# Collaborative Mobile Charging: From Abstraction to Solution 

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## Road Map

1.Need for Basic Research
2. Mobile Charging: State of the Art
3. How to Solve It

4 Collaborative Charging \& Coverage
5. Conclusions


## 1. Need for Basic Research

Z. Mao (Serve the People)

Knowledge begins with practice.
Theoretical knowledge acquired through practice, must then return to practice

G. H. Hardy (A Mathematician's Apology)

The real mathematics of the real mathematicians is almost wholly useless.
It is not possible to justify the life of any genuine mathematician on the ground of the utility of his work.


## Implications

- Politicians (when they become politically weak)
- Start new revolutions (and young people become followers)
- Mathematicians (when they become old)

Start writing books (and young people prove theorems)

- Professors (when they become seniors)

Give presentations (and students write papers)

## Power of Abstraction

Know how to make appropriate abstractions ask the right questions

Many CS students use excessive amounts of math to explain simple things


The Art of Living, Time, Sept. 23, 2013
Senior people can be creative without worry the "utility" of their work

## Energy: A Special Utility

- Limited lifetime of battery-powered WSNs
- Possible solutions
- Energy conservation
- Cannot compensate for energy depletion

Energy harvesting (or scavenging)

- Unstable, unpredictable, uncontrollable ...
- Sensor reclamation
- Costly, impractical (deep ocean, bridge surface ...)
(WSNs: Wireless Sensor Networks)


## 2. Mobile Charging: State of the Art

- The enabling technology
- Wireless power transfer
- Radiative (far-field): electromagnetic radiation

- Non-radiative (near-field): magnetic fields or electric fields

Resonant inductive coupling (2007)
Wireless Power Consortium


Rolls-Royce wireless charging


## Mobile Charger

Mobile charger (MC)
MC moves from one location to another for wireless charging
Extended from mobile sink in WSNs and ferry in DTNs

- Energy consumption
- The movement of MC
- The energy charging process

(DTNs: Delay Tolerant Networks)


## Combinatorics and Graph Models

- Traveling-Salesmen Problem (TSP)

A minimum cost tour of $n$ cities: the salesman travels from an origin city, visits each city exactly one time, then returns to the origin

- Covering Salesman Problem (CSP, Ohio State '89)

The least cost tour of a subset of cities such that every city not on the tour is within some predetermined covering distance

- Extended CSP
- Connected dominating set (FAU '99)
- Qi-ferry (UDelaware '13)



## Mobile Sinks and Chargers

- Local trees
- Data collections at all roots
- Periodic charging to all sensors
- Base station (BS)
- Objectives

- Long vocation at BS (VT'11-15)

Energy efficiency with deadline (Stony Brook '13)

## 3. How to Solve It (Poyla)

- If you can't solve a problem, then there is an easier problem you can solve: find it
- Four principles
- Understand the problem
- Devise a plan
- Carry out the plan
- Look back



## My Two Cents

- How to select a research problem
- Simple definition
- Elegant solution
- Room for imagination



## 4. Collaborative Coverage \& Charging

- Most existing methods

An MC is fast enough to charge all sensors in a cycle
An MC has sufficient energy to replenish an entire WSN (and return to BS)

- Collaborative approach using multiple MCs

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Problem 1: MCs with unrestricted capacity but limitations on speed
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Problem 2: MCs with limited capacity and speed, and have to return to BS

## Problem Description

Problem 1: Determine the minimum number of MCs (unrestricted capacity but limitations on speed) to cover a line/ring of sensors with uniform/non-uniform recharge frequencies

A toy example
A circle track with circumference 3.75 densely covered with sensors with recharge frequency $f=1$

Sensors with $\mathrm{f}=2$ at 0 and 0.5
A sensor with $f=4$ at 0.25

- What are the minimum number of MCs and the optimal trajectory planning of these MCs? (MC's max speed is 1.)


## Possible Solutions

Assigning cars for sensors with $f>1$ (a) fixed and (b) moving


- Combining odd and even car circulations (c)



## Optimal Solution (uniform frequency)

$M_{1}$ : There are $C_{1}$ MCs moving continuously around the circle

- $M_{2}$ : There are $C_{2}$ MCs moving inside the fixed interval of length $\frac{1}{2}$ so that all sensors are covered
- Combined method: It is either $M_{1}$ or $M_{2}$, so $C=\min \left\{C_{1}, C_{2}\right\}$



## Properties

## Theorem 1 (ACM MSCC'14): The combined method is optimal in terms of the minimum number of MCs used

- Scheduling

Find an appropriate breakpoint to convert a circle to a line; $M_{2}$ in the optimal solution is then followed

A linear solution, $O(n)$, is used to determine the breakpoint, assuming $n$ is the number of nodes and the circumference is a known constant

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## Linear Solution

- Directed Interval Graph

Each directed link points from the start to the end of an interval (i.e., the first sensor beyond distance 0.5)

- The number of intervals in two solutions differ by one
- Each sensor has one outgoing and multiple incoming links
- The process stops when a path with fewer or more intervals is
 found or all sensors (with their outgoing links) are examined


## Solution to the Toy Example

5 cars only, including a stop at 0.25 for $\frac{1}{4}$ time unit


- Challenges: time-space scheduling, plus speed selection


## Greedy Solution (non-uniform frequency)

- Coverage of sensors with non-uniform frequencies serve $\left(x_{1}, \ldots, x_{n}, f_{1}, \ldots, f_{n}\right)$ :

When $n \neq 0$, generate an MC that goes back and forth as far as possible at full speed (covering $x_{1}, \ldots, x_{i-1}$ ); serve $\left(x_{i}, \ldots, x_{n} ; f_{i}, \ldots, f_{n}\right)$


Theorem 2 (ACM MSCC'14): The greedy solution is within a factor of 2 of the optimal solution

## The Ant Problem: An Inspiration

## - Ant Problem, Comm. of ACM, March 2013

Ant Alice and her friends always march at $1 \mathrm{~cm} / \mathrm{sec}$ in whichever direction they are facing, and reverse directions when they collide
Alice stays in the middle of 25 ants on a 1 meter-long stick
How long must we wait before we are sure Alice has fallen off the stick?


Exchange "hats" when two ants collide

## Proof of Theorem 2



- Two cars never meet or pass each other

Partition the line into $2 k-1$ sub-regions based on different car coverage ( $k$ is the optimal number of cars)
Each sub-region can be served by one car at full speed
One extra car is used when a circle is broken to a line


[^0]
\[

$$
\begin{gathered}
2(x-a) \leq f_{x} \text { and } 2(b-x) \leq f_{x} \\
b-a \leq f_{x}
\end{gathered}
$$
\]

## Imagination

- Hilbert curve for k-D

Mapping from 2-D to 1-D for preserving distance locality


- Charging time: converting to distance
- Limited capacity: using cooperative charging
- BS to MC
- MC to MC


## Charging a Line (with limited capacity)

- Charge battery capacity: 80J
- Charger cost: 3J per unit traveling distance
- Sensor battery capacity: 2 J

- One MC cannot charge more than 10 consecutive sensors


## Problem Description

Problem 2 (IEEE MASS'12): Given k MCs with limited capacity, determine the furthest sensor they can recharge while still being able to go back to the BS

- WSN

N sensors, unit distance apart, along a line
Battery capacity for each sensor: b

- Energy consumption rate for each sensor: $r$
- MC
- Battery capacity: B

Energy consumption rate due to travelling: $c$


## Motivation Example (1)

$$
B=80 \mathrm{~J}, \mathrm{~b}=2 \mathrm{~J}, \mathrm{c}=3 \mathrm{~J} / \mathrm{m}, \mathrm{~K}=3 \mathrm{MCs}
$$




- Scheme I: (equal-charge) each MC charges all sensors b/K J (Joule)
- Conclusion: covers 12 sensors, and max distance is < $B / 2 C$ (as each MC needs a round-trip traveling cost)


## Motivational Example (2)



- Scheme II: (one-to-one) each sensor is charged by one MC
- Conclusion: covers 13 sensors, and max distance is still < $B / 2 C$ (as the last MC still needs a round-trip traveling cost)
- Scheme II reaches further than Scheme I


## Motivational Example (3)



- Scheme III: (collaborative-one-to-one-charge) each MC transfers some energy to other MCs at rendezvous points
- Conclusion: covers 17 sensors, and max distance is < $B / C$ (Last MC still needs a return trip without any charge)


## Bananas and a Hungry Camel

A farmer grows 3,000 bananas to sell at market 1,000 miles away. He can get there only by means of a camel. This camel can carry a maximum of 1,000 bananas at a time, but needs to eat a banana to refuel for every mile that he walks

What is the maximum number of bananas that the farmer can get to market?



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## Motivational Example (4): GlobalCoverage $B=80 \mathrm{~J}, \mathrm{~b}=2 \mathrm{~J}, \mathrm{c}=3 \mathrm{~J} / \mathrm{m}, \mathrm{K}=3 \mathrm{MCs}$



- "Push": limit as few chargers as possible to go forward
- "Wait": efficient use of battery "room" through two charges
- Conclusion: covers 19 sensors, and max distance is $\infty$ with unlimited number of MCs


## Properties

Theorem 3 (Optimality): GlobalCoverage has the maximum ratio of payload energy to overhead energy

## Theorem 4 (Infinite Coverage): GlobalCoverage can

 cover an infinite lineSummation of segment length $\left(L_{i}-L_{i+1}\right)$

$$
\begin{aligned}
& \sum_{i=1}^{K} \frac{B}{2 \cdot c \cdot i+b}>\sum_{i=i_{0}}^{K} \frac{B}{2 \cdot c \cdot i+b}\left(\text { let } 2 \cdot c \cdot i_{0} \geq b\right) \\
& >\sum_{i=i_{0}}^{K} \frac{B}{4 \cdot c \cdot i}=\frac{B}{4 \cdot c} \sum_{i=i_{0}}^{K} \frac{1}{i}(\text { harmonic series })
\end{aligned}
$$

## Another Solution



- Each MC moves and charges (is charged) between two adjacent rendezvous points
- Imagination: $M C_{i}$ of LocalCoverage "simulates" $M C_{i}, M C_{i-1}, \ldots$, $M C_{1}$ of GlobalCoverage


## Imagination: Extensions

- Simple extensions (while keeping optimality)
- Non-uniform distance between adjacent sensors

Same algorithm
Smaller recharge cycle (than MC round-trip time)

- Pipeline extension
- Complex extensions

Non-uniform frequency for recharging
Two- or higher-dimensional space

## Imagination: Applications

## - Robotics

Flying robots
Google WiFi Balloon

- Tesla Motors


Tesla Roadster: all-electric

- Supercharger networks



## 5. Conclusions

- Wireless energy transfer
- Collaborative mobile charging \& coverage:
- Unlimited capacity vs. limited capacity (with BS)

Charging type: BS-to-MC, MC-to-MC, and MC-to-Sensor

- Other extensions

Charging efficiency, MCs as mobile sinks for BS...
Simplicity + Elegance + Imagination = Beauty

## Real Problem I: DC Metro

Problem: The Washington, DC subway system charges fees based on travelling distance. For example, a passenger enters station $A$, stays there for $X($ say, 10$)$ hours, and exits station B. The charge is proportional to the distance between $A$ and $B$ and is irrelevant to $X$.

What are the potential flaws?
Provide possible solutions.
What happen if $X$ is limited to 4 hours as in Nanjing, P. R. China?

A
1, 2 (in)
2 (out)
2 (in)
1 (out)
1 (in)
2 (out)
2 (in)


## Real Problem II: Shanghai Taxi

> Problem: At the Shanghai int'I airport, taxi drivers have to wait for at least 4 hours. It is unfair to a driver if a passenger's destination is the Industrial Park, which is about 20 minutes away. Others will go to downtown, which is 50 minutes away.

- Find a solution so that the interests of both the drivers and the customers are protected.

Find potential flaws with the current solution at the Shanghai International Airport.


## Questions

## Collaborators

Sheng Zhang

- Richard Beigel

Huanyang Zheng


[^0]:    1-red
    2-green
    3-yellow

