maximally cover elements (e)
- Greedy algorithm with max marginal coverage has an approximation ratio of 1-1/e
- Inapproximability result: best polynomial time approximation algorithm
- Weighted version: elements have benefits, sets costs



## Our Problem

- Place RSUs on intersections to cover flows
- Different RSUs bring different detour probabilities


Intersections


## RSU Placement

## Composite Greedy Solution (CGS)

Iteratively find an intersection that can attract the maximum:
Candidate i: passengers from the uncovered flows;
Candidate ii: passenger from the covered flows, providing smaller detour distances;
Select i or ii that can attract more passengers to the shop


## RSU Placement

Theorem 1 [a]: The composite greedy solution has an approximation ratio of $1-1 / \sqrt{e}$ to the optimal solution

Time complexity: $O\left(|V|^{3}+k n|V|\right)$

- |V|: \# of intersections, k: \# of RSUs, and n: \# of flows
- Computing the detour distance takes $/ V /^{3}$ (shortest paths of all pairs using the Floyd algorithm)
- Greedy algorithm has $k$ steps; in each step, it visits each intersection to check traffic flows for coverage: $n / V /$


## Experiments

- Dataset: Dublin bus trace

Includes bus ID, longitude, latitude, and vehicle journey ID

- A vehicle journey represents a traffic flow

80,000 * 80,000 square feet, $c$ is set to be 0.001


## Experiments

- Other algorithms in comparison
- MaxCardinality: ranks intersections by \# of bus routes and places RSUs at the top- $k$ intersections
- MaxVehicles: ranks intersections by \# of passing buses and places RSUs at the top- $k$ intersections
- MaxCustomers: ranks the intersections by the \# of attracted passengers (flows) and places RSUs at the top-k intersections.
- Random: places RSUs uniform-randomly among all the intersections


## Experiments

The impact of utility function (Dublin trace)
Shop in the city with $D=20,000$


$$
f(d)=\left\{\begin{array}{cc}
0.001 \times(1-d / D) & d \leq D \\
0 & \text { otheriwse }
\end{array}\right.
$$

## Traffic flow monitoring

Coverage
Each traffic flow goes through at least one RSU

Distinguishability
RSUs used to cover each flow is unique

Objective
Minimize the number of placed RSUs


## Examples

Case 1: $f_{2}$ and $f_{3}$ are covered, but not distinguishable

$$
f_{1}:\left\{e_{5}, e_{6}\right\} \quad f_{2}:\left\{e_{3}, e_{5}\right\} \quad f_{3}:\left\{e_{3}, e_{4}\right\}
$$

Case 2: $f_{1}, f_{2}$ and $f_{3}$ are distinguishable, but $f_{1}$ is uncovered

$$
f_{1}:\left\{e_{5}, e_{6}\right\} \quad f_{2}:\left\{e_{3}, e_{5}\right\} \quad f_{3}:\left\{e_{3}, e_{4}\right\}
$$



## Model and Formulation

Graph $G=(V, E)$
$V$ : street intersections, and E: streets
$F=\left\{f_{1}, f_{2}, \ldots, f_{n}\right\}$ is a set of $n$ known flows on $G$
$S$ is a subset of $E$ on which RSUs are placed
$S(f)$ is a subset of $S$ that covers $f$

## Formulation

Objective: minimizing the number of RSUs

$$
\begin{array}{ll} 
& \text { minimize }|S| \\
\text { s.t. } & S(f) \neq \varnothing \text { for } \forall f \in F \\
& \text { (coverage) } \\
S(f) \neq S\left(f^{\prime}\right) \text { for } f \neq f^{\prime} & \text { (distinguishability) }
\end{array}
$$

## Related Work: Submodularity

$N(S)$ : \# of covered and distinguishable flows under S

Monotonicity: $N(S) \leq N\left(S^{\prime}\right)$ for $\forall S \subseteq S^{\prime}, S^{\prime} \subseteq E$
(Monotonicity enables greedy approaches)
Submodularity: $N(S \cup\{e\})-N(S) \geq N\left(S^{\prime} \cup\{e\}\right)-N\left(S^{\prime}\right)$ for $\forall e \in E$
(Submodularity ensures bounds)

## Related Work: Set Cover

Use minimum number of sets to cover all elements
Greedy algorithm with max marginal coverage has a ratio of $O(\log n)$ due to submodularity

Inapproximability result: best polynomial time approx. algo. Hitting set problem: right-vertices cover left-vertices in a bipartite graph
select 3 sets, $e_{1}, e_{3}$, and $e_{4}$


## Problem Analysis

NP-hard: reduction from the set cover problem
Non-submodularity: traditional coverage

$S=\left\{e_{1}\right\}$ and $S^{\prime}=\left\{e_{1}, e_{4}\right\}$
$N(S)=N\left(S^{\prime}\right)=1$, only $f_{1}$ is covered/distinguishable
$N\left(S \cup\left\{e_{2}\right\}\right)=1$, no change
$N\left(S^{\prime} \cup\left\{e_{2}\right\}\right)=4$, all flows are covered/distinguishable

## 3-out-of-3 Principle

To cover and distinguish an arbitrary pair of traffic flows ( $f$ and $f^{\prime}$ ), each of $f, f^{\prime}$, and $f \triangle f^{\prime}=\left(f \backslash f^{\prime}\right) \cup\left(f^{\prime} \backslash f\right)$ should include a street with a RSU placement


## Example



| subsets | $f_{1}$ | $f_{2}$ | $f_{3}$ |
| :---: | :---: | :---: | :---: |
| streets | $e_{1}, e_{2}, e_{3}, e_{6}$ | $e_{1}, e_{4}, e_{6}$ | $e_{2}, e_{5}, e_{6}, e_{7}$ |
| subsets | $f_{1} \triangle f_{2}$ | $f_{1} \triangle f_{3}$ | $f_{2} \triangle f_{3}$ |
| streets | $e_{2}, e_{3}, e_{4}$ | $e_{1}, e_{3}, e_{5}, e_{7}$ | $e_{1}, e_{2}, e_{4}, e_{5}, e_{7}$ |

$1^{\text {st }}$ iteration, $e_{1}$ is added to $S$ (appears in 4 subsets)
$2^{\text {nd }}$ iteration, $e_{2}$ is added to $S$, and terminated
$S=\{e 1, e 2\}$, with $S\left(f_{1}\right)=\left\{e_{1}, e_{2}\right\}, S\left(f_{2}\right)=\left\{e_{1}\right\}$, and $S\left(f_{3}\right)=\left\{e_{2}\right\}$

## Improved Subset-Based Greedy (ISBG)

Idea: in each greedy iteration, place an RSU that is in maximum number of subsets of $f, f^{\prime}$, and $f \triangle f^{\prime}$

Initialize $S=\varnothing$
for each pair of traffic flows (say $f$ and $f^{\prime}$ ) do Generate subsets of $f, f^{\prime}$, and $f \triangle f^{\prime}$ while there exists a subset do

Update $S$ to place an RSU that is in maximal number of subsets, remove corresponding subsets return S

## ISBG Performance

Theorem 2 [b]: ISBG has an approximation ratio $\ln [n(n+1) / 2]=O(\ln n)$ to the optimal solution, where $n$ is the number of traffic flows

Prove by converting to set cover with a ratio of In $[n(n+1) / 2]$, where $n(n+1) / 2$ is the number of subsets

Time complexity: $O\left(n^{2}|E|^{2}\right)$
Each greedy iteration visits $|E|$ RSUs for $n(n-1) / 2$ pairs of traffic flows, with $|E|$ iterations

## Experiments

Real data-driven: Seattle $10,000 \times 10,000$ square foot area
135 given traffic flows on 2,283 streets

(a) The Seattle map.

(b) The bus trace.

## Comparison Algorithms

Coverage-Oriented Greedy (COG): greedily covers all traffic flows, and then uniform-randomly place RSUs to distinguish them. $O\left(n^{2}|E|^{2}\right)$

Two Stage Placement (TSP): greedily covers all traffic flows in the $1^{\text {st }}$ stage, and then, greedily distinguishes all traffic flows in the $2^{\text {nd }}$ stage. $O\left(n^{2}|E|^{2}\right)$

Distinguishability-Oriented Greedy (DOG): greedily distinguishes pairs of traffic flows by placing an RSU at $f \triangle f^{\prime}$ until all flows are distinguishable. $O\left(n^{2}|E|^{2}\right)$

2-out-of-3 (PBG): To cover and distinguish an arbitrary pair of traffic flows ( $f$ and $f^{\prime}$ ), two RSUs should be placed on streets from two different subsets from $f \backslash f^{\prime}, f^{\prime} \backslash f$, and $f \cap f^{\prime} . O\left(n^{2}|E|^{3}\right)$

## Experiments

## Dublin (left) and Seattle (right)




Different flow patterns in Dublin and Seattle

## Conclusion

Maximum and minimum coverage using RSUs
Variation of max coverage to maximally attract passengers
Variation of min set cover to ensure coverage and distinguishability

## Future works

Extensions: Effect of multiple RSUs, multiple shops, ...
Applications: Flow monitoring/calculation in SDN networks, ...

## Q \& A

[a] H. Zheng and J. Wu, "Optimizing Roadside Advertisement Dissemination in Vehicular Cyber-Physical Systems," Proc. of IEEE ICDCS 2015.
[b] H. Zheng, W. Chang, and J. Wu, "Coverage and Distinguishability Requirements for Traffic Flow Monitoring Systems," Proc. of IEEE/ACM IWQoS2016 (Best Paper Award).


