Quantitative Analysis of the Effect of Transmitting Power on the Capacity of Wireless Ad Hoc Networks

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ABSTRACT

This paper presents a fundamental understanding regarding the effect that transmitting power has on the capacity of wireless ad hoc networks. Under the assumption that all interference is essentially regarded as noise, we carry out a quantitative analysis from the perspective of information theory. First, we answer the question, "How much information can be carried per unit bandwidth over a wireless ad hoc network under a certain power assignment and nodal distribution?" We then prove that the maximum network capacity, whether in *bps* (bits per second) or in *bmps* (bit-meters per second), strictly increases with respect to the total transmitting power under a fixed-proportion assignment, and that there is a limit as the total transmitting power goes to infinity. We further conclude that the maximum power efficiency, whether in *bpJ* (bits per Joule) or in *bmpJ* (bit-meters per Joule), strictly decreases with respect to the total transmitting power under a fixed-proportion assignment. We also show that the maximum network capacity, whether in *bps* or in *bmps*, follows an $O(n)$ scaling law, where n is the number of nodes, which coincides with previous asymptotic conclusions. Finally, we highlight the practical implications of the results for power allocation, power assignment, and transmission scheduling. The contributions of this paper may be worthy of consideration by wireless network designers.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*

General Terms

Theory, design, performance

Keywords

Capacity, energy efficiency, wireless ad hoc networks

1. INTRODUCTION

Power control plays an important role in improving the capacity of wireless ad hoc networks. A significant amount of research has been devoted to power control and capacity analysis in recent years. However, there is still limited knowledge regarding how transmitting power effects network capacity. This paper provides a quantitative analysis.

Gupta and Kumar [5] presented their seminal work on capacity analysis. Their results, and the majority of previous results [7, 8, 11, 18, 21], are based on asymptotic analysis, which yields limited information regarding understanding the exact capacity of a wireless ad hoc network composed of a certain number of nodes - particularly when the number is small. Narayanaswamy et al. [14] conducted a preliminary investigation regarding the effect that transmission power has on network capacity under a common-range model. It is concluded that the throughput capacity decreases as transmission power increases. On the contrary, Behzad and Rubin [1] concluded that higher transmission power results in increased capacity. Xie and Kumar [22] further developed an information theory to examine the capacity of wireless ad hoc networks, independent of networking protocols. But their results have a more theoretical than practical meaning. Rodoplu and Meng [17] proposed bits-per-Joule capacity in consideration of energy efficiency. Wang et al. [20] studied the energy efficiency of random wireless ad hoc networks. However, neither [17] nor [20] carried out a quantitative analysis. In summary, most previous results are asymptotic or qualitative, and hence have limited applicability in practice.

In this paper, we provide a quantitative analysis of the effect that transmitting power has on the capacity of wireless ad hoc networks. Our analysis is based on the assumption that all interference is essentially regarded as noise. It should be noted that interference is not noise, but information from the perspective of information theory. Some MAC protocols, CSMA for example, are capable of taking care of interference to some extent. However, for on-going transmissions, it is too difficult for a receiver to distill information from interference in real systems. This paper is concerned with the effect of transmitting power on network capacity at a certain time. Although how to utilize interference is not taken into account, the results of this paper can be used to evaluate interference utilization.

For an arbitrary wireless ad hoc network with a certain nodal distribution, we aim to answer the following questions:

1. *What is the greatest amount of information capable of being carried per unit bandwidth over a given wireless ad hoc network under a certain transmitting power assignment?* Theorem 1 gives the maximum capacity.

2. *How does the capacity of a wireless ad hoc network vary with the total transmitting power?* Theorem 2 shows that the maximum capacity strictly increases with the total transmitting power under a fixed-proportion assignment. However, increasing the transmitting power of one node does not necessarily increase the capacity of the entire network.

3. *Is there a limit to the maximum capacity while the total transmitting power approaches infinity?* Theorem 3 says "yes", if there are at least two nodes transmitting simultaneously. Theorem 3 also presents the limit.

4. *What is the maximum amount of information that can be carried per unit energy?* Theorem 4 provides the maximum power efficiency in the sense of network capacity, which is achieved when the total transmitting power approaches zero.

5. *In the sense of network capacity, how does the power efficiency vary with the total transmitting power?* Theorem 5 demonstrates that the power efficiency strictly decreases with respect to the total transmitting power under a fixed-proportion assignment.

6. *What is the scaling law that the quantitative results imply?* Theorem 6 shows that the maximum capacity follows an $O(n)$ scaling law, where n is the number of nodes, which coincides very well with previous asymptotic results (Gupta and Kumar [5], Xie and Kumar [22]).

The rest of the paper is organized as follows: In Section 2, we summarize related work. In Section 3, we introduce preliminaries necessary for a clear understanding of this paper, including assumptions, notations, and definitions. In Section 4, we give the main results regarding the maximum network capacity and power efficiency. Following that, we present proofs of all results in Section 5. We highlight the practical implications of those results in Section 6. Finally, we conclude this paper in Section 7.

2. RELATED WORK

Narayanaswamy et al. [14] investigated the effect that the common-range transmission power has on the capacity of wireless ad hoc networks. Their analysis was based on a Protocol Interference Model originally introduced by Gupta and Kumar [5]. A transmission from node i to j is successfully received if $d_{ij} \leq r$ and $d_{kj} \geq (1+\Delta)r$ for any other node k simultaneously transmitting with i, where d_{uv} denotes the Euclidean distance between node u and v, Δ is a real nonnegative number, r is the common transmission range, and $(1+\Delta)r$ is usually referred to as the interference range. The authors proved that the upper bound of the throughput capacity is inversely proportional to r. This conclusion paves a theoretical foundation for the COMPOW protocol, in which the common transmission power is reduced to the lowest level while preserving connectivity.

On the contrary, Behzad and Rubin [1] concluded that higher transmission power increases the capacity, independent of nodal distribution and traffic pattern. There exists a relatively maximum power vector that maximizes the capacity, i.e., at least one node uses the maximum transmission power if the maximum capacity is achieved. As a result, under the special case that the transmission power of all nodes is assumed to be identical, the capacity is maximized if all nodes use a common maximum power.

Xie and Kumar [22] carried out an information-theoretical analysis of the capacity of wireless networks without making arbitrary and preconceived assumptions about how they will operate. Suppose that n nodes are located on a two-dimensional plane with a minimum separation distance d_{min}, and that attenuation of radio signals over a distance d is modeled as $\frac{e^{-\gamma d}}{d^{\alpha}}$, where $\gamma \geq 0$ is the absorption constant, and $\alpha > 0$ is the path loss exponent. In the media with $\gamma > 0$ or $\alpha > 3$, it is concluded that the transport capacity follows an $O(n)$ scaling law under the individual power constraint, and is upper bounded by $\frac{c(\gamma,\alpha,d_{min})}{\sigma^2}P$ bit-meters per second, where $c(\gamma, \alpha, d_{min})$ is a constant determined by $(\gamma, \alpha, d_{min})$, σ^2 is the variance of Gaussian noise, and P is the total transmitted power. According to their results, the capacity may be proportional to P, and hence may be arbitrarily large as P reaches infinity. However, this is not necessarily the case in real systems.

Most previous work had not taken energy efficiency into account. Rodoplu and Meng [17] proposed bits-per-Joule capacity. Under the one-to-one traffic model in which each node sends traffic to a randomly chosen destination, the authors showed that the bits-per-Joule capacity grows as $\Omega((\frac{n}{\log n})^{(\alpha-1)/2})$. Wang et al. [20] analyzed the capacity and energy efficiency of random wireless ad hoc networks; however, the result is still asymptotic.

The past several years have seen a lot of research efforts devoted to capacity analysis [4, 6, 9, 10, 11, 12, 15, 16] since the seminal work of Gupta and Kumar [5]. However, there is still limited knowledge regarding the effect that transmitting power has on the capacity of wireless ad hoc networks.

3. PRELIMINARIES

A wireless ad hoc network can be modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices and $\mathcal{E}$ is the set of edges. A vertex $u \in \mathcal{V}$ represents a wireless node and an edge $(u, v) \in \mathcal{E}$ corresponds to a unidirectional wireless link from node u to v.

For the reader's convenience, we have listed frequently-used notations in Table 1. It is important to note that notations, such as (i, j), d_{ij}, P_{ij}, Q_{ij}, C_{ij}, etc., are legal if and only if $i \neq j$.

3.1 Radio Propagation Model

Both theoretical and measurement-based investigations of radio propagation models indicate that the average received signal power decreases with distance under the following path loss model:

$$P^r(P^t, G, d, \alpha) = \frac{G \cdot P^t}{d^\alpha}, \quad (1)$$

where P^r is a function of (P^t, G, d, α) which denotes the received power in Watts, P^t is the transmitted power, G is the gain from transmitter to receiver, d is the Euclidean distance in meters between the transmitter and receiver, and $\alpha \geq 2$ is the path loss exponent.

There are two points worth noting: First, the path loss models described by (1) are all large-scale propagation models. It is obvious that they do not hold for $d = 0$. They are only available when d is beyond some far-field distance that is related to the largest linear dimension of the transmitter antenna aperture and the carrier wavelength. Second, the well-known free space model and ground reflection model (the latter is also known as the two-ray model) can be generalized to the path loss model. As a result, any conclusion derived under (1) also holds for the free space model and ground reflection model.

3.2 Link Capacity Model

The signal-to-interference-plus-noise ratio (SINR) at the receiver of the link (u, v) can be modeled as

$$SINR_{uv} = \frac{S_{uv}}{I_{uv} + N_v},$$

where S_{uv} is the received signal power over link (u, v), I_{uv} is the interference power over (u, v), and N_v is the thermal noise power at node v's receiver. Under the path loss model, we have

$$S_{uv} = \frac{G_{uv} \cdot P_{uv}}{d_{uv}^\alpha}, \quad (2)$$

where P_{uv} is the transmitting power over link (u, v), $0 \leq P_{uv} \leq P_u$, $P_u := \sum_j P_{uj}$, G_{uv} is the gain from node u to v, and d_{uv} is the Euclidean distance between node u and v. We also have

$$I_{uv} = \sum_{j \neq v} \frac{G_{uv} \cdot P_{uj}}{d_{uv}^\alpha} + \sum_{i \neq u} \sum_j \frac{G_{iv} \cdot P_{ij}}{d_{iv}^\alpha}. \quad (3)$$

We assume that all channels in the network are Gaussian channels. It is practical to regard all interference as noise in narrow-band systems. Then, according to the well-known Shannon's formula, the capacity of link (u, v) in *bps* (bits per second) per unit bandwidth, denoted by $C_{uv}(bps)$, is given by

$$C_{uv}(bps) = \log_2(1 + SINR_{uv}).$$

We also consider the capacity measured in terms of *bmps* (bit-meters per second), originally introduced by Gupta and Kumar [5]. The capacity of link (u, v) in *bmps* per unit bandwidth is given by

$$C_{uv}(bmps) = d_{uv} \cdot C_{uv}(bps).$$

Table 1: Frequently-used notations.

Notations	Descriptions
C_{uv}	Capacity of link (u, v).
C_u	Capacity of node u. $C_u := \sum_v C_{uv}$.
C	Network capacity. $C := \sum_{uv} C_{uv}$.
C_u^{max}	$C_u^{max} := \max(C_u)$.
C^{max}	$C^{max} := \max(C)$.
d_{uv}	Euclidean distance between node u and v.
d_u	$d_u := \sum_v d_{uv}$.
G_{uv}	Gain from node u to v.
I_{uv}	Interference power over link (u, v). $I_{uv} = \sum_{j \neq v} \frac{G_{uv} \cdot P_{uj}}{d_{uv}^\alpha} + \sum_{i \neq u} \sum_j \frac{G_{iv} \cdot P_{ij}}{d_{iv}^\alpha}$.
K_{uv}^i	$K_{uv}^i := \frac{d_{uv}^\alpha G_{iv}}{G_{uv} d_{iv}^\alpha}$.
M_{uv}	$M_{uv} := \frac{d_{uv}^\alpha}{G_{uv}} N_v$.
N_v	Thermal noise power at node v.
n	Number of nodes.
P_{uv}	Transmitting power over link (u, v).
P_u	Transmitting power of node u. $P_u := \sum_v P_{uv}$.
P	Total transmitting power. $P := \sum_u P_u$.
Q_{uv}	$Q_{uv} := \frac{d_{uv}^\alpha}{G_{uv}} \left(\sum_i \frac{G_{iv}}{d_{iv}^\alpha} P_i + N_v \right) = \frac{d_{uv}^\alpha}{G_{uv}} T_v$.
S_{uv}	Received signal power over link (u, v).
T_v	Total power at node v's receiver. $T_v := S_{uv} + I_{uv} + N_v = \sum_i \frac{G_{iv}}{d_{iv}^\alpha} P_i + N_v$.
α	Path loss exponent.
η	Power efficiency or energy efficiency. $\eta := C/P$.
η^{max}	$\eta^{max} := \max(\eta)$.
ζ	$\zeta := (\zeta_1, \zeta_2, \cdots, \zeta_n)$, where $\zeta_i = P_i/P$, $i = 1 \cdots n$.

3.3 Definitions

Definition 1. Power Allocation: The power vector $(P_{uv_1}, P_{uv_2}, \cdots, P_{uv_{(n-1)}})$ is said to be a transmitting power allocation at node u, where $P_{uv} \geq 0$ denotes the power allocated to link (u, v), $v \neq u$, $v = v_1, v_2, \cdots, v_{(n-1)}$.

Definition 2. Power Assignment: We assume $P = \sum_i P_i$ and $\zeta_i = P_i/P$, i.e., $0 \leq \zeta_i \leq 1$ and $\sum_i \zeta_i = 1$. Then $P\zeta = (P_1, P_2, \cdots, P_n)$ is said to be a transmitting power assignment, where $\zeta = (\zeta_1, \zeta_2, \cdots, \zeta_n)$. For simplicity, we also refer to ζ as the same power assignment.

Definition 3. Fixed-Proportion Power Assignment: $P\zeta$ is said to be a fixed-proportion power assignment if ζ is fixed while P scales.

Definition 4. Non-monopolized Power Assignment: We refer to ζ_i as the power assignment ratio for node i. ζ is said to be a non-monopolized power assignment if there are at least two non-zero power assignment ratios in ζ. Otherwise, ζ is said to be a monopolized power assignment.

Definition 5. Network Capacity: The capacity of a wireless ad hoc network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, measured in terms of *bps* and in *bmps*, are respectively defined as

$$C(bps) := \sum_{(u,v)\in\mathcal{E}} C_{uv}(bps),$$
$$C(bmps) := \sum_{(u,v)\in\mathcal{E}} C_{uv}(bmps).$$

Definition 6. Power Efficiency or Energy Efficiency: The power efficiency of a network, measured in terms of *bpJ* (bits per Joule) and in *bmpJ* (bit-meters per Joule), is respectively defined as

$$\eta(bpJ) := \frac{C(bps)}{P},$$
$$\eta(bmpJ) := \frac{C(bmps)}{P},$$

where $P = \sum_u P_u$ is the total transmitting power.

For brevity, we use C to denote either $C(bps)$ or $C(bmps)$, and η to denote either $\eta(bpJ)$ or $\eta(bmpJ)$. Assume $C^{max} := \max(C)$ and $\eta^{max} := \max(\eta)$. For a given wireless ad hoc network, this paper aims to provide a fundamental understanding of C^{max} and η^{max}.

4. MAIN RESULTS

In this section, we present the main results of this paper on C^{max} and η^{max} over a given wireless ad hoc networks. The results are concluded from quantitative information-theoretical analysis under the assumption that all interference is essentially regarded as noise.

THEOREM 1. *Given a wireless ad hoc network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, its maximum capacity per unit bandwidth is*

$$C^{max}(bps) = \sum_{u\in\mathcal{V}} \max_v \left\{ \log_2 \left(\frac{Q_{uv}}{Q_{uv} - P_u} \right) \right\}, \quad (4)$$

$$C^{max}(bmps) = \sum_{u\in\mathcal{V}} \max_v \left\{ d_{uv} \log_2 \left(\frac{Q_{uv}}{Q_{uv} - P_u} \right) \right\}, \quad (5)$$

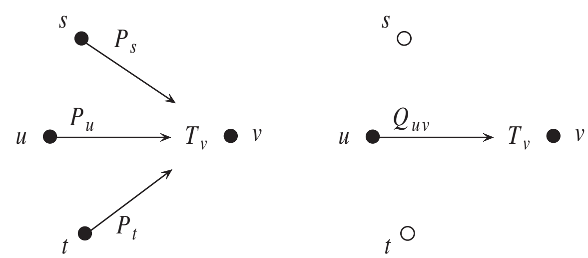
where

$$Q_{uv} := \frac{d_{uv}^{\alpha}}{G_{uv}} \Big(\sum_i \frac{G_{iv}}{d_{iv}^{\alpha}} P_i + N_v \Big).$$

Since Q_{uv} is one of the most important notations in this paper, we explain its physical meaning as follows: We use $T_v := S_{uv} + I_{uv} + N_v$ to denote the total power at node v's receiver. It is obvious that all nodes contribute to T_v, as shown in Fig.1 (a). In reality, $T_v = \frac{G_{uv}}{d_{uv}^{\alpha}} Q_{uv}$. In other words, if T_v were assumed to be totally from node u, the transmitting power of u would have been Q_{uv}, as shown in Fig.1 (b).

Theorem 1 reveals the maximum amount of information that can be carried per unit bandwidth over a given wireless ad hoc network under a certain transmitting power assignment. It also implies what the best power allocation is in the sense of maximizing network capacity. The following results are based on Theorem 1.

THEOREM 2. *Given a wireless ad hoc network $\mathcal{G}$ under a fixed-proportion power assignment ζ, the maximum capacity C^{max}, whether in bps or in bmps, strictly increases with respect to the total transmitting power P.*



(a) All nodes contribute to T_v. (b) Q_{uv} attenuates to T_v.

Figure 1: The physical meaning of Q_{uv}.

Theorem 2 is somewhat similar to the conclusion drawn by Xie and Kumar[22], and even more similar to Behzad and Rubin's conclusion in [1]. However, neither [1] nor [22] show whether there is a limit to the maximum capacity as P approaches infinity; Theorem 3 answers this question.

THEOREM 3. *Given a wireless ad hoc network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ under a non-monopolized fixed-proportion power assignment ζ, there is a limit to C^{max} as P goes to infinity.*

$$\lim_{P\to\infty} C^{max}(bps) = \sum_{u\in\mathcal{V}} \max_v \left\{ \log_2 \left(\frac{\sum_i K_{uv}^i \zeta_i}{\sum_i K_{uv}^i \zeta_i - \zeta_u} \right) \right\},$$

$$\lim_{P\to\infty} C^{max}(bmps) = \sum_{u\in\mathcal{V}} \max_v \left\{ d_{uv} \log_2 \left(\frac{\sum_i K_{uv}^i \zeta_i}{\sum_i K_{uv}^i \zeta_i - \zeta_u} \right) \right\},$$

where

$$K_{uv}^i := \frac{d_{uv}^{\alpha} G_{iv}}{G_{uv} d_{iv}^{\alpha}}.$$

It is rational to believe that the power assignment is non-monopolized in real systems. Therefore, there is a limit to the maximum capacity, as given in Theorem 3. As a result, the power efficiency in the sense of the capacity per Watt approaches zero as P goes to infinity.

THEOREM 4. *Given a wireless ad hoc network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ under a fixed-proportion power assignment ζ, the maximum power efficiency η^{max} approaches η_0^{max} while P approaches zero, where η_0^{max} in bpJ and that in bmpJ can be computed respectively as follows:*

$$\eta_0^{max}(bpJ) = \frac{1}{\ln 2} \sum_{u\in\mathcal{V}} \zeta_u \max_v \left\{ \frac{1}{M_{uv}} \right\},$$

$$\eta_0^{max}(bmpJ) = \frac{1}{\ln 2} \sum_{u\in\mathcal{V}} \zeta_u \max_v \left\{ \frac{d_{uv}}{M_{uv}} \right\},$$

where

$$M_{uv} := \frac{d_{uv}^{\alpha}}{G_{uv}} N_v.$$

Theorem 4 gives an upper bound of the maximum power efficiency for a given wireless ad hoc network under a fixed-proportion power assignment. η_0^{max} is referred to as the upper bound because Theorem 5 shows that the maximum power efficiency strictly decreases with respect to the total transmitting power.

THEOREM 5. *Given a wireless ad hoc network $\mathcal{G}$ under a fixed-proportion power assignment ζ, the maximum power efficiency η^{max}, whether in bpJ or in bmpJ, strictly decreases with respect to the total transmitting power P.*

It is interesting to derive an asymptotic result from our quantitative analysis. For comparison with previous work, the following theorem gives a scaling law.

THEOREM 6. *Suppose that n nodes are located on a plane with a minimum separation distance d_{min}. If we assume that $P_u \leq P_{max}$ for any node u, $G_{uv} = 1$ for any link (u, v), and $N_v = N$ for any node v, then we have*

$$C^{max}(bps) \leq \frac{1}{\ln 2} \cdot \frac{P_{max} \cdot n}{d_{min}^{\alpha} N} = O(n),$$
$$C^{max}(bmps) \leq \frac{1}{\ln 2} \cdot \frac{P_{max} \cdot n}{d_{min}^{\alpha-1} N} = O(n).$$

Gupta and Kumar [5] showed that the transport capacity is of order $O(\sqrt{An})$, where A is the area of the deployment region. Under the reasonable assumption that $A = \Theta(n)$, the scaling law is $O(\sqrt{An}) = O(n)$. Xie and Kumar [22] also concluded that the transport capacity follows an $O(n)$ scaling law. Therefore, our quantitative results coincide very well with previous asymptotic conclusions. In comparison with asymptotic results, quantitative results provide more insights. We highlight the practical implications in Section 6.

5. PROOFS

In this section, we give proofs of the results presented in Section 4. We begin with three lemmas, followed by detailed proofs of all the theorems.

5.1 Three Lemmas

LEMMA 1. *The capacity of a link (u, v) per unit bandwidth in bps is*

$$C_{uv}(bps) = \log_2\Big(\frac{T_v}{T_v - S_{uv}}\Big) = \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_{uv}}\Big),$$

where

$$T_v := S_{uv} + I_{uv} + N_v = \sum_i \frac{G_{iv}}{d_{iv}^{\alpha}} P_i + N_v,$$
$$Q_{uv} := \frac{d_{uv}^{\alpha}}{G_{uv}}\Big(\sum_i \frac{G_{iv}}{d_{iv}^{\alpha}} P_i + N_v\Big).$$

PROOF. Since $T_v := S_{uv} + I_{uv} + N_v$, substituting (2) and (3) for S_{uv} and I_{uv}, respectively, yields

$$\begin{aligned} T_v &= \frac{G_{uv} \cdot P_{uv}}{d_{uv}^{\alpha}} + \sum_{j \neq v} \frac{G_{uv} \cdot P_{uj}}{d_{uv}^{\alpha}} + \sum_{i \neq u} \sum_j \frac{G_{iv} \cdot P_{ij}}{d_{iv}^{\alpha}} + N_v \\ &= \frac{G_{uv} \cdot \sum_j P_{uj}}{d_{uv}^{\alpha}} + \sum_{i \neq u} \frac{G_{iv} \cdot \sum_j P_{ij}}{d_{iv}^{\alpha}} + N_v \\ &= \sum_i \frac{G_{iv}}{d_{iv}^{\alpha}} P_i + N_v. \end{aligned}$$

Then, we have

$$T_v = \frac{G_{uv}}{d_{uv}^{\alpha}} Q_{uv}. \tag{6}$$

Under the assumption that all interference is essentially regarded as noise, we have

$$\begin{aligned} C_{uv}(bps) &= \log_2(1 + SINR_{uv}) = \log_2\Big(1 + \frac{S_{uv}}{I_{uv} + N_v}\Big) \\ &= \log_2\Big(\frac{S_{uv} + I_{uv} + N_v}{S_{uv} + I_{uv} + N_v - S_{uv}}\Big). \end{aligned}$$

Since $S_{uv} + I_{uv} + N_v = T_v$, we have

$$C_{uv}(bps) = \log_2\Big(\frac{T_v}{T_v - S_{uv}}\Big). \tag{7}$$

Substituting (2) and (6) in (7) yields

$$C_{uv}(bps) = \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_{uv}}\Big). \tag{8}$$

□

LEMMA 2. *If P_u is fixed for each node u, then we have $C^{max} = \sum_u C_u^{max}$, whether in bps or in bmps, where $C_u^{max} := \max(C_u)$, $C_u := \sum_v C_{uv}$.*

PROOF. For each node u, if P_u is fixed, according to (7), we have the observation that power allocation at any other node $w \neq u$ does not affect the capacity of link (u, v) because T_v is independent of power allocation. That is to say, C_u is independent of any other $C_{w \neq u}$ under a certain transmitting power assignment. Therefore, $C^{max} = \sum_u C_u^{max}$, whether in *bps* or in *bmps*. □

LEMMA 3. *If ζ is a non-monopolized power assignment, then $\sum_i K_{uv}^i \zeta_i - \zeta_u > 0$ for any u.*

PROOF. Because $K_{uv}^i := \frac{d_{uv}^{\alpha} G_{iv}}{G_{uv} d_{iv}^{\alpha}}$, we have

$$K_{uv}^u = \frac{d_{uv}^{\alpha} G_{uv}}{G_{uv} d_{uv}^{\alpha}} = 1.$$

Therefore,

$$\sum_i K_{uv}^i \zeta_i - \zeta_u = \sum_{i \neq u} K_{uv}^i \zeta_i + K_{uv}^u \zeta_u - \zeta_u = \sum_{i \neq u} K_{uv}^i \zeta_i. \tag{9}$$

Since ζ is a non-monopolized power assignment, according to Definition 4, there are at least two non-zero power assignment ratios, say ζ_{i_1} and ζ_{i_2}. Then,

$$\sum_{i \neq u} K_{uv}^i \zeta_i \geq \min\{K_{uv}^{i_1} \zeta_{i_1}, K_{uv}^{i_2} \zeta_{i_2}\} > 0. \tag{10}$$

Combining (9) and (10) yields $\sum_i K_{uv}^i \zeta_i - \zeta_u > 0$ for any node u. □

5.2 Proof of Theorem 1

PROOF. At first, we maximize $C_u(bmps)$ for each node u subject to $\sum_v P_{uv} = P_u$. Suppose

$$f_{uv}(x) := d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - x}\Big), \tag{11}$$
$$f_u(x) := \max_v \big(f_{uv}(x)\big), \tag{12}$$

where $0 \leq x < Q_{um}$, $Q_{um} := \min_v\{Q_{uv}\}$. Because

$$\frac{\partial^2 f_{uv}(x)}{\partial x^2} = \frac{d_{uv}}{\ln 2 \cdot (Q_{uv} - x)^2} > 0,$$

$f_{uv}(x)$ is a convex function of x. It is not difficult to prove that $f_u(x)$ is also convex with respect to x.

Suppose that $l(x)$ is the line across point $(0, f_u(0))$ and point $((x_1+x_2), f_u(x_1+x_2))$. In reality, $f_u(0) = 0$. Then, the equation of $l(x)$ is

$$l(x) = \frac{f_u(x_1+x_2)}{x_1+x_2} x,$$

where $x_1 \geq 0$, $x_2 \geq 0$, $0 < x_1 + x_2 < Q_{um}$. Because $f_u(x)$ is convex, we have $f_u(x) \leq l(x)$. Since

$$\begin{aligned} f_u(x_1) + f_u(x_2) &\leq l(x_1) + l(x_2) \\ &= \frac{f_u(x_1+x_2)}{x_1+x_2} x_1 + \frac{f_u(x_1+x_2)}{x_1+x_2} x_2 \\ &= f_u(x_1+x_2), \end{aligned}$$

we have $f_u(x_1) + f_u(x_2) \leq f_u(x_1+x_2)$.

By inductive reasoning, it is easy to conclude

$$\sum_{i=1}^{k} f_u(x_i) \leq f_u\Big(\sum_{i=1}^{k} x_i\Big). \tag{13}$$

According to the definition of f_{uv} (11), we have

$$C_u(bmps) = \sum_v d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_{uv}}\Big) = \sum_v f_{uv}(P_{uv}).$$

According to (12) and (13), we have

$$C_u(bmps) \leq \sum_v f_u(P_{uv}) \leq f_u\Big(\sum_v P_{uv}\Big) = f_u(P_u).$$

It is easy to know that $f_u(P_u)$ is the tight upper bound of C_u. That is

$$C_u^{max}(bmps) = f_u(P_u). \tag{14}$$

Substituting the definition of f_u in (14) yields

$$C_u^{max}(bmps) = \max_v \left\{ d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}.$$

Since Lemma (2) says $C^{max} = \sum_u C_u^{max}$, we have

$$C^{max}(bmps) = \sum_{u \in \mathcal{V}} \max_v \left\{ d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}.$$

Similarly, we have

$$C^{max}(bps) = \sum_{u \in \mathcal{V}} \max_v \left\{ \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}.$$

□

5.3 Proof of Theorem 2

PROOF. According to Theorem 1, we have the maximum per-unit-bandwidth capacity in *bmps* as follows:

$$C^{max}(bmps) = \sum_{u \in \mathcal{V}} \max_v \left\{ d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}.$$

For each node u, we assume that m_u is such a node that

$$d_{um_u} \log_2\Big(\frac{Q_{um_u}}{Q_{um_u} - P_u}\Big) = \max_v \left\{ d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}.$$

Hence,

$$C^{max}(bmps) = \sum_{u \in \mathcal{V}} d_{um_u} \log_2\Big(\frac{Q_{um_u}}{Q_{um_u} - P_u}\Big). \tag{15}$$

It is obvious that $C^{max}(bmps)$ is continuous and piecewise derivable. If $C^{max}(bmps)$ is derivable at P, then we have

$$\frac{\partial C^{max}(bmps)}{\partial P} = \sum_u d_{um_u} \frac{Q_{um_u} \frac{\partial P_u}{\partial P} - P_u \frac{\partial Q_{um_u}}{\partial P}}{\ln 2 \cdot Q_{um_u}(Q_{um_u} - P_u)}. \tag{16}$$

According to the definition of Q_{uv}, we have

$$\begin{aligned} Q_{uv} &= \frac{d_{uv}^{\alpha}}{G_{uv}} \Big(\sum_i \frac{G_{iv}}{d_{iv}^{\alpha}} P_i + N_v \Big) \\ &= \sum_i \frac{d_{uv}^{\alpha} G_{iv}}{G_{uv} d_{iv}^{\alpha}} \zeta_i P + \frac{d_{uv}^{\alpha}}{G_{uv}} N_v. \end{aligned}$$

Since $K_{uv}^i := \frac{d_{uv}^{\alpha} G_{iv}}{G_{uv} d_{iv}^{\alpha}}$ and $M_{uv} := \frac{d_{uv}^{\alpha}}{G_{uv}} N_v$, we have

$$Q_{uv} = \sum_i K_{uv}^i \zeta_i P + M_{uv}. \tag{17}$$

Therefore,

$$Q_{um_u} = \sum_i K_{um_u}^i \zeta_i P + M_{um_u}, \tag{18}$$

$$\frac{\partial Q_{um_u}}{\partial P} = \sum_i K_{um_u}^i \zeta_i. \tag{19}$$

Hence, we have

$$\begin{aligned} & Q_{um_u} \frac{\partial P_u}{\partial P} - P_u \frac{\partial Q_{um_u}}{\partial P} \\ &= Q_{um_u} \zeta_u - \zeta_u P \sum_i K_{um_u}^i \zeta_i \\ &= \zeta_u \big(Q_{um_u} - \sum_i K_{um_u}^i \zeta_i P \big). \end{aligned} \tag{20}$$

Substituting (18) in (20) yields

$$Q_{um_u} \frac{\partial P_u}{\partial P} - P_u \frac{\partial Q_{um_u}}{\partial P} = \zeta_u M_{um_u}. \tag{21}$$

Substituting (21) in (16) yields

$$\frac{\partial C^{max}(bmps)}{\partial P} = \frac{1}{\ln 2} \sum_u \frac{\zeta_u d_{um_u} M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)}. \tag{22}$$

It is easy to see that $(Q_{um_u} - P_u) > 0$. Therefore,

$$\frac{\partial C^{max}(bmps)}{\partial P} > 0.$$

Similarly, if $C^{max}(bps)$ is derivable at P, we have

$$\frac{\partial C^{max}(bps)}{\partial P} > 0.$$

As a result, both $C^{max}(bps)$ and $C^{max}(bmps)$ strictly increase at each derivable piece. Since $C^{max}(bps)$ and $C^{max}(bmps)$ are continuous and piecewise derivable, the maximum capacity, whether in *bps* or in *bmps*, strictly increases with respect to the total transmitting power P. □

5.4 Proof of Theorem 3

PROOF. Substituting (17) in (5) yields

$$C^{max}(bmps) = \sum_{u \in \mathcal{V}} \max_v \left\{ d_{uv} \log_2\Big(\frac{Q_{uv}}{Q_{uv} - P_u}\Big) \right\}$$

$$= \sum_{u\in\mathcal{V}} \max_v \left\{ d_{uv} \log_2 \Big(\frac{\sum_i K_{uv}^i \zeta_i P + M_{uv}}{\sum_i K_{uv}^i \zeta_i P + M_{uv} - \zeta_u P} \Big) \right\}$$

$$= \sum_{u\in\mathcal{V}} \max_v \left\{ d_{uv} \log_2 \Big(\frac{\sum_i K_{uv}^i \zeta_i + \frac{M_{uv}}{P}}{\sum_i K_{uv}^i \zeta_i + \frac{M_{uv}}{P} - \zeta_u} \Big) \right\}.$$

Therefore,

$$\lim_{P\to\infty} C^{max}(bmps) = \sum_{u\in\mathcal{V}} \max_v \left\{ d_{uv} \log_2 \Big(\frac{\sum_i K_{uv}^i \zeta_i}{\sum_i K_{uv}^i \zeta_i - \zeta_u} \Big) \right\}. \quad (23)$$

Similarly, we have

$$\lim_{P\to\infty} C^{max}(bps) = \sum_{u\in\mathcal{V}} \max_v \left\{ \log_2 \Big(\frac{\sum_i K_{uv}^i \zeta_i}{\sum_i K_{uv}^i \zeta_i - \zeta_u} \Big) \right\}. \quad (24)$$

Since ζ is a non-monopolized power assignment, according to Lemma 3, we have $\sum_i K_{uv}^i \zeta_i - \zeta_u > 0$ for any u, which implies that the denominator in (23) and (24) is not zero, i.e., there is a limit. □

5.5 Proof of Theorem 4

Proof.

$$\eta_0^{max}(bmpJ) = \lim_{P\to 0} \frac{C^{max}(bmps)}{P} = \lim_{P\to 0} \frac{\partial C^{max}(bmps)}{\partial P}. \quad (25)$$

Substituting (22) in (25) yields

$$\eta_0^{max}(bmpJ) = \lim_{P\to 0} \frac{1}{\ln 2} \sum_u \frac{\zeta_u d_{um_u} M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)}.$$

Because

$$\lim_{P\to 0} Q_{um_u} = \lim_{P\to 0} \big(\sum_i K_{um_u}^i \zeta_i P + M_{um_u} \big) = M_{um_u},$$

$$\lim_{P\to 0} P_u = 0,$$

we have

$$\eta_0^{max}(bmpJ) = \frac{1}{\ln 2} \sum_u \frac{\zeta_u d_{um_u}}{M_{um_u}}.$$

When $P \neq 0$, m_u is such a node that

$$f_{um_u} = \max_v \{f_{uv}\},$$

where

$$f_{uv} = d_{uv} \log_2 \frac{Q_{uv}}{Q_{uv} - P_u}.$$

It is obvious that $f_{uv} = 0$ if $P = 0$. Therefore, while $P \to 0$, m_u is such a node that

$$\frac{\partial f_{um_u}(P=0)}{\partial P} = \max_v \left\{ \frac{\partial f_{uv}(P=0)}{\partial P} \right\}.$$

Because

$$\frac{\partial f_{uv}(P=0)}{\partial P} = \frac{\zeta_u d_{uv}}{\ln 2 \cdot M_{uv}},$$

we have

$$\eta_0^{max}(bmpJ) = \frac{1}{\ln 2} \sum_u \zeta_u \max_v \left\{ \frac{d_{uv}}{M_{uv}} \right\}.$$

Similarly, we have

$$\eta_0^{max}(bpJ) = \frac{1}{\ln 2} \sum_u \zeta_u \max_v \left\{ \frac{1}{M_{uv}} \right\}.$$

□

5.6 Proof of Theorem 5

Proof. It is easy to see that η^{max} is also continuous and piecewise derivable since $\eta^{max} = \frac{C^{max}}{P}$. If η^{max} is derivable at P, we have

$$\frac{\partial \eta^{max}}{\partial P} = \frac{1}{P} \cdot \frac{\partial C^{max}}{\partial P} - \frac{1}{P^2} \cdot C^{max}. \quad (26)$$

Substituting (15) and (22) in (26) yields

$$\begin{aligned}
\frac{\partial \eta^{max}(bmpJ)}{\partial P} &= \frac{1}{\ln 2 \cdot P} \sum_u \frac{\zeta_u d_{um_u} M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)} \\
&\quad - \frac{1}{P^2} \sum_u d_{um_u} \log_2 \Big(\frac{Q_{um_u}}{Q_{um_u} - P_u} \Big) \\
&= \frac{1}{\ln 2 \cdot P^2} \sum_u \frac{P_u d_{um_u} M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)} \\
&\quad + \frac{1}{\ln 2 \cdot P^2} \sum_u d_{um_u} \ln \Big(1 - \frac{P_u}{Q_{um_u}} \Big).
\end{aligned}$$

That can be expressed as

$$\frac{\partial \eta^{max}(bmpJ)}{\partial P} = \frac{1}{\ln 2 \cdot P^2} \sum_u P_u d_{um_u} h_{um_u}, \quad (27)$$

where

$$h_{um_u} = \frac{M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)} + \frac{1}{P_u} \ln \Big(1 - \frac{P_u}{Q_{um_u}} \Big). \quad (28)$$

According to the well-known Taylor's Formula, we have

$$\begin{aligned}
\ln(1-x) &= -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^k}{k} - o(x^k) \\
&= -x\Big(1 + \frac{x}{2} + \frac{x^2}{3} + \cdots + \frac{x^{k-1}}{k} + o(x^{k-1})\Big) \\
&\le -x\Big(1 + \frac{x}{2} + \frac{x^2}{2^2} + \cdots + \frac{x^{k-1}}{2^{k-1}} + \cdots\Big) \\
&= -\frac{2x}{2-x}.
\end{aligned}$$

That is to say, for any $x \in [0,1)$, we have

$$\ln(1-x) \le -\frac{2x}{2-x}. \quad (29)$$

It is easy to know that $0 \le \frac{P_u}{Q_{um_u}} < 1$. Then, substituting $x = \frac{P_u}{Q_{um_u}}$ in (29) yields

$$\ln \Big(1 - \frac{P_u}{Q_{um_u}} \Big) \le -\frac{2P_u}{2Q_{um_u} - P_u}. \quad (30)$$

Combining (28) and (30) yields

$$\begin{aligned}
h_{um_u} &\le \frac{M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)} - \frac{2}{2Q_{um_u} - P_u} \\
&= \frac{2Q_{um_u}(P_u + M_{um_u} - Q_{um_u}) - P_u M_{um_u}}{Q_{um_u}(Q_{um_u} - P_u)(2Q_{um_u} - P_u)}. \quad (31)
\end{aligned}$$

According to the definition of Q_{um_u}, we have

$$\begin{aligned} & P_u + M_{um_u} - Q_{um_u} \\ &= P_u + M_{um_u} - \sum_i K^i_{um_u} P_i - M_{um_u} \\ &= -\sum_{i \neq u} K^i_{um_u} P_i. \end{aligned} \quad (32)$$

Since $P_i \geq 0$, $K^i_{um_u} > 0$, and $\sum_i P_i = P > 0$, we have

$$\begin{array}{ll} \sum_{i \neq u} K^i_{um_u} P_i > 0 & \text{if } P_u = 0, \\ \sum_{i \neq u} K^i_{um_u} P_i \geq 0 & \text{if } P_u > 0. \end{array} \quad (33)$$

Combining (32) and (33) yields

$$\begin{array}{ll} P_u + M_{um_u} - Q_{um_u} < 0 & \text{if } P_u = 0, \\ P_u + M_{um_u} - Q_{um_u} \leq 0 & \text{if } P_u > 0. \end{array}$$

Since $Q_{um_u} > 0$, we have

$$\begin{array}{ll} 2Q_{um_u}(P_u + M_{um_u} - Q_{um_u}) < 0 & \text{if } P_u = 0, \\ 2Q_{um_u}(P_u + M_{um_u} - Q_{um_u}) \leq 0 & \text{if } P_u > 0. \end{array} \quad (34)$$

Since $M_{um_u} > 0$, we also have

$$\begin{array}{ll} -P_u M_{um_u} = 0 & \text{if } P_u = 0, \\ -P_u M_{um_u} < 0 & \text{if } P_u > 0. \end{array} \quad (35)$$

According to (34) and (35), we have

$$2Q_{um_u}(P_u + M_{um_u} - Q_{um_u}) - P_u M_{um_u} < 0. \quad (36)$$

Combining (31) and (36) yields

$$h_{um_u} < 0. \quad (37)$$

Because $P > 0$, a node i exists such that $P_i > 0$. Then, combining (27) and (37), we have

$$\frac{\partial \eta^{max}(bmpJ)}{\partial P} < 0.$$

Similarly, if $\eta^{max}(bpJ)$ is derivable at P, we have

$$\frac{\partial \eta^{max}(bpJ)}{\partial P} < 0.$$

As a result, both $\eta^{max}(bpJ)$ and $\eta^{max}(bmpJ)$ strictly decrease at each derivable piece. Since $\eta^{max}(bpJ)$ and $\eta^{max}(bmpJ)$ are continuous and piecewise derivable, the maximum power efficiency, whether in bpJ or in $bmpJ$, strictly decreases with respect to the total transmitting power P. □

5.7 Proof of Theorem 6

PROOF.

$$\begin{aligned} \log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right) &= -\log_2\left(\frac{Q_{uv} - P_u}{Q_{uv}}\right) \\ &= -\log_2\left(1 - \frac{P_u}{Q_{uv}}\right) \\ &= -\frac{1}{\ln 2} \cdot \ln\left(1 - \frac{P_u}{Q_{uv}}\right). \end{aligned} \quad (38)$$

According to the well-known Taylor's formula, we have

$$\ln(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots. \quad (39)$$

Substituting $x = \frac{P_u}{Q_{uv}}$ in (39) yields

$$\ln\left(1 - \frac{P_u}{Q_{uv}}\right) = -\frac{P_u}{Q_{uv}} - \frac{P_u^2}{2Q_{uv}^2} - \frac{P_u^3}{3Q_{uv}^3} - \frac{P_u^4}{4Q_{uv}^4} - \cdots. \quad (40)$$

Substituting (40) in (38) yields

$$\begin{aligned} \log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right) &= \frac{1}{\ln 2} \cdot \left(\frac{P_u}{Q_{uv}} + \frac{P_u^2}{2Q_{uv}^2} + \frac{P_u^3}{3Q_{uv}^3} + \cdots\right) \\ &\leq \frac{1}{\ln 2} \cdot \left(\frac{P_u}{Q_{uv}} + \frac{P_u^2}{Q_{uv}^2} + \frac{P_u^3}{Q_{uv}^3} + \cdots\right). \end{aligned}$$

Since $\frac{P_u}{Q_{uv}} < 1$, we have

$$\log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right) \leq \frac{1}{\ln 2} \cdot \left(\frac{\frac{P_u}{Q_{uv}}}{1 - \frac{P_u}{Q_{uv}}}\right) = \frac{1}{\ln 2} \cdot \frac{P_u}{Q_{uv} - P_u}.$$

Therefore,

$$d_{uv} \log_2 \frac{Q_{uv}}{Q_{uv} - P_u} \leq \frac{1}{\ln 2} \cdot \frac{d_{uv} P_u}{Q_{uv} - P_u}. \quad (41)$$

Substituting the definition of Q_{uv} in (41) yields

$$d_{uv} \log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right) \leq \frac{1}{\ln 2} \cdot \frac{d_{uv} P_u}{\frac{d_{uv}^\alpha}{G_{uv}}\left(\sum_i \frac{G_{iv}}{d_{iv}^\alpha} P_i + N_v\right) - P_u}.$$

Since we assume $G_{uv} = 1$ for any link (u, v) and $N_v = N$ for any node v, we have

$$\begin{aligned} d_{uv} \log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right) &\leq \frac{1}{\ln 2} \cdot \frac{d_{uv} P_u}{d_{uv}^\alpha\left(\sum_i \frac{1}{d_{iv}^\alpha} P_u + N\right) - P_u} \\ &= \frac{1}{\ln 2} \cdot \frac{d_{uv} P_u}{\left(\sum_i \frac{d_{uv}^\alpha}{d_{iv}^\alpha}\right) P_u - P_u + d_{uv}^\alpha N} \\ &\leq \frac{1}{\ln 2} \cdot \frac{d_{uv} P_u}{d_{uv}^\alpha N} \\ &= \frac{1}{\ln 2} \cdot \frac{P_u}{d_{uv}^{\alpha-1} N}. \end{aligned}$$

Since $d_{uv} \geq d_{min}$ and $P_u \leq P_{max}$, we have

$$\max_v \left\{ d_{uv} \log_2\left(\frac{Q_{uv}}{Q_{uv} - P_u}\right)\right\} \leq \frac{1}{\ln 2} \cdot \frac{P_{max}}{d_{min}^{\alpha-1} N}. \quad (42)$$

Combining (42) and (5) yields

$$\begin{aligned} C^{max}(bmps) &\leq \sum_{u \in \mathcal{V}} \left(\frac{1}{\ln 2} \cdot \frac{P_{max}}{d_{min}^{\alpha-1} N}\right) \\ &= \frac{1}{\ln 2} \cdot \frac{P_{max} \cdot n}{d_{min}^{\alpha-1} N} \\ &= O(n). \end{aligned} \quad (43)$$

Similarly, we have

$$C^{max}(bps) \leq \frac{1}{\ln 2} \cdot \frac{P_{max} \cdot n}{d_{min}^\alpha N} = O(n). \quad (44)$$

□

6. PRACTICAL IMPLICATIONS

The theoretical analysis has drawn some conclusions on the effect of transmitting power on the capacity of wireless ad hoc networks composed of a certain number of nodes. The quantitative results are more meaningful than the previous asymptotic results. In this section, we highlight the practical implications of our results regarding power allocation, power assignment, and transmission scheduling.

6.1 Power Allocation

Definition 1 states that the power allocation at a node u refers to the allocation of u's transmitting power P_u for each link (u, v), where $v \in \mathcal{V}, v \neq u$. Under some ideal assumptions, one node might be capable of unicasting to more than one neighbor, which would pose the power allocation problem. Although this is not necessarily the case at present, insights into power allocation may shed some light on the study of transmission scheduling and packet routing.

Theorem 1 gives C^{max}, the upper bound on network capacity for a certain power assignment, which implies the best power allocation.

Corollary 1. *For each node u in a given wireless ad hoc network, the best power allocation insofar as maximizing network capacity in bps is concerned is:*

$$P_{um} = P_u,$$
$$P_{uv} = 0 \text{ for any } v \neq m,$$

where m is such a node that $Q_{um} = \min_v\{Q_{uv}\}$.

Corollary 2. *For each node u in a given wireless ad hoc network, the best power allocation insofar as maximizing network capacity in bmps is concerned is:*

$$P_{um} = P_u,$$
$$P_{uv} = 0 \text{ for any } v \neq m,$$

where m is such a node that

$$d_{um} \log_2 \left(\frac{Q_{um}}{Q_{um} - P_u} \right) = \max_v \left\{ d_{uv} \log_2 \left(\frac{Q_{uv}}{Q_{uv} - P_u} \right) \right\}.$$

According to the results on optimal power allocations, we observe that 1) it would not be a good idea to unicast simultaneously to more than one neighbor in narrow-band systems, even though the node is capable of doing it; and 2) the best next-hop (i.e. node m in the corollaries) insofar as maximizing network capacity is concerned is not necessarily the nearest neighbor. This may be worthy of further consideration from wireless network designers.

6.2 Power Assignment

Power assignment, range assignment, power control, and topology control all have much in common. These issues have attracted a significant amount of research interest over the past decade. Even so, there is still limited information concerning our understanding of the effect that transmitting power has on the capacity of wireless ad hoc networks.

Behzad and Rubin [1] argued that higher transmission power increases the capacity of wireless ad hoc networks. In reality, their result is the same as Theorem 2 in this paper. So, does higher transmitting power always increase network capacity? We will proceed to clarify this statement by using an example to show that this is not necessarily the case. We consider a network on the plane, as shown in Fig.2. Nodes 1 to 8 are uniformly placed on the circle centered at node 0 with $r = 1000m$ as its radius. We assume $\alpha = 3$, $G_{uv} = 1$ for any link (u, v), $P_u = 100mW$, and $N_u = 10^{-7}mW$ for any node u. According to Theorem 1, it is easy to write a program to compute the maximum per-unit-bandwidth capacity. For our example, we have $C^{max}(bps) = 4.206507$ and $C^{max}(bmps) = 3267.208872$. If we increase P_0 from $100mW$ to $120mW$, we have $C^{max}(bps) = 4.122036$

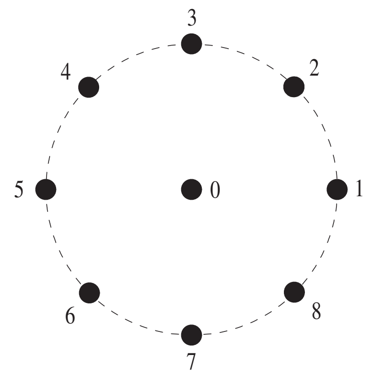


Figure 2: Higher transmitting power does not necessarily increase the capacity of the entire network.

and $C^{max}(bmps) = 3211.339561$. Therefore, increasing the transmitting power of one node does not necessarily increase the capacity of the entire wireless ad hoc network.

We want to determine what the best power assignment is. Unfortunately, according to our results, no practical power assignment exists insofar as maximizing C^{max} or maximizing η^{max} is concerned. Theorem 2 states that C^{max} strictly increases with respect to the total transmitting power under a fixed-proportion assignment, while Theorem 5 shows that η^{max} strictly decreases. There is a tradeoff between C^{max} and η^{max}. Power assignment needs further research.

6.3 Transmission Scheduling

According to the quantitative study of C^{max}, we believe that transmission scheduling plays a more important role than power assignment in improving the capacity of wireless ad hoc networks. This is intuitively the case because transmission scheduling can significantly reduce radio interference. In reality, transmission scheduling has drawn an increasing amount of research interest [2, 3, 13, 19, 23]. However, to the best of our knowledge, no previous work on transmission scheduling is based on the computation of the maximum network capacity. The quantitative results of this paper could potentially be utilized for the optimization of transmission scheduling.

Given a certain transmission scheduling and power assignment, Theorem 1 provides a centralized computation of C^{max} (and hence, η^{max}) in $O(n^3)$ time, where n is the number of nodes. Since its applicability is not very obvious due to the computational complexity, we present a further discussion. In the theoretical analysis, we assume that there is a link between any two nodes, regardless of how bad the link is. In practice, we do not need to take all links into account. C^{max} can be approximately computed by neglecting bad links. Furthermore, not every node needs to consider the interference from distant nodes. In other words, the capacity of each node can be maximized locally. Since Lemma 2 states that C^{max} is the sum of the maximum capacity of all nodes, the C^{max} (and hence, η^{max}) of a given network can be optimized in a localized manner. This observation sheds light on some practical implications. Therefore, the results of this paper may be of interest to designers seeking to develop transmission scheduling algorithms as well as MAC protocols.

7. CONCLUSIONS

Under the assumption that all interference is essentially regarded as noise, we have quantitatively investigated the effect that transmitting power has on network capacity from a perspective of information theory. We have given the maximum capacity of a wireless ad hoc network under a certain power assignment and nodal distribution, disregarding the manner in which the network operates. For a fixed-proportion power assignment, we have shown that the maximum capacity strictly increases with respect to the total transmitting power, and that there is a limit as the total transmitting power reaches infinity. Although higher transmitting power may well increase the capacity of wireless ad hoc networks, we demonstrate that this is not necessarily the case if transmitting powers are not proportionally increased. We have also proved that the maximum power efficiency in the sense of network capacity strictly decreases with the total transmitting power under a fixed-proportion assignment. We have further highlighted the practical implications for power allocation, power assignment, and transmission scheduling. Since the maximum capacity at any time can be optimized in a localized manner, the results of this paper may be of interest to designers seeking to develop networking algorithms and protocols.

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