Active Opinion-Formation in Online Social Networks

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***Abstract*—Recommendation systems usually try to "guess" a user's preferences from the system's view. We study another side of recommendation: active opinion-formation from the perspective of the user. In real life, a user's opinion evolves with time and refines when new evidence occurs. Then, how does an online user form his/her own opinion actively in large social networks? The problem has three challenges: the factor, the effect and the open environment. To address those challenges, we investigate: (1) what factors or channels a user will consider, (2) how those channels will take effect, and (3) an incremental approach to incorporate multiple channels. We explore three types of channels: the internal opinion of an individual user, influences from trusted friends, and influences from public channels. A novel simulator, $OpinionFormer$, is proposed to incorporate those channels incrementally. It differentiates the effects of friends and public channels as well as positive and negative opinions. We validate the performance of $OpinionFormer$ by predicting users' opinions using real-world data sets. Experimental results show that our model can improve accuracy over other models that ignore some channels or that neglect the evolving features.**

***Index Terms*—opinion formation, internal opinion, trusted friend, public channel, fluid dynamics.**

I. Introduction

Online social networks (OSNs) are the most popular tools for individuals' daily communication and interactions. To better understand the principles behind these social systems behavior and evolution, it is necessary to know how people form their opinions in online systems. Opinion formation can be involved in a variety of application domains including opinion dynamics [1], [2], rating prediction [3], or identifying opinion leaders [4]. In this paper, we investigate the individual's active opinion-formation process. A basic model in literature is DeGroot's averaging model [5] in which individuals update their opinion using the weighted average of their own opinion and their neighbors'. Based on [5], Dandekar et al. propose the biased assimilation model [6] which introduces a biased parameter that gives more weights to confirming opinions. Friedkin and Johnsen [7] differentiate the internal opinion and the expressed opinion; the former is an inherent opinion that cannot be changed while the latter is an explicit opinion that can influence others or be influenced by others. We focus on the following question based on concepts in [7]: *how do people form their expressed opinions in online systems*?

This problem has three main challenges. (1) *Challenge of factors*: There is large body of information in online systems (and also off-line) that people refer to, and it is difficult to identify which reference sources (or reference for short) a user takes advice from. (2) *Challenge of effects*: The effect of each reference is difficult to measure. (3) *Challenge of open environments*: Before a user makes the final decision, new references may occur and need incremental treatment.

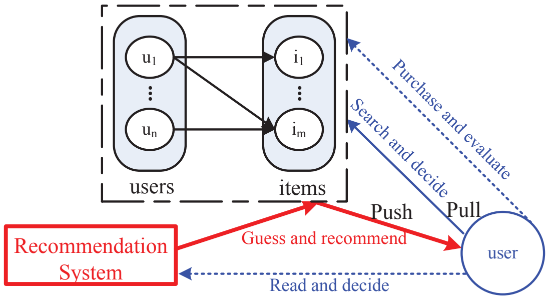


Fig. 1. An illustration of an online system consisting of users and items. A recommendation system is incorporated to "guess" users' preferences and to make recommendations. On the other side, users actively search information and make decisions (e.g., to purchase or not).

Many factors can impact users' opinions. Individuals are more likely to adopt recommendations from their *friends* [8], [9]. A *public channel* also shows similarly significant effects [10]. Last but not least, a user's *internal opinion*, born by nature or by previous experiences, can also impact the final decision [11]. Existing works usually only consider one or two types of channels, such as friend channel in [5], [6]; and overlooking the impacts of the others. We seek to incorporate all three types of channels in one model.

Our motivations are threefold: (1) *from passive to active*: many existing works, particularly those in recommendation systems, try to "guess" a user's preferences from a system's view in which a user "passively" receives influences. Unlike these works, we strive to study "active" opinion formation from an individual's view in which a user can selectively listen to some opinions and then, integrate their own biases and preferences. Fig. 1 shows an illustration of an online system and the role of recommendation systems and users. (2) *comprehensive study on three types of channels*: we try to study the elements and effects of the three types of channels so as to better understand the fundamental principles behind peoples' opinion formations. (3) *exploiting incremental approaches to deal with new references*. Existing works usually consider a fixed group or community. Our work is different in that, we are considering an open system where new references may occur and need to be treated.

Without the loss of generality, we primarily focus on the online environments where users' opinions are expressed by numeric ratings. We consider a setting where there is a single item of interest (e.g., a product). A special user, the *sink* (denoted as a), is the one who is trying to form his expressed opinion (denoted as O_a) about the item. A subset of users,

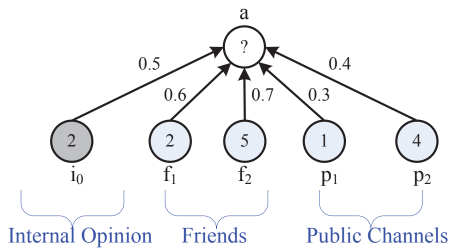


Fig. 2. An example of a reference network for sink a where nodes represent users, numbers on nodes represent ratings on a given target item, and weighted edges represent influence relations.

friends (denoted as F) and *public channels* (denoted as P), have prior opinions about this item and their opinions can be seen by the sink (which is the expressed opinion, if not already specified). The collective set of those reference sources from three types of channels and influence edges from them to the sink construct a *reference network*.

Fig. 2 shows an example reference network for sink a. The user is considering some item and has $\{i_0, f_1, f_2, p_1, p_2\}$ for reference. i_0 is a virtual node representing a's internal opinion, f_1 and f_2 are his trusted friends, and p_1 and p_2 are other non-friend users. f_1, f_2, p_1, and p_2 have rated the item. The number associated with a node corresponds to a rating/opinion. In general, a rating higher than a threshold (e.g., 3 for range [1,5]) indicates a positive opinion, and a lower rating indicates a negative one. The number associated with each edge represents the influence strength from a reference source to a.

In order to solve the active opinion-formation problem, we identify essential elements for each channel and explore the way those channels take effect. We also propose a simulator to integrate those channels incrementally, which differentiates the effects of different channels. Our contributions are fourfold:

1. *We systemically study the active opinion-formation problem (AOFP) in OSNs*. In our work, a user searches for information and forms his own opinion on his own initiative. We believe this is a promising direction for research about online systems. We identify four tasks in AOFP, analyze its hardness, and prove the NP-completeness of task 2 (i.e., reference subset selection). We, then, propose a heuristic solution. (Section III)

2. *We identify elements for three types of channels*. Based on empirical evidence, we identify essential elements for each channel (i.e., the value of the opinion and its influence strength to the sink). Then, construct a reference network for the sink's opinion formation. (Section IV)

3. *Keeping the incremental treatment in mind, we propose a novel simulator, $OpinionFormer$*. It can incorporate all the channels in a more fine-grained approach and incrementally deal with new references. It differentiates effects from friends and from public channels, and also differentiates effects from positive opinions and from negative opinions. We also conduct a comprehensive analysis. (Section V and Section VI)

4. *We test the performance with two real-world data sets: Epinions (Epinions.com) and Ciao (Ciao.com)* [12]. The results show improvements in predicting users' opinions, indicating the effectiveness of our work. (Section VII)

II. Related Work

We briefly review related works and identify the connections and differences from our work, as follows.

Researches on Opinion Dynamics. Most of the opinion dynamics models study the collective opinions in a fixed group or a community [2]. We briefly introduce three typical and commonly studied models: the *Ising model* [13], the *DeGroot model* [5], and the *Biased assimilation model* [6].

Ising model [13]. In this model, each agent has one opinion represented as a spin that can be up or down, which determines a choice between two options. Spin couplings represent peer interactions, and external information is the magnetic field. This model links opinion formation with physical phenomena, which shows a powerful representation ability.

DeGroot model [5]. This model studies how a consensus is formed in a fixed network. At each time step, individuals simultaneously update their opinions using the weighted average of their own opinion and their neighbors'. Taking Fig. 2 for instance, a's opinion can be calculated as $O_a = (2*0.5+2*0.6+5*0.7)/(0.5+0.6+0.7) \approx 3.17$.

Biased assimilation model [6]. This model considers *biased assimilation*, where individuals weigh "confirming" evidence more heavily relative to disconfirming evidence. It introduces a bias parameter b_a ($b_a \geq 0$). Suppose $b_a = 0.5$ and normalize opinions into $[0,1]$ (i.e., $\tilde{O}_{b_0} = \tilde{O}_{f_1} = 2/5 = 0.4, \tilde{O}_{f_2} = 5/5 = 1$). Then, Fig. 2 will get $\tilde{O}_a = \frac{0.4\cdot0.5+0.4^{b_a}(0.4\cdot0.6+1\cdot0.7)}{0.5+0.4^{b_a}(0.4\cdot0.6+1\cdot0.7)+(1-0.4)^{b_a}\cdot(0.6+0.7-0.5)} \approx 0.4635$. Finally, $O_a = \tilde{O}_a \cdot 5 = 2.3175$.

Das et al. [14] observe three common user behavioral types: stubborn behavior is where users do not change their opinions, compromising behavior is when a user chooses an opinion in between his own opinion and the average of his neighbors', and biased conforming behavior is where users give more weight to opinions that are closer to their own initial opinion. It validates the necessity of considering active opinion-formation.

A Novel Approach: Fluid Dynamics. Jiang et al. first introduced fluid dynamics theory into trust-based recommendation systems in [3], [15], where a user is seen as a "container", and their opinion as fluid in the container; their opinion is measured by the fluid temperature, and fluid height as the persistence of their opinion. Containers are connected via pipes and the influences among users are modeled as fluid exchanges and mixtures. Zheng et al. make an extension to this model by considering a public channel in [10]; however, their public channel represents the average rating of all users.

We study the above models carefully and we exploit the fluid dynamics theory in our work because it can meet the request of the incremental treatment and reflect the evolving features of opinions. Our work differs from all the above works in three aspects: (1) We are considering an open system where new references may occur, and therefore, creates the necessity of incremental treatment. (2) We consider three types of channels that include almost all possible references. However, it is important to note that we take each public available rating

TABLE I

NOTATIONS.

Symbol	Description
$G=(V,E)$	reference network with nodes V and edges E
$R=\{i_0\}\bigcup F\bigcup P$	references $\{i_0,f_1,...,f_{m_1},p_1,...,p_{m_2}\}$
i_0	internal opinion
F	friends $\{f_1,...,f_{m_1}\}$
P	public channels $\{p_1,...,p_{m_2}\}$
$a/O_{i_0}/O_a$	sink and his internal/expressed opinion
f/O_f	a friend f and his expressed opinion
p/O_p	a public channel p and his expressed opinion
$w_{vv'}$	influence strength from v to v', $w_{vv'}\in[0,1]$

as a public channel, and don't only use the average rating as in [10]. In this way, channels are treated more precisely. (3) We differentiate the effects of friends and public channels, as well as the effects of positive and negative opinions.

III. PROBLEM FORMULATION

We define the basic system settings and the problem that we will solve. Notations are described in Table I.

A. System Settings

Definition 1: Reference Network. A reference network is a directed graph, $G=(V,E)$, where V is a set of nodes and $E\subseteq V^2$ is a set of directed edges. Each edge, $e_{vv'}$, has the direction from node v to node v' associated with a weight, $w_{vv'}$, indicating the influence strength from v to v'.

The node set $V=\{i_0,f_1,...,f_{m_1},p_1,...,p_{m_2},a\}$ consists of four types of nodes: a virtual node, i_0, that provides the internal opinion, *friends* $F=\{f_1,f_2,...,f_{m_1}\}$, and *public channels* $P=\{p_1,...,p_{m_2}\}$, which have formed their opinions/ratings, and *sink*, a, who is trying to form his own expressed opinion. We let $R=\{i_0\}\bigcup F\bigcup P$, representing the set of references. It is worth noting that the friends in this definition are 1-hop friends who are directly trusted by a.

Definition 2: Internal Opinion. An *internal opinion* of a user is endogenous opinion about certain content.

The internal opinion is modeling the inherent beliefs of a person, which may be related to personality, background, and education, and is unlikely to be changed [4]. In this paper, we only consider the internal opinion of the sink, and all references' opinions are expressed opinions.

Definition 3: Expressed Opinion. An *expressed opinion* is the opinion that a user gives explicitly on certain content. Examples include ratings/reviews in e-commerce web sites.

Definition 4: Public Channel. A *public channel* is a channel providing a rating that can be seen by all users.

B. Active Opinion-Formation Problem (AOFP)

Given a reference network $G=(V,E)$ with three types of channels as references (i.e., $R=\{i_0\}\bigcup F\bigcup P$) and a sink, a, whose expressed opinion on a target item, O_a, needs to be formed, active opinion formation has the following tasks:

Task 1: Identify essential elements. For each reference, determine the value of an opinion (i.e., O_{i_0},O_f,O_p) and its influence strength to sink a (i.e., w_{i_0a},w_{fa},w_{pa}).

Task 2: Select a proper subset of references. With limited time and energy, it is almost impossible for a user to check all references for decision making. Therefore, it is necessary to select a subset $R'\subseteq R$, which is expected to minimize the estimation error with a given budget (it can be time or energy).

Task 3: Explore the way those channels take effect. In order to accurately model the opinion formation process, it is expected to differentiate the effects of different channels.

Task 4: Design a model to incorporate the selected channels. People's opinions evolve with time, and new references may occur now and then. Therefore, the model is expected to treat channels incrementally and reflect the evolving features.

The objective is to efficiently and effectively simulate the process with which a forms his expressed opinion O_a, and thus, to predict it accurately. We set the goal to minimize the prediction error with a given budget (see Definition 5 below).

C. The Hardness

Task 2 of the AOFP problem can be converted to the Jury Selection Problem (JSP) in [16], which tries to gather answers for decision making questions from micro-blog followers. In [16], each juror has an individual error rate; the goal is to select a subset of jurors with a minimum Jury error rate and a given budget. They prove the NP-completeness by reducing the nth-order Knapsack Problem (nOKP) to the JSP problem. We make the following conversions and then follow their proof showing that the AOFP is also NP-complete.

(1) We convert the scenario of a's expressed opinion formation to be this: the selected references in R' are trying to answer "what is the rating of a?" (2) We simplify the question to be "is a's expressed opinion a positive one?" to get a decision problem as in [16]. Then, each reference, R_l, in the AOFP can be denoted as a juror which has an individual error rate, ε_l. R' can be taken as a jury. The total error is:

Definition 5: Jury Error Rate. It is the probability that the Carelessness, C, is greater than $\frac{|R'|+1}{2}$ for a jury R', namely

$$JER(R')=\sum_{o=\frac{|R'|+1}{2}}^{|R'|}\sum_{A\in F_o}\prod_{l\in A}\varepsilon_l\prod_{j\in A^c}1-\varepsilon_j=Pr(C\geq\frac{|R'|+1}{2}|R')$$

where F_o is all the subsets of R with size o, A is a such subset, and A^c is A's complement; ϵ_l is the error rate of R_l.

Theorem 1: AOFP is NP-complete.

Proof: We prove Theorem 1 by proving the NP-completeness of its decision version, the Decision AOFP (DAOFP), i.e., given an AOFP instance and a value ξ, decide whether a subset, R', can be selected so that their total error is equal to ξ. Definition 5 shows the objective function of AOFP. According to this definition, this optimization problem is an nth-order Knapsack Problem, which is a Knapsack problem (KP) with the following objective function:

optimize $\sum_{i_1\in n}\sum_{i_2\in n}...\sum_{i_n\in n}V[i_1,i_2,...,i_n]\cdot x_1x_2...x_n$

where $V[i_1,i_2,...,i_n]$ is an n-dimensional vector indicating the profit achieved if items $[i_1,i_2,...,i_n]$ are selected simultaneously. Given an instance of a traditional KP, we can construct an $nOKP$ instance by defining the profit n-dimensional vector as $V[i,i,...,i]=\pi_i$ and $V[otherwise]=0$ for all i, where π_i is the profit in the traditional KP. The weight vector and objective value remain the same. ■

Heuristic Solution: We provide a heuristic solution for Task 2, reference subsect selection. First, based on real-life experiences, we know that a more close relation usually leads to a lower error rate. Therefore, the internal opinion has the smallest error rate, friend channels have the second, and public channels have the last. Next, we select the references according to this order, until there is no budget or reference. Solutions for the other three tasks are given below.

IV. Elements for Opinion Formation

We provide the solution for Task 1, identifying the elements. We first study the theoretical background and exploit some empirical evidence. This evidence can serve as the general rules for element identification. Next, we identify essential elements for three types of channels. Keeping incremental treatment in mind, we treat each channel independently. To be specific, we consider two elements for each channel: the value of the opinion (i.e., $O_{i_0}, O_f, O_p \in [1, 5]$) and its influence strength to the sink (i.e., $w_{i_0a}, w_{fa}, w_{pa} \in [0, 1]$).

A. Empirical Evidence

Empirical Evidence 1: *Influence from a friend usually weighs more than that from a public channel*. This is because of the "social influence" where users feel attracted to content liked by their friends [17]. The roots lie in two theories: the theory of "conformity" [18], which states that people are apt to act as their friends do, and the theory of "biased assimilation" [6], which states that people will give more weight to opinions that are similar to their own, and vise versa.

Two extensions from Evidence 1 are as follows:

(1) *Empirical Evidence 1.1: A higher interaction frequency indicates more influence*. In real life, if two people interact frequently, they are more likely to be mutually influenced. It reflects the "mere-exposure effect" in psychology [17].

(2) *Empirical Evidence 1.2: More common friends indicates more influence*. This is because having common friends will increase the chance of mutual interactions.

Empirical Evidence 2: *The influence of a negative opinion usually weighs more than that of a positive one*. This is because of the brilliant-but-cruel hypothesis [19] where "negative reviewers are perceived as more intelligent and expert than positive reviewers." Moreover, since most ratings in online systems are positive ones, and even some of that is false-praise, people are more sensitive to negative ones.

Empirical Evidence 3: *Helpfulness indicates confidence*. A review with a larger helpfulness rating is usually given more weight, indicating more confidence [18].

B. Elements of internal opinion

A user's internal opinion can be formed by his personality or by past experiences. Due to its special nature, internal opinions are usually hidden and can only be inferred [4].

In our work, we take the expressed opinions as the final opinions, and we classify all factors into three types of channels. Let the average rating represent a summary of friends' and public channels' opinions. Then, we can estimate the internal opinion by considering the distance from a user's expressed rating to the average rating. For example, suppose a has rated item i. The distance is calculated as $d_{ai} = r_{ai} - \overline{r_i}$, where r_{ai} (or r_a if there is no need to distinguish item) is the rating by a on item i in some category and $\overline{r_i}$ is the average rating expressed before a by friend/public channels.

In addition, if a has rated several items $I_a = \{i_1, i_2, ..., i_l\}$ in this category, then the average distance can be used to calculate the internal opinion on item i, as follows:

$$O_{i_0i} = \overline{r_i} + \sum\nolimits_{i \in I_a} d_{ai}/|I_a|,$$

where $\overline{r_i}$ is the average rating that item i receives. In fact, O_{i_0i} may also indicate the habit of a's rating expression in this category, e.g., he is likely to give higher ratings. The category can help to improve the estimation accuracy since a user may have different biases or habits in different categories; moreover, it can decrease the calculation cost.

The above description helps us to determine the value of the opinion. Next, we analyze its influence strength, w_{i_0a}, to a, which indicates the sink's confidence in his internal opinion. $w_{i_0a} = 0.5$ represents a neutral confidence. In this case, the user has some self-confidence. Note that $w_{i_0a} = 1$ means that he feels the most confidence in himself; however, it does not mean that he will not listen to others.

C. Elements of Friends Channels

We consider the opinion of a trusted friend f on item i (i.e., O_{fi}) and its influence strength to the sink a (i.e., w_{fa}). The former is already explicitly expressed as a rating, i.e., $O_{fi} = r_{fi}$, while the latter requires some effort to explore.

In some OSNs (e.g., Epinions and Ciao), users can add another user as a trusted friend if their reviews are usually considered helpful. However, the values of all trust relations are uniformly taken as 1, which overlooks the difference from one trust relation to another. With *Empirical Evidence 1.1* and *1.2*, we can refine the influence strength by considering users' interaction frequencies, and their number of common friends.

Let F_a be the trustee set of a and I_{fa} be the interaction set between a friend f and a. Then, the interaction frequency can be calculated as $w^1_{fa} = |I_{fa}|/max\{I_{fa}, f \in F\}$. Similarly, the impact from common friends can be calculated with $w^2_{fa} = |F_a \bigcap F_f|/max\{|F_a \bigcap F_f|, f \in F_a\}$. Then, the two parts can be integrated with $w_{fa} = \lambda w^1_{fa} + (1-\lambda) w^2_{fa}$, where $\lambda \in [0, 1]$.

D. Elements of Public Channels

We consider the opinion of a public channel $p \in P$ on item i (i.e., O_{pi}) and its influence strength to the sink a (i.e., w_{pa}). The former is already explicitly expressed as a rating r_{pi}, i.e., $O_{pi} = r_{pi}$. As for the latter, we consider the helpfulness of p's rating/review (denoted as r^r_{pi}).

With *Empirical Evidence 3*, "Helpfulness indicates confidence," we can calculate the influence strength as $w_{pa} = r^r_{pi}/maxH$, where $maxH$ is the maximum helpfulness (e.g., $maxH = 5$ in Epinions and Ciao).

Now all three types of channels and their elements (opinion and its influence strength) can be determined. Based on this, a

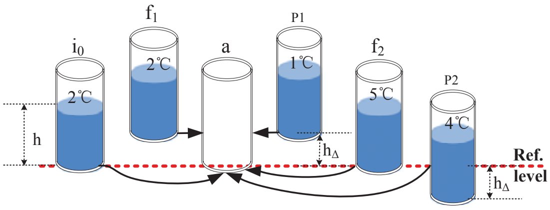


Fig. 3. The illustration of $OpinionFormer$, which maps a reference network in Fig. 2 to a fluid system. A container represents a user, and fluid represents opinion (height as confidence, temperature as the rating).

reference network can be constructed for the sink. In the next section, we provide the solution for Task 3 and Task 4, where we design a simulator with the resulting reference network.

Algorithm 1 Initialization (G, a, R)

Input: G, a reference network; a, sink; R, reference set.
Output: G', a fluid system for sink a.

Let a's container be empty.
for each reference source $v \in R$ **do**
    Set up a container with enough volume in G'. $O_v \leftarrow r_v$, $h_v \leftarrow h$.
    Containers of negative opinions are put higher than the reference level with h_Δ (indicating more effects).
    Containers of public channels are put lower than the reference level with h_Δ (indicating less effects).
for each influence edge $e_{vv'}$ in G **do**
    Set up a single-direction pipe from v to v' in G'.

V. A Simulator: $OpinionFormer$

We propose a novel simulator, $OpinionFormer$, to simulate active opinion-formation. It explores the fluid dynamics theory to differentiate the effects of different channels (Task 3) and incorporate all channels incrementally (Task 4).

A. Overview

We take an active opinion-formation as follows: sink a considers some references one-by-one and refines his opinion step-by-step. $OpinionFormer$ maps the reference network into a fluid system (e.g., from Fig. 2 to Fig. 3) with three components: containers, fluid, and pipes. The reference level represents the horizontal level of placing sink's container.

(1) *The container represents the user*. Each user is modeled as a container. At the very beginning, the container of sink a is empty, indicating that a has no opinion. Each of his references(e.g. v)'s container, has some fluid with height h. The cross-sectional area of all containers is taken as 1.

(2) *Fluid indicates an opinion*. Fluid temperature in v's container is equal to a rating r_{vi} on item i, and fluid height h_v indicates the persistency/confidence of v's opinion.

(3) *Pipes pass influence*. There are pipes connecting v and a with a direction from v to a. Pipes' positions in containers may be different according to their height differences with the reference level. Fluid can flow from other containers to that of a if a chooses to listen to the advice. The cross-sectional area of a pipe is equal to w_{va}, i.e., the influence strength from v to a. The final fluid temperature of a is taken as his opinion.

With the *Empirical Evidence* 1 and 2, we differentiate the effects of friends and public channels and also negative and positive opinions (Task 3). This is accomplished by putting the containers higher or lower than the reference level. More specifically, the containers of the public channels (i.e., p_2) are set lower than the reference level, indicating smaller effects. The containers of the negative ratings (i.e., f_1 and p_1) are set higher than the reference level, indicating larger effects. The height difference is denoted as h_Δ. Then, the fluid height over the reference level falls in the range of $[h - h_\Delta, h + h_\Delta]$.

B. Assumptions and Initialization

We assume that (1) All users are honest, and their expressed ratings are true. (2) A reference network on which the fluid system can be set up is already available. (3) The fluid system meets mass and energy conservation. (4) Each pipe is installed with a valve, and sink a has the right to open the valve to allow fluid to enter. (5) Fluid heights of reference sources remain unchanged (this can be done by injecting fluid to those containers with large enough sources).

Algorithm 1 describes the initialization process in which each reference source is considered once, taking $O(|V|)$ time. Each edge is transformed into a pipe, with time complexity $O(|E|)$. Therefore, the time complexity is $O(|V| + |E|)$.

C. Algorithm Details of Fluid Updating

Inspired by what people do in real life, we take the individual's opinion formation process in discrete steps (in fact, it is the same as many existing models including [5], [6]), i.e., a checks the ratings of his references one-by-one. Whenever he checks a rating, he takes part of the advice and refines his opinion. As the "confirming" theory [18] suggests, if the new reference's opinion is consistent with a, a will increase his confidence about the previous idea. Otherwise, he will doubt either the target item, the reference, or in some cases, even both. In this case, we can decrease the confidence. The process can be repeated (i.e., in the same way that a reference can be considered many times in real life) and continued until a has enough confidence and forms an expressed opinion, or until there is no budget or reference left.

We design Algorithm 2 to apply the $OpinionFormer$ model to sink a's opinion formation. We take a discrete and asynchronous approach. Each time that a wants to listen to some advice, he will open a valve and allow fluid to flow in for a duration of Δ. There are two basic operations:

(1) *Allowing new fluid to come in when the sink listens to some advice*. This is implemented using Torricelli's law [20], with the equation $\sigma = \sqrt{2gh}$. σ is the speed of efflux, h is the height from the bottom, and g is the acceleration due to gravity. Applying this law to our case, the speed of the flowing fluid will be $\sigma_{va} = \sqrt{2gh_v}$. Considering the cross-sectional area w_{va} of the pipe and the duration of a time slot Δ, the volume of flowing fluid S_{va} can be calculated as follows:

$$S_{va} = \sqrt{2gh_v} \cdot w_{va} \cdot \Delta \quad (1)$$

Algorithm 2 $OpinionFormer(G', a, R)$

Input: G', fluid system; a, sink; R, reference set.
Output: O_{ai} and h_a, a's opinion and confidence.

Let h^* be a's predefined confidence threshold, B be the budget, and $curcost$ be the cost for treating the current source.
SubProcess: *AllowFluidIn(a,v)*
  $\{a$ opens the valve to allow v's fluid to come in. The volume is calculated using Eq. 1.
  a will conduct fluid mixing using Eqs. 2 and 3.
  $R \leftarrow R - v$. $B \leftarrow B - curcost.\}$
Suppose $u \in R$ is the first user who is being considered.
Call *AllowFluidIn(a,u)* and a forms an opinion.
while $h_a \leq h^*$ and $R \neq \emptyset$ and $B \neq 0$ **do**
  a chooses to listen to a user v' in R.
  Call *AllowFluidIn(a,v')*.
  if the opinion of v' is opposite to that of a **then**
    a drops fluid to decrease his confidence (Eq. 4).

(2) *Mixing fluids to simulate the sink's opinion update*. Suppose that at the k^{th} step, a already has the amount of fluid $S_a(k)$ and the new, incoming fluid is S_{va}. The mixed fluid volume will be

$$S_a(k+1) = S_a(k) + S_{va}. \tag{2}$$

Since the cross-sectional area is 1, we have $h_a(k+1) = S_a(k+1)$. According to the law of energy conservation, the fluid temperature after mixing is calculated as follows:

$$O_a(k+1) = \frac{O_a(k) \cdot S_a(k) + O_v \cdot S_{va}}{S_a(k+1)}, \tag{3}$$

where O_v is the fluid temperature in v's container.

In real life, if we hear a different opinion, we may doubt and rethink our current one, which causes our confidence regarding our current opinion to, more or less, decrease. To reflect this point, in $OpinionFormer$, if the new, incoming opinion is different from the current one (e.g., a positive opinion meets a negative one, or vise versa), we will decrease a's confidence with a ratio $\eta \in [0, 1]$ after mixing the new opinion. That is,

$$h_a = h_a \cdot (1 - \eta). \tag{4}$$

Let h^* be a's confidence threshold. The process continues until $h_a = h^*$ or $R = \emptyset$ or $B = 0$. In Algorithm 2, a visits at most all references, $V - \{a\}$. Each time a visits a user, only a constant time is taken. Therefore, the complexity is $O(|V|)$.

It is worth noting that the durations of time slots can also be non-uniform. For instance, a longer time slot of an internal opinion can indicate a more biased assimilation [6].

D. Case Study

We apply $OpinionFormer$ to the example in Fig. 2. Initially, $S_a(0) = 0$. Let $h = 10, h_\Delta = 5, \eta = 0.1, \Delta = 0.04$. Table II shows a sample calculation process in which each reference is considered once. There are five references and thus five steps of opinion evolution. At each step, we first calculate the fluid volume that will flow into a using Eq. 1 (e.g., S_{i_0a}). Next, we update fluid volume S_a using Eq. 2; and update the fluid height h_a. Finally, we update the fluid temperature O_a using Eq. 3. $O_a(5)$ is the final expressed opinion.

TABLE II
CASE STUDY OF THE EXAMPLE IN FIG. 2.

Parameter	Calculation	Value
S_{i_0a}	$\sqrt{2gh} \cdot 0.5 \cdot \Delta$	0.28
$S_a(1)$/$h_a(1)$	$S_a(0) + S_{i_0a}$	0.28
$O_a(1)$	O_{i_0}	2
S_{f_1a}	$\sqrt{2g(h + h_\Delta - h_a(1))} \cdot 0.6 \cdot \Delta$	0.408
$S_a(2)$	$S_a(1) + S_{f_1a}$	0.688
$O_a(2)$	$(S_a(1) \cdot O_a(1) + S_{f_1a} \cdot O_{f_1})/S_a(2)$	2
S_{f_2a}	$\sqrt{2g(h - h_a(2))} \cdot 0.7 \cdot \Delta$	0.378
$S_a(3)$	$S_a(2) + S_{f_2a}$	1.066
$O_a(3)$	$(S_a(2) \cdot O_a(2) + S_{f_2a} \cdot O_{f_2})/S_a(3)$	3.064
$h_a(3)$,$S_a(3)$	$h_a(3) \cdot (1 - \eta)$	0.959
S_{p_1a}	$\sqrt{2g(h + h_\Delta - h_a(3))} \cdot 0.3 \cdot \Delta$	0.199
$S_a(4)$	$S_a(3) + S_{p_1a}$	1.158
$O_a(4)$	$S_a(3) \cdot O_a(3) + S_{p_1a} \cdot O_{p_1})/S_a(4)$	2.71
$h_a(4)$,$S_a(4)$	$h_a(4) \cdot (1 - \eta)$	1.042
S_{p_2a}	$\sqrt{2g(h - h_\Delta - h_a(4))} \cdot 0.4 \cdot \Delta$	0.141
$S_a(5)$	$S_a(4) + S_{p_2a}$	1.299
$O_a(5)$	$S_a(4) \cdot O_a(4) + S_{p_2a} \cdot O_{p_2})/S_a(5)$	2.85
$h_a(5)$,$S_a(5)$	$h_a(5) \cdot (1 - \eta)$	1.169

VI. ANALYSES

We comprehensively analyze the convergence, the main advantages, and the desirable properties of $OpinionFormer$.

A. Convergence Analysis

We provide two theorems to analyze the convergence.

Theorem 2: In $OpinionFormer$, a's fluid height, h_a, will not be larger than the upper-bound h_{max} in a reference set.

Proof: We prove this by contradiction. Initially, a is empty and $h_a = 0$. Suppose that at some time, $h_a > h_{max}$. Then, the fluid in a must come from some reference(s). Without loss of generality, we denote the last reference as b. Now, we have $h_a > h_{max} \geq h_b + h_\Delta > h_b$. However, we also have $h_b + h_\Delta > h_a$ or $h_b > h_a$ before b's fluid flows into a (because fluid can flow only in these cases). According to the basic physical theory, it will never happen that $h_a > h_b + h_\Delta$. It contradicts the statement. ∎

Theorem 3: In $OpinionFormer$, suppose sink a continuously listens to the references' opinions. Then, after a sufficient time period of opinion refinement, a's fluid height will be equal to the upper-bound h_{max} in the reference set.

Proof: Suppose $h_a \neq h_{max}$. According to Theorem 2, it cannot be that $h_a > h_{max}$. Then it must be that $h_a < h_{max}$. Since the fluid in a must come from some reference(s), without loss of generality, we denote the last reference as b. Because $h_a < h_{max}$ and a continuously allows fluid to come in, then the fluid will flow from b to a, until $h_a = h_{max}$. ∎

B. Main Advantages

(1) *$OpinionFormer$ comprehensively incorporates three types of channels into a fluid system.* Each reference is modeled as a container, the opinion is modeled as fluid, and the influence strength is modeled as the width of the pipes.

TABLE III
STATISTICS OF DATA SETS.

	#U	#P	#C	#R	#TR	$\overline{R}$	$\overline{H}$
Epinions	22,166	296,277	27	922,267	355,813	3.97	1.94
Ciao	2,378	16,861	6	36,065	57,544	4.22	1.43

U: Users, P: Products, C: Category, R: Rating, TR: Trust relations, $\overline{R}/\overline{H}$: Average Rating/Helpfulness

Different effects of negative opinions over positive ones and public channels over self or over friends are reflected by a height difference, $h_\triangle$. All elements are properly embedded.

(2) *$OpinionFormer$ simulates the opinion formation process naturally and flexibly*. At the very beginning, a person has no idea of a given target item (and is represented by the "empty container"). Upon receiving opinions, he formulates and refines his opinion, which is represented by the analogy of "mixing fluid." As time passes on, the opinion of a person becomes more mature, which can be demonstrated by an increased fluid height (confidence). Furthermore, at first, the opinion matures quickly, but slows down as it progresses (because the height difference with the references decrease). The process is consistent with our real-world experiences.

C. Desirable Properties

Property 1: *Evolution Compatibility*. $OpinionFormer$ is evolution-compatible, i.e., it provides nodes with timely fluid updating/mixing in response to opinion refinements.

In real life, a user forms an opinion gradually. For instance, in an online shopping scenario, a user may go through the most recent records for the target item. For each record, a user spends time thinking and analyzing, and then, refines his opinion. $OpinionFormer$ can reflect this process by conducting fluid updating via channels one-by-one. In addition, the sink can always refine his opinion before making a final decision, whenever new records appear.

Property 2: *Incremental Treatment*. In $OpinionFormer$, each channel takes its effects independently so that new channels can be processed incrementally.

There are generally two basic steps in opinion formation: (1) collecting evidence, and (2) integrating it to make a final result. In $OpinionFormer$, we separate each channel and treat them independently. Thus, $OpinionFormer$ can treat new channels efficiently using an incremental approach.

VII. EXPERIMENTAL EVALUATION

We conduct extensive experiments in real-world data sets. We try to validate: (1) how each type of channel impacts the effects of $OpinionFormer$ and (2) what its advantages are.

A. Basic Settings

Data Sets. We use two data sets from Epinions and Ciao [12]. In both websites, users can read/write reviews, and express ratings on products or reviews (i.e., the helpfulness). Both data sets provide trust relations from one user to another and users' ratings on items. The values of ratings and helpfulness fall in [1,5], where 1 represents unsatisfied/not helpful and 5 is excellent. Statistics are shown in Table III.

TABLE IV
PARAMETER SETTINGS.

Parameter	**Description**	**Range**	**Default**
λ	weight of interaction frequency	[0,1]	0.5
h	fluid height	10	
$h_\triangle$	height difference with ref. level	[1,10]	5
k	number of rounds	[1, 100]	
Δ	time slot	0.04	
η	confidence decrease ratio	[0,1]	0.1

Evaluation Method. We use the leave-one-out method to evaluate the performance [3]. First, we mask the original rating for a given test pair. Then, we construct a reference network and map it into a fluid system. Next, we conduct multiple steps of fluid updating using $OpinionFormer$. The temperature of the user is collected and deemed the user's final opinion. Finally, we compare this value with the masked one.

Accuracy Metrics. We consider four metrics for opinion prediction accuracy [21]. Suppose A_h is the number of users whose rating is higher than 3 (the default threshold for the range [1, 5]); and B_h is the number of that by prediction through the algorithm. A_l and B_l have the same meanings but have ratings lower than 3. Then, the metrics are defined as follows: (1) *Precision*: $P_h = A_h \cap B_h / B_h$, $P_l = A_l \cap B_l / B_l$. (2) *Recall*: $R_h = A_h \cap B_h / A_h$, $R_l = A_l \cap B_l / A_l$. (3) *FScore*: $F_h = 2R_h P_h/(R_h + P_h)$, $F_l = 2R_l P_l/(R_l + P_l)$. *FScore* is used to measure the accuracy using *Recall* and *Precision* jointly. (4)*The root mean squared error*: $RMSE = \sqrt{\sum(r_{vi} - \widehat{r_{vi}})^2/D}$, where D is the total number of user/item pairs that can be predicted and r_{vi} and $\widehat{r_{vi}}$ denote the real and predicted ratings on item i by user v, respectively. A higher *Precision*/*Recall*, or a smaller RMSE indicates a higher accuracy. Table IV shows the parameter settings.

Algorithms for Comparison. We compare our model with (1) FR: FluidRating [3], which considers friends' recommendations and (2) DF: DynFluid [10], which considers friends and a public channel, both FR and DF use the fluid dynamics theory; (3) RW: Random Walk, for which we set different thresholds on the number of steps; (4) RSTE: recommendation with the social trust ensemble [22]; (5) SMF: SocialMF [23], which integrates trust and matrix factorization.

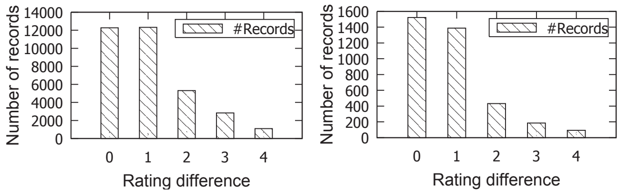
B. Experimental Results and Analysis

We analyze the effects of three types of channels and several key parameters. We also compare our model with others.

The Effects of Three Types of Channels. We first test the effects of three types of channels. Fig. 5 shows the accuracy results of considering all channels and overlooking one of the three types of channels. The main findings are as follows:

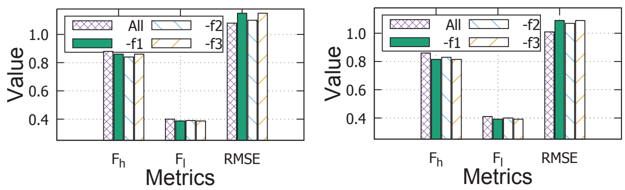
(1) *Each type of channel has its impact*. When overlooking one of the channels, the accuracy will decrease, indicating that each channel has an impact on improving prediction accuracy.

(2) *The public channel has the largest impact, the internal opinion has the second, and the friend channel has the last.* The accuracy decreases the most when we overlook the public channels and decreases slightly less when overlooking the internal opinion. Friends' influences cause the least decrease (a bit surprisingly).



(a) Epinions (b) Ciao

Fig. 4. Rating differences among users and their trustees.



(a) Epinions (b) Ciao

Fig. 5. The effects of three types of channels (-f1, -f2, and -f3 represent neglecting internal opinion, friends' and public channels' influences).

To better understand the above findings, we look deep into the data sets. We find that public channels are available for most items. We further check for the existence of friend channels. By studying trust relations among users who have co-rated the same items, we gain the following two observations:

Observation 1: There do exist trust relations among users who have rated the same item, but it is quite sparse. We count the number of items that have been rated by a user and their trustee (e.g., a user on whom he or she puts trust). The percentages of co-rating items are 0.1% in Epinions and 5% in Ciao, which is quite sparse. This indicates that friend channels do exist, but they are not as common as public channels.

Observation 2: Ratings of a user and their trustee on the same item are close to each other's; however, users' ratings are not always consistent with those of their trustees. Fig. 4 shows rating differences between a user and a trustee on the same item. Taking Ciao for instance, among user and trustee pairs, 42.4% of users give the same rating with their trustees. 80.3% of users and trustees give ratings with a difference of ≤ 1. It indicates that in general, the ratings of a user and those of their trustee on the same item are close to each other. However, some users give quite different ratings from their trustees, e.g., 7.7% users-and-trustees' rating differences are ≥ 3. Moreover, if a user gives the same rating as their trustee, the two users usually have higher interaction frequency and share more common friends. This finding is consistent with the Empirical Evidence 1.1 and 1.2.

The above analysis indicates that the two main references available to users are the public channels and the user himself.

Fig. 5 also shows that the accuracy of predicting high ratings is much higher than that of low ratings. F_h is over 80% while F_l is around 40%. We analyze the reason to be that there are more high ratings than low ones in the data sets (with the average> 4). Hence, for any user whose rating is being predicted, most of their references are giving high ratings, which leads to difficulty when predicting low ratings. It thus indicates the importance of emphasizing negative opinions.

The Effects of Parameters. We test the impact of the influence strength of internal opinion w_{i_0a}, the height difference with reference level $h_\triangle$, and the confidence decrease ratio η when meeting inconsistent opinions. Fig. 6 shows the result. The sub-figures on the top show the Fscore of the high rating prediction (rating≥ 3), while those in the middle show that of the low rating prediction (rating< 3), and the sub-figures at the bottom show the RMSE. We have several findings: (1) *The larger the influence strength of internal opinion w_{i_0a} is, the better the performance is.* As shown in Fig. 6(a), with the increase of w_{i_0a}, F_h and F_l increase, and the RMSE decreases. This indicates the importance of considering the internal opinion. Note that the influence strengths of different channels are independent in our model. Hence, a larger w_{i_0a} does not mean smaller influence strengths of other channels.

(2) *There is a turning point of* $h_\triangle$. Fig. 6(b) shows the trends of changing $h_\triangle$ from 1 to 10. When $h_\triangle \leq 5$, the accuracy increases with $h_\triangle$. After that, it decreases. At the point of $h_\triangle \leq 5$, we get the highest accuracy with $F_h = 0.86, F_l = 0.41, RMSE = 1.01$. We analyze the reason: the essential meaning of $h_\triangle$ is to weaken the influence of public channels and enhance that of negative ratings. When it becomes too large, the influence of public channels may not be able to pass on to the sink or the negative ratings will be amplified.

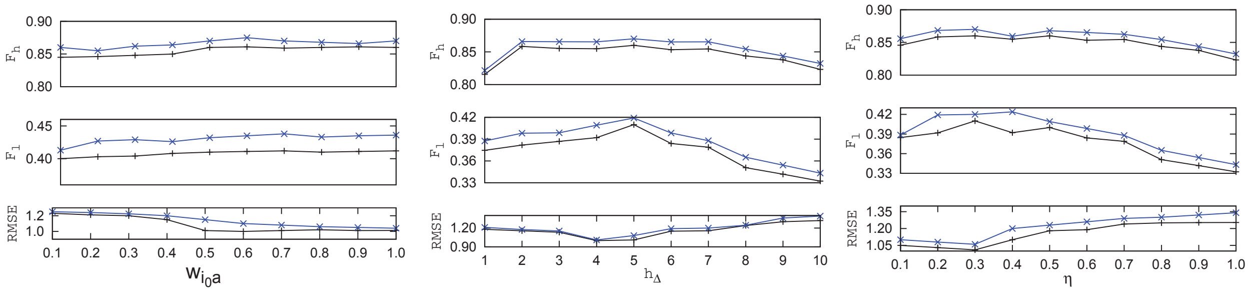
(3) *The confidence decrease ratio η cannot be too large*. Fig. 6(c) shows the trends of changing η. We find that when η changes from 0.1 to 0.3, the accuracy increases gradually. After that, it decreases sharply. This indicates that even when meeting different opinions, users will still hold most of their current opinions. This finding is consistent with the "first impression" [3] and the "biased assimilation" [6] phenomenons.

We also check the weight of interaction frequency, λ. This makes a slight difference on the final accuracy, again due to the few cases in which a user has co-rated with his trustees.

Comparison Study. Fig. 7 shows the comparison results. $OpinionFormer$ beats all the other methods in Fscore and RMSE. SocialMF [23] produces the performance closest to ours; the next closest two are DyFluid [10] and FluidRating [3]. To mention a few improvements: the RMSE of using $OpinionFormer$ is 6.09% less than that of using DyFluid in Epinions while the improvement of F_h is 2.33% and that of F_l is 3.46%. In Ciao, the improvements are 7.34% for RMSE, 2.38% for F_h, and 4.8% for F_l. That is to say, the proposed method shows a better prediction accuracy. This achievement indicates its reasonability and advantages.

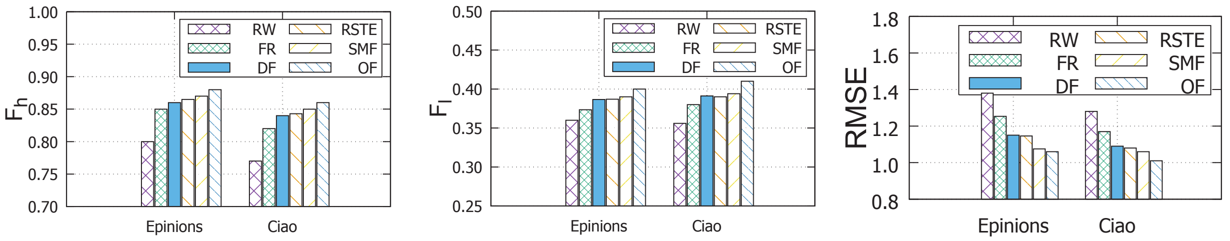
C. Summary of Experiments

Experimental results show that three types of channels can impact a user's opinion formation with different effects. Internal opinions and public channels impact the accuracy more than trusted friends do, based on the few cases in which a user and their trustees co-rate the same items. We also test the effects of parameters The results indicate the importance of the internal opinion and the necessity of careful treatment of public channels. $OpinionFormer$ can flexibly handle those factors and thus it performs better than the models that only consider partial channels or overlook the evolving features.



(a) Changing $w_{i_0 a}$ (b) Changing h_Δ (c) Changing η

Fig. 6. The effects of impact factors (Blue: Epinions; Black: Ciao): (a) the influence strength of internal opinion, $w_{i_0 a}$; (b) the height difference with reference level, h_Δ; and (c) the confidence decrease ratio when meeting inconsistency, η.



(a) F_h (b) F_l (c) RMSE

Fig. 7. A comparison with other methods (the 6th bar 'OF' is short for our model $OpinionFormer$).

VIII. Conclusion & Future Work

We study active opinion-formation at the individual level. We consider three types of channels and identify their essential elements and effects. Based on this, we propose a novel simulator using fluid dynamics to incorporate those channels incrementally. We validate its advantages via analysis and experiment with two real-world product review data sets. We will investigate other types of OSNs in future. We will also compare our method with other graphical models [24].

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