## Collaborative Mobile Charging and Coverage

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## Road Map

1. Need for Basic Research
2. Mobile Charging: State of the Art
3. Collaborative Charging \& Coverage
4. Conclusions


## 1. Need for Basic Research

## John F. Kennedy

... progress in technology depends on progress in theory... The vitality of a scientific community springs from its passion to answer science's most fundamental questions.

## Ronald Reagan

- ... although basic research does not begin with a particular practical goal ..., it ends up being one of most practical things government does.


## Mao vs. Hardy

Z. Mao (Serve the People)

- Knowledge begins with practice.


Theoretical knowledge acquired through practice must then return to practice.
G. H. Hardy (A Mathematician's Apology)


The real mathematics of the real mathematicians is almost wholly useless.

It is not possible to justify the life of any genuine professional mathematician on the ground of the utility of his work.

## Implications

- Politicians (when they become politically weak)

Start new revolutions
(and young people become followers)

- Mathematicians (when they become old)

Start writing books
(and young people prove theorems)

- Professors (when they become seniors)

Give presentations (and students write papers)

## Picasso \& Matisse

Know how to make appropriate abstractions ask the right questions

Many CS students use excessive amounts of math to explain simple things

Le Rêve (the Dream)


- The Art of Living, Time, Sept. 23, 2013 Senior people can be creative without worry the "utility" of their work


## My Two Cents

- How to select a research problem
- Simple definition
- Elegant solution
- Room for imagination



## Energy: A Special Utility

- Limited lifetime of battery-powered WSNs
- Possible solutions
- Energy conservation
- Cannot compensate for energy depletion
- Energy harvesting (or scavenging)
- Unstable, unpredictable, uncontrollable ...
- Sensor reclamation
- Costly, impractical (deep ocean, bridge surface ...)
(WSNs: Wireless Sensor Networks)


## 2. Mobile Charging: State of the Art

- The enabling technology
- Wireless energy transfer (Kurs '07)

Wireless Power Consortium

- Mobile chargers (MC)


MC moves from one location to another for wireless charging
Extended from mobile sink in WSNs and ferry in DTNs

- Energy consumption
- The movement of MC
- The energy charging process


## Combinatorics and Graph Models

- Traveling-Salesmen Problem (TSP)

A minimum cost tour of $n$ cities: the salesman travels from an origin city, visits each city exactly one time, then returns to the origin

- Covering Salesman Problem (CSP, Ohio State '89)

The least cost tour of a subset of cities such that every city not on the tour is within some predetermined covering distance

- Extended CSP
- Connected dominating set (FAU '99)
- Qi-ferry (UDelaware '13)



## Mobile Sinks and Chargers

- Local trees
- Data collections at all roots
- Periodic charging to all sensors
- Base station (BS)
- Objectives

- Long vocation at BS (VT'11-13)

Energy efficiency with deadline (Stony Brook '13)

## 3. Collaborative Coverage \& Charging

- Most existing methods

An MC is fast enough to charge all sensors in a cycle
An MC has sufficient energy to replenish an entire WSN (and return to $B S$ )

- Collaborative approach using multiple MCs Problem 1: MCs with unrestricted capacity but limitations on speed

Problem 2: MCs with limited capacity and speed, and have to return to BS

## Problem Description

Problem 1: Determine the minimum number of MCs (unrestricted capacity but limitations on speed) to cover a line/ring of sensors with uniform/non-uniform recharge frequencies

A toy example
A circle track with circumference 3.75 densely covered with sensors with recharge frequency $f=1$
Sensors with $\mathrm{f}=2$ at 0 and 0.5
A sensor with $f=4$ at 0.25


- What are the minimum number of MCs and the optimal trajectory planning of these MCs? (MC's max speed is 1.)


## Possible Solutions

- Assigning cars for sensors with f>1 (a) fixed and (b) moving

- Combining odd and even car circulations (c)



## Optimal Solution (uniform frequency)

$M_{1}$ : There are $C_{1}$ MCs moving continuously around the circle

- $M_{2}$ : There are $C_{2}$ MCs moving inside the fixed interval of length $\frac{1}{2}$ so that all sensors are covered

Combined method: It is either $M_{1}$ or $M_{2}$, so $C=\min \left\{C_{1}, C_{2}\right\}$


## Properties

Theorem 1: The combined method is optimal in terms of the minimum number of MCs used

- Scheduling
- Find an appropriate breakpoint to convert a circle to a line: $M_{2}$ in the optimal solution is then followed

A linear solution is used to determine the breakpoint

## Linear Solution

- Directed Interval Graph

Each directed link points from the start to the end of an interval (i.e., the first sensor beyond distance 0.5)

- The number of intervals in two solutions differ by one

Each sensor has one outgoing and multiple incoming links

- The process stops when a path with fewer or more intervals is
 found or all sensors (with their outgoing links) are examined


## Solution to the Toy Example

- 5 cars only, including a stop at 0.25 for $\frac{1}{4}$ time unit

- Challenges: time-space scheduling, plus speed selection


## Greedy Solution (non-uniform frequency)

- Coverage of sensors with non-uniform frequencies
serve $\left(x_{1}, \ldots, x_{n} ; f_{1}, \ldots, f_{n}\right)$ :
When $n \neq 0$, generate an MC that goes back and forth as far as possible at full speed (covering $x_{1}, \ldots, x_{i-1}$ );
$\operatorname{serve}\left(x_{i}, \ldots, x_{n} ; f_{i}, \ldots, f_{n}\right)$

Theorem 2: The greedy solution is within a factor of 2 of the optimal solution

## The Ant Problem: An Inspiration

## - Ant Problem, Comm. of ACM, March 2013

Ant Alice and her friends always march at $1 \mathrm{~cm} / \mathrm{sec}$ in whichever direction they are facing, and reverse directions when they collide
Alice stays in the middle of 25 ants on a 1 meter-long stick
How long must we wait before we are sure Alice has fallen off the stick?


Exchange "hats" when two ants collide

## Proof of Theorem 2



- Two cars never meet or pass each other
- Partition the line into $2 k-1$ sub-regions based on different car coverage ( $k$ is the optimal number of cars)
Each sub-region can be served by one car at full speed
One extra car is used when a circle is broken to a line

$2(x-a) \leq f_{x}$ and $2(b-x) \leq f_{x}$


## Imagination

- Hilbert curve for K-D

Mapping from 2-D to 1-D for preserving distance locality


- Charging time: converting to distance
- Limited capacity: using cooperative charging
- BS to MC
- MC to MC


## Charging a Line (with limited capacity)

- Charge battery capacity: 80J
- Charger cost: 3J per unit traveling distance
- Sensor battery capacity: 2J

- One MC cannot charge more than 10 consecutive sensors


## Problem Description

Problem 2 (IEEE MASS'12): Given k MCs with limited capacity, determine the furthest sensor they can recharge while still being able to go back to the BS

USN
N sensors, unit distance apart, along a line
Battery capacity for each sensor : b

- Energy consumption rate for each sensor: $r$
- MC

Battery capacity: B
Energy consumption rate due to travelling: $c$


## Motivation Example (1)

$$
B=80 \mathrm{~J}, \mathrm{~b}=2 \mathrm{~J}, \mathrm{c}=3 \mathrm{~J} / \mathrm{m}, \mathrm{~K}=3 \mathrm{MCs}
$$



- Scheme I: (equal-charge) each MC charges all sensors b/K J (Joule)
- Conclusion: covers 12 sensors, and max distance is < B/2C (as each MC needs a round-trip traveling cost)


## Motivational Example (2)



- Scheme II: (one-to-one) each sensor is charged by one MC
- Conclusion: covers 13 sensors, and max distance is still < $B / 2 c$ (as the last MC still needs a round-trip traveling cost)
- Scheme II reaches further than Scheme I


## Motivational Example (3)



- Scheme III: (collaborative-one-to-one-charge) each MC transfers some energy to other MCs at rendezvous points
- Conclusion: covers 17 sensors, and max distance is < $B / C$ (Last MC still needs a return trip without any charge)


## Details on Scheme III

- $M C_{i}$ charges battery to all sensors between $L_{i+1}$ and $L_{i}$
- $M C_{i}(1 \leq i \leq K)$ transfers energy to $M C_{i-1}, M C_{i-2, \ldots} M C_{1}$ to their full capacity at $L_{i}$

Each $M C_{i}$ has just enough energy to return to the BS


## Bananas and a Hungry Camel

- A farmer grows 3,000 bananas to sell at market 1,000 miles away. He can get there only by means of a camel. This camel can carry a maximum of 1,000 bananas at a time, but needs to eat a banana to refuel for every mile that he walks

What is the maximum number of bananas that the farmer can get to market?

Start


## Motivational Example (4): GlobalCoverage

 $B=80 \mathrm{~J}, \mathrm{~b}=2 \mathrm{~J}, \mathrm{c}=3 \mathrm{~J} / \mathrm{m}, \mathrm{K}=3 \mathrm{MCs}$

- "Push": limit as few chargers as possible to go forward
- "Wait": efficient use of battery "room" through two charges
- Conclusion: covers 19 sensors, and max distance is $\infty$ with unlimited number of MCs


## Details on Push-and-Wait

- $M C_{i}$ charges battery to all sensors between $L_{i+1}$ and $L_{i}$
- $M C_{i}(1 \leq i \leq K)$ transfers energy to $M C_{i-1}, \ldots M C_{1}$ to their full capacity at $L_{i}$
- $M C_{i}$ waits at $L_{i}$, while all other MCs keep moving forward
- After $M C_{i}, M C_{i-1}, \ldots M C_{1}$ return to $L_{i}, M C_{i}$ evenly balances energy among them (including itself)
- Each $M c_{i}, M C_{i-1}, . . M C_{1}$ has just enough energy to return to $L_{i+1}$


## Another Solution



- Each MC moves and charges (is charged) between two adjacent rendezvous points
- Imagination: $M C_{i}$ of LocalCoverage "simulates" $M C_{i}, M C_{i-1}, \ldots$, $M C_{1}$ of GlobalCoverage


## Properties

Theorem 3 (Optimality): GlobalCoverage has the maximum ratio of payload energy to overhead energy

Theorem 4 (Infinite Coverage): GlobalCoverage can cover an infinite line

Summation of segment length $\left(L_{i}-L_{i+1}\right)$

$$
\begin{aligned}
& \sum_{i=1}^{K} \frac{B}{2 \cdot c \cdot i+b}>\sum_{i=i_{0}}^{K} \frac{B}{2 \cdot c \cdot i+b}\left(\text { let } 2 \cdot c \cdot i_{0} \geq b\right) \\
& >\sum_{i=i_{0}}^{K} \frac{B}{4 \cdot c \cdot i}=\frac{B}{4 \cdot c} \sum_{i=i_{0}}^{K} \frac{1}{i}(\text { harmonic series })
\end{aligned}
$$

## Imagination: Extensions

- Simple extensions (while keeping optimality)

Non-uniform distance between adjacent sensors

- Same algorithm

Smaller recharge cycle (than MC round-trip time)

- Pipeline extension
- Complex extensions

Non-uniform frequency for recharging
Two- or higher-dimensional space

## Imagination: Applications

- Robotics

Flying robots
Google WiFi Balloon


- Tesla Moters

Tesla Roadster: all-electric

- Supercharger networks



## 4. Conclusions

- Wireless energy transfer
- Collaborative mobile charging \& coverage:
- Unlimited capacity vs. limited capacity (with BS)

Charging type: BS-to-MC, MC-to-MC, and MC-to-Sensor

- Other extensions

Charging efficiency, MCs as mobile sinks for BS...
Simplicity + Elegance + Imagination = Beauty

