Collaborative Mobile Charging for Sensor Networks

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Abstract—The limited battery capacity of sensor nodes has become the biggest impediment to wireless sensor network (WSN) applications. Two recent breakthroughs in the areas of wireless energy transfer and rechargeable lithium batteries promise the use of mobile vehicles, with high volume batteries, as mobile chargers that transfer energy to sensor nodes wirelessly. In this paper, for the first time, we envision a novel charging paradigm: collaborative mobile charging, where mobile chargers are allowed to charge each other. We investigate the problem of scheduling multiple mobile chargers, which collaboratively recharge sensors, to maximize the ratio of the amount of payload energy to overhead energy, such that every sensor will not run out of energy. We first consider the uniform case where all sensors consume energy at the same rate, and propose a scheduling algorithm, PushWait, which is proven to be optimal in this case and can cover a one-dimensional WSN of infinite length. Then, in the non-uniform case, which is conjectured to be NP-hard, we first present two observations from space and time aspects to remove some impossible scheduling choices, and we propose our heuristic algorithm, ClusterCharging(β), which clusters sensors into groups and divides a scheduling cycle into charging rounds. Its approximation ratio is also presented. Extensive evaluations confirm the efficiency of our algorithms.

Index Terms—Collaborative mobile charging, wireless energy transfer, wireless sensor networks.

I. Introduction

Many applications of wireless sensor networks (WSNs) [1], such as structural health monitoring for the Golden Gate Bridge [2], agricultural rain-fed farming decisions [3], and forest fire detection [4], desire a long-lived WSN. However, sensor nodes are typically supplied by batteries that can only store a limited amount of energy, which has become the biggest impediment. Therefore, a lot of efforts, including energy conservation [5–7], energy harvesting [8, 9], and sensor reclamation [10], have been devoted to prolonging the lifetime of WSNs. However, energy conservation cannot compensate for energy depletion; energy harvesting is neither controllable nor predictable; sensor reclamation is costly and impractical when sensors are deployed in the deep ocean, on bridge surfaces, or in containers of hazardous materials.

We recently observed two particular breakthroughs in the areas of wireless energy transfer [11, 12] and rechargeable lithium batteries [13]. Wireless energy transfer is the transmission of electric energy from a power source to a receiver without any interconnecting conductors. Rechargeable lithium batteries with high energy density and high charge/discharge capability are identified in [13]. Armed with these two technologies, some studies [14–17] employed mobile vehicles of high volume batteries as mobile chargers to deliver energy to sensors. However, most of them [14–16] assume that a mobile charger has a sufficient amount of battery to cover the entire WSN and to make a round trip back to the base station. This model will become invalid when there is a remote area where even a dedicated charger with full battery energy cannot reach before running out of energy.

In this paper, we introduce a novel charging paradigm: collaborative mobile charging, where mobile chargers are allowed to charge each other. That is to say, multiple mobile chargers start from the base station with full energy, and after some time, some of them can intentionally gather at a rendezvous point to recharge others or to be recharged. We shall see that this collaborative paradigm not only enlarges the charging coverage, but also improves the energy efficiency since chargers in existing methods may return to the base station with residual energy. The scheduling problem of collaborative mobile charging in a general WSN is very complicated. As a first step, to initiate a meaningful study, this paper narrows the scope of this problem to a manageable extent: we consider one-dimensional (1-D) WSNs and leave 2-D as future work. The linear structure of 1-D WSNs can be utilized to reduce maintenance costs, increase routing efficiency, and improve network reliability [18]; therefore, these kind of WSNs have a broad array of applications, ranging from oil/gas/water pipeline monitoring [19] to driver-alert systems [18] to bridge and international border protection [20].

This paper focuses on the following problem: given a 1-D WSN and battery capacity constraints, how can we schedule multiple mobile chargers, which collaboratively recharge sensors, to maximize the ratio of the amount of payload energy to overhead energy, such that every sensor will not run out of energy? To gain a better understanding, we first consider the uniform case of this problem, where all sensors consume energy at the same rate, for which we propose an algorithm called PushWait. We prove the optimality of PushWait in this case. Then, in the non-uniform case of this problem, we conjecture that the problem becomes NP-hard and propose a heuristic algorithm called ClusterCharging(β) with guaranteed performance. We evaluate the performance of our algorithms with extensive simulations. The contributions of this paper are summarized as follows:

1. To our best knowledge, we are the first to consider the collaborative mobile charging paradigm. By means of examples, theoretical analysis and experimental evaluations, this paper demonstrates the advantages of this novel paradigm in coverage and energy efficiency.

2. For the uniform case of the scheduling problem, we propose a scheduling algorithm, PushWait, which is proven...
to be optimal and can cover a 1-D WSN of any length. A variation of *PushWait* that uses dedicated chargers to substitute roundtrip chargers is also presented.

(3) For the non-uniform case, which is conjectured to be NP-hard, we first present two observations from space and time aspects to remove some impossible scheduling choices. Then, we propose our heuristic algorithm, *ClusterCharging*($\delta$), which clusters sensors into groups and divides a scheduling cycle into charging rounds. Its approximation ratio is also presented.

II. PROBLEM DESCRIPTION

A. Network Model

We consider a set of sensor nodes that are uniformly distributed, unit distance apart, along a one-dimensional straight line to the east of a base station (BS), as shown in Fig. 1. There are, in all, $N$ sensor nodes, say $s_1, s_2, \ldots, s_N$. Sensor $s_i$ consumes $r_i$ amount of energy per unit time. All nodes are assumed to have the same battery capacity, say $b$. The recharging cycle of a sensor is defined as the time period that this sensor of full energy can survive without being charged. Denote the recharging cycle of $s_i$ as $\tau_i$; we have $\tau_i = b/r_i$.

B. Charging Model

A mobile charger (MC) has a maximum battery capacity of $B$ and consumes $c$ amount of energy per unit distance. The base station BS serves as data sink as well as the energy source. Mobile chargers start from the BS with full batteries, charge sensors, finally come back to the BS, and then get themselves recharged by the BS. Both the movement of the mobile chargers and the process of wireless charging share the same pool of battery energy.

The energy transfer efficiencies of BS-to-MC, MC-to-MC, and MC-to-sensor are all assumed to be 1, i.e., there is no energy loss. The corresponding charging time is negligible compared to the traveling time of mobile chargers.

Typically, the recharging cycle (or the lifetime) of a sensor is several months; while the time for a charger traveling from the BS to the farthest sensor in a WSN is usually several hours, or at most several days. Thus, in this paper, we assume that any two charging rounds have no intersection, i.e., mobile chargers can always accomplish a charging round, return to the BS, and wait for another charging round$^1$.

C. Performance Measure

When scheduling, we must decide the actions (such as, recharging a sensor or another charger, being charged, waiting, etc.) of each mobile charger in its time-space trajectory. A scheduling is said to be feasible if (i) all sensor nodes do not die, i.e., each sensor node will get charged before running out of energy, and (ii) all MCs are able to return to the BS to be serviced (e.g., replacing or recharging its battery).

$^1$We make this assumption for the brevity of presentation. As we shall see shortly, when the recharging cycle is smaller than the MC round-trip time, pipeline-like parallel *PushWait* could still achieve its optimality.

We define the scheduling cycle of a scheduling to be the time interval between two consecutive points of time when all sensors are fully charged. Although this definition seems strange at a first glance, we shall see its generality. It can be applied to the uniform case problem (Section III), where the scheduling cycle equals the recharging cycle of each sensor, and it can also be applied to the non-uniform case problem (Section IV), where the scheduling cycle contains more than one recharging cycle of a sensor.

In a scheduling cycle, denote the energy eventually obtained by sensors as payload energy ($E_{\text{payload}}$), and the energy consumed by MCs’ movements as overhead energy ($E_{\text{overhead}}$). The efficiency ratio of the scheduling can be defined as:

$$\text{ratio} = \frac{E_{\text{payload}}}{E_{\text{overhead}}}$$

A feasible scheduling cyclically charges sensor nodes to make a sensor network long-lived, so this definition characterizes the long-term efficiency of a scheduling well.

D. Scheduling as an Optimization Problem

**Problem 1:** (Collaborative mobile charging scheduling problem (CMCS)) Given a 1-D WSN with parameters $b$ and $r_i$, how can we find a feasible scheduling of chargers, with parameters $c$ and $B$, so as to maximize the ratio defined above.

In order to have a better understanding of the CMCS problem, we first consider a uniform case where all sensors consume energy at the same rate in Section III; then, we study the non-uniform case in Section IV with the knowledge obtained from the uniform case.

III. CMCS WITH UNIFORM ENERGY CONSUMPTION RATE

In the uniform case, sensors consume energy at the same rate, which is denoted as $r$, then all sensors have the same recharging cycle, i.e., $\tau = b/r$. This uniformity makes the scheduling become simple: in each recharging cycle, we let the MCs charge all of the sensors and then wait for the next recharging cycle. Therefore, in this case, the scheduling cycle equals the recharging cycle. We then have $E_{\text{payload}} = N \cdot b$ is fixed, so the objective of maximizing the ratio of $E_{\text{payload}}$ to $E_{\text{overhead}}$ is reduced to minimizing $E_{\text{overhead}}$.

A. Motivational Examples

We use the following examples to demonstrate the benefits of collaborative mobile charging and to motivate our algorithm design. Three different scheduling schemes are shown in Fig. 2. The former two schemes do not consider collaboration, while the third does. We denote $K$ as the number of MCs, $L_i$ ($1 \leq i \leq K$) as the farthest point that $MC_i$ reaches, and also let $L_{K+1} = 0$ for compatibility. Figs. 2(a), 2(b), and 2(c) illustrate the time-space view as well as the maximum coverage of these three scheduling schemes, respectively. The settings are $B = 80J, b = 2J, c = 3J/m$, and $K = 3$. 

![Fig. 1: Problem description](image-url)
Scheme I: Each MC charges each sensor an amount of $b/K$ energy, and each sensor is charged by all passing MCs. As shown in Fig. 2(a), 12 sensors can be covered.

Scheme II: Each sensor is charged by only one MC. MC$_i$ (1 $\leq i \leq K$) charges sensors from $L_{i+1}$ to $L_i$, its residual energy at $L_i$ is just enough for it to return to the BS. Fig. 2(b) shows the entire process, where 13 sensors can be covered.

Scheme III: Each sensor is charged by only one MC. MC$_i$ (2 $\leq i \leq K$) charges sensors from $L_{i+1}$ to $L_i$, and it transfers energy to MC$_{i-1}$, MC$_{i-2}$, ..., and MC$_1$ until they are at their full energy capacity at $L_i$, and then it just has enough energy to return to the BS. Fig. 2(c) illustrates this scheme, where MC$_3$ charges MC$_2$ and MC$_1$ at A, and MC$_2$ charges MC$_1$ at B. This time, 17 sensors can be covered.

In summary, given a fixed number of MCs, scheme III can cover more sensors than Schemes I and II; collaboration makes scheduling more energy-efficient in the sense that scheduling with collaboration consumes less $E_{\text{overhead}}$ than scheduling without collaboration to deliver the same amount of $E_{\text{payload}}$.

B. PushWait

Recall the objective of our scheduling is to minimize $E_{\text{overhead}}$, which is consumed by MCs’ movement. The basic idea of PushWait is to use as less MCs as possible to carry the residual energy of all MCs through letting some MCs charge others at some rendezvous points. PushWait is illustrated as:

- MC$_i$ charges sensors between $L_{i+1}$ and $L_i$ to their full batteries. At $L_i$, MC$_i$ transfers energy to MC$_{i-1}$, MC$_{i-2}$, ..., and MC$_1$ until they are at their full energy capacity. Then MC$_i$ waits at $L_i$, and all of the other $i-1$ MCs keep moving forward.
- After MC$_{i-1}$, MC$_{i-2}$, ..., and MC$_1$ return to $L_i$, where MC$_i$ waits for them, MC$_i$ evenly distributes its residual energy among $i$ MCs (including MC$_i$ itself) and moves towards the BS. The reason of naming this scheduling after “PushWait” is clear: from the point of view of MC$_i$, it pushes the other MCs to move forward and waits for their return.

Fig. 2(d) depicts the time-space view of applying PushWait to the aforementioned settings. Three chargers start from the BS with full battery, gets fully charged at locations $L_9$, $L_{K-1}$,..., and $L_{i+1}$, charges sensor nodes between $L_{i+1}$ and $L_i$, charges MC$_{i-1}$, MC$_{i-2}$, ..., and MC$_1$ at $L_i$, waits for these MCs to return, then evenly distributes its residual energy among these $i$ MCs (including MC$_i$ itself) and moves towards the BS. The reason of naming this scheduling after “PushWait” is clear: from the point of view of MC$_i$, it pushes the other MCs to move forward and waits for their return.

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C. Rendezvous Points

To make PushWait work, $L_i$ (1 $\leq i \leq K$) should be chosen carefully to guarantee that MC$_1$, MC$_2$,..., and MC$_{i-1}$ have zero energy when they return to $L_i$. Let’s take the interval between $L_{i+1}$ and $L_i$ as an example to illustrate how to determine the values of these $K$ rendezvous points.

MC$_i$ gets fully charged at $L_{i+1}$ and comes back to $L_{i+1}$ with zero energy. The energy consumption of the full battery $B$ includes the following five parts: (i) energy transferred to the sensors between $L_{i+1}$ and $L_i$; (ii) energy consumed by MC$_i$ to travel from $L_{i+1}$ to $L_i$; (iii) energy transferred to MC$_1$, MC$_2$, etc. etc. in Fig. 2(d).
..., and $MC_{i-1}$ at $L_2$ for the first time. Note that these $i - 1$ $MC$s are fully charged at $L_{i+1}$, thus the energy transferred to them at $L_i$ is exactly the energy consumed by them to travel from $L_{i+1}$ to $L_i$; (iv) energy consumed by $MC_i$ to travel from $L_1$ to $L_{i+1}$; and (v) energy transferred to $MC_1$, $MC_2$, ..., and $MC_{i-1}$ at $L_3$ for the second time, which is exactly the energy used by them to travel from $L_1$ to $L_{i+1}$.

Therefore, we have the following equations.

$$
2 \cdot c \cdot (L_1 - L_2) \cdot 1 + b \cdot (L_1 - L_2) = B \\
2 \cdot c \cdot (L_{i-1} - L_i) \cdot (1 - b) \cdot (L_{i-1} - L_i) = B \quad (2 \leq i \leq K) \\
2 \cdot c \cdot (L_K - 0) \cdot K + b \cdot (L_K - 0) \leq B
$$

The last formula is an inequality, since $PushWait$ cannot use up the total amount of energy of $K$ $MC$s precisely. It is straightforward to see that:

$$
\begin{align*}
L_1 &= N \\
L_i &= N - \sum_{j=1}^{i-1} B \frac{c + d}{d \cdot d + b} \\
&\quad (2 \leq i \leq K)
\end{align*}
$$

$K$ can be determined by: $L_K > 0$, $L_{K+1} \leq 0$. Then, we have:

$$
E_{payload} = N \cdot b \\
E_{overhead} = 2c \cdot \sum_{i=1}^{K} L_i
$$

We note in passing that, as $MC_K$ may have some residual energy when it returns to the BS, we can further improve $PushWait$ through the following trick. Let another $MC'$ stay at $L_K$ to collect the residual energy that $MC_K$ would take back to the BS. In doing so, after enough scheduling cycles, only $(K-1)$ $MC$s are required to start from the BS in the subsequent scheduling cycle.

Fig. 2(d) shows an example, $MC_3$ has 14J residual energy when it comes back to the BS; another $MC'$ can be used to collect 14J energy at $L_3$; after five scheduling cycles, $MC'$ will have 70J energy. As the reader can verify, in the sixth scheduling cycle, only two $MC$s are needed.

**D. Optimality**

**Theorem 1:** For the uniform case of the CMCS problem, $PushWait$ achieves the maximum ratio of $E_{payload}$ to $E_{overhead}$.

**Proof:** Given a 1-D WSN where sensors consume energy at the same rate, $E_{payload}$ is fixed in a recharging cycle. Hence, it is sufficient to prove that $PushWait$ uses the minimum $E_{overhead}$, which is proportional to the distance traveled by all of the $MC$s. Denote the distance traveled by all of the $MC$s in a scheduling scheme as $Distance(scheme)$. Suppose that $PushWait$ requires $K$ $MC$s to charge the given WSN. We prove the theorem by induction on $K$.

**Base cases:** $K=1$ and $K=2$

- $K=1$: this case is trivial. $Distance(PushWait) = 2 \cdot L_1$. Note that $L_1$ equals the length of the given WSN. Any scheduling scheme must have at least one $MC$ to reach the farthest sensor in the WSN and then turn back, thus $Distance(\text{any scheme}) \geq 2 \cdot L_1 = Distance(PushWait)$.

- $K=2$: (by contradiction) suppose that $PushWait$ is not optimal, and the optimal scheduling scheme is $OPT$. As one $MC$ is not enough to cover the entire WSN, there are at least two $MC$s in the $OPT$. One of them, say $MC'$, must reach the farthest sensor, thus it travels $2 \cdot L_1$ distance. Since $OPT$ is the optimal scheduling scheme, i.e., $Distance(\text{OPT}) < Distance(PushWait) = 2 \cdot L_1 + 2 \cdot L_2$. Hence, all of the other $MC$s in $OPT$ should not reach $L_2$; otherwise, $OPT$ is not optimal. However, according to our calculation of $L_2$ in $PushWait$, a fully charged $MC$ at $L_2$ only charges the sensors from $L_2$ to $L_1$ and returns to $L_2$ with zero energy, then we know $MC'$ in $OPT$ can by no means reach $L_1$. A contradiction! Therefore, no such $OPT$ exists. $PushWait$ is optimal.

**I.H.: PushWait is optimal when $K=n$.**

- $K=n+1$: (by contradiction) suppose that $PushWait$ is not optimal, and there are $n+1$ rendezvous points $L_{n+1}$, $L_n$, $L_1$ in $PushWait$. The optimal scheduling scheme is $OPT$.

We can divide the WSN into two parts, the BS-to-$L_{n+1}$ part, and the $L_{n+1}$-to-$L_1$ part. Suppose that a virtual base station $BS'$ is located at $L_{n+1}$. $PushWait$ needs precisely $n \cdot B$ energy to cover the sensors between $L_{n+1}$ and $L_1$. By I.H., $OPT$ will require more than $n \cdot B$ energy to cover the same part. Denote this energy as $Q > n \cdot B$.

Therefore, the task of $PushWait$ is to cover the sensors from BS to $L_{n+1}$ to deliver $n \cdot B$ energy to $L_{n+1}$. According to $PushWait$, $n+1$ $MC$s that start from the BS can accomplish this task. Correspondingly, the task of $OPT$ is to cover the sensors from BS to $L_{n+1}$ to deliver $Q$ energy to $L_{n+1}$. We know that $OPT$ requires at least $n + 1$ $MC$s to reach $L_{n+1}$ (otherwise, the total residual energy of less than $n+1$ $MC$s at $L_{n+1}$ is definitely less than $n \cdot B$).

Considering $Q > n \cdot B$, $PushWait$ is optimal.

Remarks: This theorem still holds in some some general settings, for example, the distance between adjacent sensors is non-uniform, the sensor battery capacity is non-uniform, the recharging cycle is smaller than MC round-trip time, and so on. The corresponding proofs follow a similar routine as the above proof and are left to the reader.

**E. Coverage**

**Theorem 2:** Given infinite $MC$s, the maximum numbers of sensors that can be covered by scheme I, II, III, and $PushWait$ are $< B/2c$, $< B/2c$, $< B/c$, and infinite, respectively.

**Proof:** Scheme I: Each $MC$ needs to contribute $b/K$ amount of energy to each sensor. When $K$ approaches infinity, the share $b/K$ approaches 0. However, every $MC$ still needs to return to the BS, thus the maximum number of sensors that can be covered by this scheme is less than $B/2c$.

Scheme II: When $i$ increases, $MC_i$ needs to travel a longer distance to reach the sensors that it should cover. Also, every $MC$ needs to return to the BS, so the maximum number of sensors that can be covered by this scheme is less than $B/2c$.

Scheme III: When an $MC$ begins to turn back, it can no longer get energy from others. Thus, the maximum number of sensors that can be covered is less than $B/c$. 
PushWait: According to Equ. (3), we have:

\[ L_i - L_{i+1} = \frac{B}{2 \cdot c \cdot i + b}, \forall i \geq 1 \]  

(4)

Then the distance covered by \( K \) MCs is:

\[
\sum_{i=1}^{K} \frac{B}{2 \cdot c \cdot i + b} > \sum_{i=0}^{K} \frac{B}{2 \cdot c \cdot i + b} \text{ (let } 2 \cdot c \cdot i_0 \geq b) 
\]

\[
> \frac{K}{4} \cdot c \cdot \sum_{i=1}^{K} \frac{1}{i} \text{ (harmonic series)}
\]

which approaches infinity as \( K \) approaches infinity.

\[ \square \]

A. Examples of Scheduling Schemes

Denote \( \tau_i = b/r_i \) as the recharging cycle of sensor \( s_i \). To avoid the messy details and focus on the main problem, we assume that \( \tau_i \) is an integer (in fact, \( \tau_i \) is typically large enough for us to let \( \tau_i = \lceil \tau_i \rceil \)). Fig. 4 shows two feasible scheduling schemes. At time \( t_0 \), four sensors are full of energy; as \( \tau_1 = 4 \), \( s_1 \) should be recharged no later than time \( t_0 + 4 \), otherwise it will die; similar statements hold for other sensors. Recall our definition of scheduling cycle in Section II-C; in these examples, the consecutive time points when all sensors get fully charged are \( t_0 \) and \( t_0 + 6 \). Therefore, the scheduling cycle of both of Scheme IV and V is 6, and there are 6 rounds of charging in a scheduling cycle in both of them.

We notice from these examples that the number of possible scheduling solutions could be extremely large, because any charging round in a solution has exponential choices of a set of sensors to recharge. To find the optimal scheduling for a given WSN, we must determine both the length of the scheduling cycle and the set of sensors to be recharged in each round. With all that said, we conjecture that the non-uniform case of the CMCS problem is NP-hard. In the following subsections, we will present a heuristic with an approximation ratio after introducing two observations.

B. Observation from Space Aspect

In Fig. 4(a), \( \tau_1 = 4 > \tau_2 = 2 \), whenever we recharge \( s_2 \), we can recharge \( s_1 \) incidentally. Considering that the objective is to maximize the ratio of payload energy to overhead energy, we see that there is no need to recharge \( s_1 \) individually, i.e., the recharging at point A in Fig. 4(a) is not cost-efficient. In doing so, we can take \( \tau_1 \) as 2. For the same reason, we can take \( \tau_3 \) as 3, and recharge \( s_3 \) at time \( t_0 + 3 \) (as point C shows) instead of \( t_0 + 5 \) (as point B shows). This observation enables us to only consider the following setting in the rest of this paper: \( \tau_1 \leq \tau_2 \leq \cdots \leq \tau_N \).
C. Observation from Time Aspect

The following theorem indicates that we only need to start a charging round when there is at least one sensor node that will die if we do not. For example, in Fig. 4(b), we only plan possible rounds at \( t = t_0 + i \), where \( i \) is an integer; we do not need to start the round at \( t_0 + 2.5 \) because, at that time, there is no sensor node that will die if it is not charged.

**Theorem 3:** Given a base station \( BS \) and a sensor node \( s \), with a battery capacity of \( b \), that are \( d \) distance apart, then it is better to deliver \( b \) amount of energy to \( s \) using \( PushWait \) one time than twice (the total energy \( s \) gets is still \( b \)).

**Proof:** Note that \( d \) is not restricted to being small, thus one \( MC \) may not be enough. Suppose that \( k \) \( MCs \) are needed to deliver \( b \) amount of energy to \( s \) using \( PushWait \) one time; according to Eq. (4), \( k \) should satisfy:

\[
\frac{B}{2kc} + \frac{B}{2(k-1)c} + \cdots + \frac{B-b}{2c} = \frac{1}{2c} \sum_{i=1}^{k} \frac{b}{i} = d
\]

Equivalently:

\[
\sum_{i=1}^{k} \frac{1}{i} = \frac{2cd + b}{B}
\]  

(5)

Similarly, if we use \( PushWait \) twice, suppose that \( k_1 \) \( MCs \) are needed to deliver \( \epsilon \) amount of energy to \( s \) for the first time, and \( k_2 \) \( MCs \) are needed to deliver \( b-\epsilon \) amount of energy to \( s \) for the second time, then \( k_1 \) and \( k_2 \) should satisfy:

\[
\frac{B}{2k_1c} + \frac{B}{2(k_1-1)c} + \cdots + \frac{B-\epsilon}{2c} = \frac{1}{2c} \sum_{i=1}^{k_1} \frac{1}{i} - \frac{\epsilon}{2c} = d
\]

\[
\frac{B}{2k_2c} + \frac{B}{2(k_2-1)c} + \cdots + \frac{B-(b-\epsilon)}{2c} = \frac{1}{2c} \sum_{i=1}^{k_2} \frac{b-\epsilon}{i} = d
\]

Equivalently:

\[
\sum_{i=1}^{k_1} \frac{1}{i} = \frac{2cd + \epsilon}{B}, \quad \sum_{i=1}^{k_2} \frac{1}{i} = \frac{2cd + b - \epsilon}{B}
\]  

(6)

Then it is sufficient to prove that \( k_1 + k_2 < k \) cannot be true, subject to Eqs. (5) and (6). The harmonic series [21] can be represented as:

\[
\sum_{i=1}^{k} \frac{1}{i} \approx \ln k + \frac{1}{2k} + \gamma
\]  

(7)

where \( \gamma \) is the Euler-Mascheroni constant. As we know, if \( k_1 + k_2 \) is fixed, \( \sum_{i=1}^{k_1} 1/i + \sum_{i=1}^{k_2} 1/i \) achieves its maximum when \( k_1 = k_2 \). Therefore, we let \( k_1 = k_2 = k/2 \) to see what condition \( k \) should satisfy to ensure Eqs. (5) and (6). By combining Eqs. (5), (6), and (7), we have:

\[
k = \frac{B}{2ln2 \cdot B - b} < \frac{B}{2 \cdot B - B} = 1
\]

(8)

which is impossible. Thus, we prove that \( k_1 + k_2 > k \), indicating that using \( PushWait \) one time is more cost-efficient.

It is worth mentioning that the above proof is based on the following assumption: the total amount of energy of \( k \) \( MCs \), \( k_1 \) \( MCs \), or \( k_2 \) \( MCs \) is completely used up. The worst case is when the amount of energy of \( k \) \( MCs \) is completely used up while there are two \( MCs \) among \( k_1 \) and \( k_2 \) whose energy is nearly unused. Then, the two times of \( PushWait \) costs the total energy of \( k_1 - 1 + k_2 - 1 = k_1 + k_2 - 2 \) \( MCs \). As \( k_1 + k_2 \geq k + 1 \), the only bad situation is \( k_1 + k_2 = k + 1 \), which is rare compared to all possible cases.

D. ClusterCharging(\( \beta \))

1) Basic idea: The basic idea of ClusterCharging(\( \beta \)) is to cluster sensors into groups such that the ratio of the maximum recharging cycle to the minimum recharging cycle in each group is less than the clustering threshold \( \beta \). In each charging round, our heuristic selects the groups that satisfy the following condition as the charging targets, and employs \( PushWait \) to recharge the sensors in these groups. The condition for a group to be selected is that there is at least one sensor in this group that is going to die if our heuristic does not recharge it.

Take Fig. 5 for example. In Fig. 5(a), \( \beta = 1 \), thus each sensor itself forms a group; each group (or sensor) gets recharged only before running out of energy.

In Fig. 5(b), \( \beta = 2 \). Since \( \tau_1 = 1 \) and \( \tau_2 = 2 \), \( \tau_2/\tau_1 \geq 2 = \beta \), then \( s_1 \) itself forms a group. Also \( \tau_3/\tau_2 < \beta \) and \( \tau_4/\tau_2 \geq \beta \), thus sensors \( s_2 \) and \( s_3 \) form a group, and so forth. In summary, when \( \beta = 2 \), there are three groups, \( (s_1) \), \( (s_2,s_3) \), and \( (s_4,s_5,s_6) \). At time \( t_0 + 1 \), only the first group is recharged; at time \( t_0 + 2 \),

\[ \text{Note that, in each round of this scenario, different sensors may need to be charged a different amount of energy. For example, at time } t_0 + 4 \text{ in Fig. 5(b), the six sensors need } b, b, 2b/3, b, 4b/5, \text{ and } 4b/6 \text{ amount of energy, respectively. PushWait still achieves the optimality in each round according to the remarks in Section III-D.} \]
as $s_2$ is going to die if it is not recharged, the second group together with the first group are recharged; at time $t_0 + 3$, all groups are selected as the charging targets.

Similarly, when $\beta = 3$ in Fig. 5(c), there are three groups, $(s_1, s_2)$, $(s_3, s_4, s_5)$, and $(s_6)$. When $\beta = \infty$ in Fig. 5(d), there is only one group that contains all sensors.

2) Scheduling Cycle: $\beta = 1$. ClusterCharging(1) lazily charges each sensor just before it runs out of energy. The scheduling cycle is the least common multiple of $\tau_1, \ldots, \tau_N$. Denote it as $lcm$. Then, the $E_{\text{payload}}$ in a scheduling cycle is:

$$E_{\text{payload}} = \frac{lcm}{\tau_1} b + \frac{lcm}{\tau_2} b + \cdots + \frac{lcm}{\tau_N} b = \sum_{i=1}^{N} \left( \frac{r_i}{lcm} \right)$$

$\beta = 2, 3, \ldots, n$. Suppose that there are $x$ groups; since the ratio of the maximum recharging cycle to the minimum recharging cycle in each group is less than $\beta$, we have $\tau_1 \cdot \beta^x \leq \tau_N$, then we know $x = \lceil \log_\beta(\tau_N/\tau_1) \rceil$. Thus, the scheduling cycle of ClusterCharging($\beta$) is:

$$\beta^{\lceil \log_\beta(\tau_N/\tau_1) \rceil} \cdot \tau_1$$

For example, the scheduling cycles in Figs. 5(b) and 5(c) are 4 and 6, respectively. Correspondingly, we can calculate $E_{\text{payload}}$ and $E_{\text{overhead}}$ in a scheduling cycle using PushWait.

$\beta = \infty$, ClusterCharging($\infty$) charges all sensors to their full battery capacity every $\tau_1$ time, i.e., the minimum recharging cycle among all sensor nodes. Obviously, the scheduling cycle is $\tau_1$. Then, the $E_{\text{payload}}$ in a scheduling cycle is:

$$E_{\text{payload}} = \frac{\tau_1}{\tau_1} b + \frac{\tau_1}{\tau_2} b + \cdots + \frac{\tau_1}{\tau_N} b = \sum_{i=1}^{N} \frac{\tau_i}{\tau_1} b$$

Different values of $\beta$ lead to different performances. In our simulations, we will show that when the parameters of the problem instance change, the optimal $\beta$ also changes.

3) Approximation Ratio of ClusterCharging($\beta$): Denote by ratio($S_1$) the ratio of $E_{\text{payload}}$ to $E_{\text{overhead}}$ in a scheduling scheme $S$. Denote by OPT the optimal scheduling scheme for the non-uniform case of the CMCS problem.

**Theorem 4:**

$$\frac{\text{ratio(ClusterCharging($\beta$))}}{\text{ratio(OPT)}} > \frac{2c}{k_{min}Brt_N - b}$$

where,

$$k_{min} = \arg\min \left( \sum_{i=1}^{N} \frac{1}{r_i} \geq \frac{(2cB + b)rt_N}{Br_N} \right)$$

**Proof:** The main line of this proof is to construct a scheme $S_1$ so that ratio($S_1$) < ratio(ClusterCharging($\beta$)), and to construct another scheme $S_2$ so that ratio(OPT) < ratio($S_2$). We then have ratio(ClusterCharging($\beta$))/ratio(OPT) > ratio($S_1$)/ratio($S_2$).

Constructing $S_1$. Consider the following charging round: we use $MC$ to charge only one sensor, which is at the farthest point of the WSN and only needs the least possible energy amount, i.e., $b/rt_N$. In this round, $E_{\text{payload}} = b/rt_N$ and $E_{\text{overhead}}$ can be obtained as follows. The number of $MC$ used in this round is: $k_{min} = \arg\min(\sum_{i=1}^{N} 1/i \geq (2cB + b)/Br_N)$ (similar to Equ. (5)). Therefore, $E_{\text{overhead}} = k_{min} \cdot B - b/rt_N$. As this round is the worst round we can imagine, we have ratio($S_1$) > $E_{\text{payload}}/E_{\text{overhead}} = b/(k_{min}Brt_N - b)$.

Constructing $S_2$. Suppose that an $MC$ has an infinite amount of energy, then we only need one $MC$ in each round. Obviously, ratio(OPT) < ratio($S_2$). Remember that whenever a sensor needs to be recharged, this $MC$ must travel at least one unit distance to recharge it and then travel at least another one unit distance to come back to its original point. Therefore, ratio($S_2$) < $b/2c$.

The theorem follows immediately.

V. PERFORMANCE EVALUATION

In this section, we primarily focus on evaluating ClusterCharging($\beta$) in different settings with respect to various parameters, and will not evaluate PushWait since PushWait provides the optimal solution for the uniform case of the CMCS problem. We first introduce the evaluation settings, then present the results.

A. Evaluation Setup

In order to see the impact of the recharging cycles, $\tau_1, \tau_2, \ldots, \tau_N$, on the performance of ClusterCharging($\beta$), we use two different settings to generate these cycles.

Random-Setting: The recharging cycles are randomly generated from a bounded range, i.e., [lb$\text{bound}$, ub$\text{bound}$] = [2, 8]. We then sort them to guarantee that $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_N$. For evaluations based on this setting, we ran experiments 100 times and averaged the results.

Power-Setting: The recharging cycles are generated based on a power function, i.e., $\tau_i = \text{base}^{(i+1)/2} = 2^{(i+1)/\text{base}}$.

These two settings reflect two extremes of the mathematical variances of recharging cycles: Random-Setting generates cycles with a small variance, while Power-Setting generates cycles with a relatively large variance. Therefore, we can observe the impacts of the non-uniform recharging cycles on our proposed heuristic more clearly.

In each setting, we try to evaluate the effects of the number of sensors, $N$, the energy cost per unit distance, $c$, the battery capacity of a sensor node, $b$, and the battery capacity of a mobile charger, $B$, separately. We are also interested in the impacts of the bounded range and the power function in each setting. Hence, we ran experiments with the $\text{ubound}$ varying from 4 to 12 while keeping $\text{lb$\text{bound}$} = 2$, and we ran experiments with $\text{base}$ varying from 2 to 6.

The optimal solution to the non-uniform case of the CMCS problem requires exhaustive searching, which is infeasible even when the number of sensors is a little large. Considering that the approximation ratio of ClusterCharging($\beta$) is given, we do not implement the optimal solution for comparison.

B. Evaluation Results

1) Random-Setting: Fig. 6 shows the results of different setups for the Random-Setting. In general, ClusterCharging($\infty$) (red line with circle markers) achieves almost the same performance as ClusterCharging(2) (green line with cross
(a) Varying $N, (c = 5, b = 5, B = 60, lbound = 2, ubound = 8)$
(b) Varying $c, (N = 20, b = 5, B = 60, lbound = 2, ubound = 8)$
(c) Varying $b, (N = 20, c = 5, B = 60, lbound = 2, ubound = 8)$
(d) Varying $B, (N = 20, c = 5, B = 60, lbound = 2, ubound = 8)$
(e) Varying $ubound, (N = 20, c = 5, b = 5, B = 60, lbound = 2)$
(f) Partial derivative of the ratio with respect to each parameter

Fig. 6: Impact of various parameters in Random-Setting

markers), and they outperform ClusterCharging(1) (blue line with square markers). In detail, when the other parameters are fixed, the performance metric, i.e., the ratio of $E_{\text{payload}}$ to $E_{\text{overhead}}$, goes down as the number of sensors increases (Fig. 6(a)), goes down as the energy cost per unit distance increases (Fig. 6(b)), goes up as the battery capacity of a sensor node increases (Fig. 6(c)), and goes up as the battery capacity of the MC increases (Fig. 6(d)).

In Fig. 6(e), when the $ubound$ increases, the ratio increases. This is because a larger range incurs a larger variance of the recharging cycles, which leads to a longer scheduling cycle, and only a few sensors need to be charged in each round. This sparsity causes a performance reduction in each round.

Fig. 6(f) shows the partial derivative of the ratio with respect to each parameter in this setting. For example, when $N$ increases by 1, the ratio of ClusterCharging($\infty$) decreases by 0.00684. We notice that the impact of $c$ is the greatest; $B$ and $ubound$ have the least impacts on the ratio. This is reasonable, as the change of $c$ influences every moving segment between any pair of adjacent sensor nodes.

2) Power-Setting: Fig. 7 shows the results of different setups for the Power-Setting. In general, ClusterCharging(1) (blue line with square markers) achieves almost the same performance as ClusterCharging(3) (green line with cross markers), while ClusterCharging($\infty$) (red line with circle markers) has the worst performance. Most of the observations from Random-Setting still hold in Figs. 7(a) to 7(d).

In Fig. 7(e), when the $base$ increases, the ratio decreases. The main reason is that a larger $base$ makes the length of the consecutive recharging cycles of the same value become longer, which further leads to a smaller variance. For example, if $base = 2$, the generated sequence is $\{1, 2, 2, 4, 4, 8, \ldots\}$; if $base = 4$, the sequence is $\{1, 1, 1, 4, 4, 4, 4, 16, \ldots\}$. Fig. 7(f) illustrates the partial derivative of the ratio with respect to each parameter in this setting. Like the partial derivatives in Power-Setting, $c$ has the greatest impact on the ratio; $B$ have the least impact.

In summary, our simulations show that the proposed algorithms perform well in a variety of settings. Specifically, when the variance of the recharging cycles becomes larger, ClusterCharging($\beta$) performs worse; ClusterCharging(1) and ClusterCharging($\infty$) are sensible to the variance of the recharging cycles, while ClusterCharging($\beta$) with other value of $\beta$ is robust in both settings.
VI. RELATED WORK

Energy conservation, harvesting and node reclamation. Energy conservation was proposed to slow down the energy consumption rate. Bhattacharya et al. [5] proposed to cache mutable data at some locations to control data retrieval rate. Dunkels et al. [6] incorporated cross-layer information-sharing in the their proposed adaptive communication architecture. Wang et al. [7] proposed to use resource rich mobile nodes as sinks or relays to prolong the lifetime of WSNs. However, conservation cannot compensate for energy depletion in the end. Energy harvesting [8, 9] tries to harvest energy (such as solar, wind, and vibration) directly from the environment to replenish sensors, but it is neither controllable nor predictable, which hinders WSNs from providing the desired level of performance. Sensor reclamation [10] periodically replaces sensors of no or low energy with fully charged ones; however, it requires either human intervention or advanced robotic mechanisms, which can be costly in various situations.

Wireless energy transfer. The wireless power consortium [22] defines the inter-operability standards of wireless energy transfer based on magnetic induction. Kurs et al. [11] demonstrated it to be efficient and non-radiative in Science. Peng et al. [14] proposed the use of a mobile charger with sufficient energy to charge the entire network and formulated it as a TSP-like (Traveling salesman problem [23]) problem. Li et al. [15] took both mobile charger scheduling and touring into consideration with the assumption that the movement of a mobile charger costs zero energy. Tong et al. [17] found that: when the number of sensor nodes being charged simultaneously increases, the average power received at each sensor remains approximately the same. Using this observation, the authors tried to determine the optimal node deployment and routing strategies to improve energy efficiency. Shi et al. [16] also assumed that the mobile charger has unbounded energy and investigated the problem of periodically charging sensors to maximize the free time of the mobile charger over a cycle. In contrast to these works, we investigate the problem with a more realistic condition: a mobile charger may not have enough energy to cover the entire network.

VII. CONCLUSIONS AND FUTURE WORK

This paper introduces a novel charging paradigm, i.e., collaborative mobile charging. We investigate the collaborative mobile charging scheduling problem in 1-D WSNs. We first consider the uniform case and propose an algorithm, PushWait, which is proven to be optimal in this case and can cover a 1-D WSN of any length. A variation of PushWait that uses dedicated chargers to substitute roundtrip chargers is also presented. We then develop a heuristic, ClusterCharging(β), with guaranteed performance for the non-uniform case. Extensive simulations validate the advantages of our algorithms.

Our future work will focus on two parts. One part involves investigating the impact of wireless transfer efficiency, which is assumed to be one in this paper. The other part involves extending our algorithms to 2-D networks.