## Optimizing Roadside Advertisement Dissemination in Vehicular CPS



## 1. Introduction

- Roadside Advertisement Dissemination
- Passengers, shopkeeper, and Roadside Access Point (RAP)

Shopkeeper disseminates ads to passing vehicles through RAPs

- Passengers may go shopping, depending on detour distance



## 1. Introduction

- Roadside Advertisement Dissemination

RAP placement optimization: Given a fixed number of RAPs and traffic flows, maximally attract passengers to the shop

- Tradeoff between traffic density and detour probability



## 2. Scenario

- City graph $G=(V, E)$

V: a set of street intersections (nodes)

- One shop and RAPs located at street intersections

E: streets (directed edges)


## 2. Scenario

- City graph G = (V, E)

Traffic flows on streets (known a priori)

- Vehicles that travel daily from office to home
- Traveling path is the shortest path



## 2. Scenario

- Detour model

Shopkeeper disseminates ads to passengers through RAPs

- Passengers in a flow may detour to the shop

Detour probability depends on detour distance: $d_{1}+d_{2}-d_{3}$


## 2. Scenario

Theorem 1: For a given traffic flow, the first RAP on its path always provides the best detour option compared to all the other RAPs on the path

Insight: it provides the
highest traveling flexibility

The first RAP dominates the others $d_{l}(x)$


- Redundant ads do not provide extra attraction


## 2. Scenario

- Detour probability
- Non-increasing with respect to the detour distance

For a traffic flow, $T$, with a detour distance, $d$

- $p(d)$ : the detour probability of each passenger
- An expectation of $p(d)^{*} / T /$ passengers detour to the shop
- Two utility functions to describe $p(d)$
$\square$ Threshold utility function
$\square$ Decreasing utility function


## 2. Scenario

## Detour probability

- Threshold utility function

$$
p(d)=\left\{\begin{array}{cc}
c & d \leq D \\
0 & \text { otherwise }
\end{array}\right.
$$

$\square$ c: constant, D: distance threshold

detour

- Decreasing utility function (example) probability

$$
p(d)=\left\{\begin{array}{cc}
c \times(1-d / D) & d \leq D \\
0 & \text { otherwise }
\end{array}\right.
$$



## 3. Related Work

- Maximum coverage problem

Elements, $e_{1}, e_{2}, e_{3}, e_{4}, e_{5}$ and sets, $s_{1}=\left\{e_{1}, e_{2}\right\}, s_{2}=\left\{e_{2}, e_{3}\right\}, s_{3}=\left\{e_{3}, e_{4}\right\}$, $s_{4}=\left\{e_{5}\right\}$

Use a given number of sets to maximally cover elements
Greedy algorithm with max marginal coverage has an approximation ratio of 1-1/e
select $s_{1}$ and $s_{3}$ to
cover 4 elements


## 3. Related Work

- Weighted version
- Elements have benefits, sets have costs
- Use a given cost budget to maximize benefit of covered elements

Iteratively select the set with a maximal benefit-to-cost ratio


## 3. Related Work

- Our problem
- Place RAPs on intersections to cover traffic flows

Different RAPs bring different detour probabilities


## 4. General RAP Placement

- RAP placement with threshold utility
- Passengers have a fixed probability of going shopping, when the detour is smaller than a threshold $D$

$$
p(d)=\left\{\begin{array}{cc}
c & d \leq D \\
0 & \text { otheriwse }
\end{array}\right.
$$

Reduce to the maximum coverage problem

> Algorithm 1 (max marginal coverage): iteratively place an RAP at an intersection that can attract the maximum number of passengers from uncovered traffic flows

## 4. General RAP Placement

- RAP placement with threshold utility

Time complexity: $O\left(/ V /^{3}+k / V / F\right)$

- /V/: \# of intersections, k. \# of RAPs, and F: \# of traffic flows
- Computing the detour distance takes $/ V / 3$ (shortest paths between all pairs of intersections via the Floyd algorithm)
- Greedy algorithm has ksteps; in each step, it visits each intersection to check traffic flows for coverage: /V/*F


## 4. General RAP Placement

- RAP placement with decreasing utility
- Example (any decay function)

$$
p(d)=\left\{\begin{array}{cc}
c \times(1-d / D) & d \leq D \\
0 & \text { otheriwse }
\end{array}\right.
$$

- Key observation:
- Place an RAP to cover an uncovered traffic flow
- Place an RAP to provide a smaller detour distance for a covered traffic flow


## 4. General RAP Placement

## Algorithm 2 (composite greedy solution):

Iteratively find an intersection that can attract the maximum:
Candidate i: passengers from the uncovered traffic flows;
Candidate ii: passenger from the covered traffic flows, providing smaller detour distances;
Select i or ii that can attract more passengers to the shop:


## 4. General RAP Placement

- A composite greedy solution

Theorem 2: The composite greedy solution has an approximation ratio of $1-1 / \sqrt{e}$ to the optimal solution

Compare \# of passengers in the optimal solution (OPT) and the greedy solution at the i-th step $\left(\mathcal{G}_{i}\right)$

- $w_{i}$ : of passengers in the covered flows in OPT, but uncovered in $G_{i}$
$w_{2}$ : \# of passengers in OPT that have smaller detour distances than $G_{i}$
- Same complexity as Algorithm 1


## 4. General RAP Placement

Proof


Let $w(\cdot)$ : \# of attracted passengers in the corresponding solution

$$
w(O P T)-w\left(G_{i}\right) \leq w_{1}+w_{2}
$$

$G_{i}$ may also cover some uncovered traffic flows or provide smaller detour distances compared with OPT

## 4. General RAP Placement

## Proof (Cont'd)

OPT places at most $k$ RAPs to obtain $w_{1}$

- In the greedy approach, there exists a placement of a RAP that attracts $w_{1} / k$ passengers from the uncovered flows
- Therefore,

$$
w\left(G_{i+1}\right)-w\left(G_{i}\right) \geq \frac{w_{1}}{k}
$$

- Similarly,

$$
w\left(G_{i+1}\right)-w\left(G_{i}\right) \geq \frac{w_{2}}{k}
$$

## 4. General RAP Placement

Proof (Cont'd)

$$
w\left(G_{i+1}\right)-w\left(G_{i}\right) \geq \frac{w_{1}+w_{2}}{2 k}
$$

Therefore,

$$
\begin{aligned}
& \frac{w(O P T)-w\left(G_{i}\right)}{2 k} \leq \frac{w_{1}+w_{2}}{2 k} \leq w\left(G_{i+1}\right)-w\left(G_{i}\right) \\
& w(O P T)-w\left(G_{i}\right) \geq \frac{2 k}{2 k-1}\left[w(O P T)-w\left(G_{i+1}\right)\right]
\end{aligned}
$$

## 4. General RAP Placement

Proof (Cont'd)
Through iteration
$w(O P T)=w(O P T)-w\left(G_{0}\right) \geq\left(\frac{2 k}{2 k-1}\right)^{k} \times\left[w(O P T)-w\left(G_{k}\right)\right]$
Since, $\quad\left(\frac{2 k-1}{2 k}\right)^{k}=\sqrt{\left(1-\frac{1}{2 k}\right)^{2 k}} \leq \sqrt{\frac{1}{e}}=\frac{1}{\sqrt{e}}$

$$
w\left(G_{k}\right) \geq\left(1-\left(\frac{2 k-1}{2 k}\right)^{k}\right) \times w(O P T) \geq\left(1-\frac{1}{\sqrt{e}}\right) \times w(O P T)
$$

## 5. Manhattan RAP Placement

## Manhattan Streets (grid structure)



## 5. Manhattan RAP Placement

## Manhattan Streets

Multiple shortest paths exist: my Taizhong story
Passengers travel through the shortest path with RAPs


3 shortest paths from $V_{3}$ to $V_{4}$


Passengers choose $\mathrm{V}_{3}-\mathrm{V}_{1}-\mathrm{V}_{4}$

## 5. Manhattan RAP Placement

## Manhattan Streets

All traffic flows go through these Manhattan streets
Passengers will detour once receiving ads


## 5. Manhattan RAP Placement

- Key observation

Straight traffic flows: $\rightarrow$

- Optimal RAP placement, as they are independent

Turned traffic flows: $\nabla \underset{\sim}{\boldsymbol{A}}$

- Four RAPs at the four corners cover all the turned traffic

Other traffic flows: $\square$

- Reduce to the classic maximum coverage problem with an approximation ratio of 1-1/e


## 5. Manhattan RAP Placement

Algorithm 3 (two-stage solution):
If we have more than five RAPs
Place an RAP at each corner of the grid (for turned traffic flows)
Use remaining RAPs to maximally cover remaining traffic flows
Otherwise, exhaustive search can be used
ok. the number of RAPs
Approximation ratio of $1-\frac{1}{3 e}-\frac{4}{3 k}$

## 5. Manhattan RAP Placement

Two-stage solution

- Turned traffic flows
- A fraction of $\frac{2}{3}$
- Covered by 4 RAPs
- Remaining traffic flows
- A fraction of $\frac{1}{3}$

- Covered with an approximation ratio of $1-\frac{1}{e}$
- Covered by $k-4$ RAPs, leading to a ratio of $1-\frac{4}{k}$

Overall approximation ratio

$$
\frac{2}{3}+\frac{1}{3} \times\left(1-\frac{1}{e}\right) \times\left(1-\frac{4}{k}\right) \geq 1-\frac{1}{3 e}-\frac{4}{3 k}
$$

## 6. Experiments

- Dataset: Dublin bus trace (general scenario)

Includes bus ID, longitude, latitude, and vehicle journey ID
A vehicle journey represents a traffic flow
80,000 * 80,000 square feet


## 6. Experiments

- Dataset: Seattle bus trace (Manhattan streets)

Includes bus ID, $x$-coordinate, $y$-coordinate, and route ID
10,000 * 10,000 square feet
Manhattan streets


## 6. Experiments

- Settings

The location of the shop is classified into the city center, city, or suburb, depending on the amount of passing traffic flows

In the utility functions, $c$ is set to be 0.001
Each bus in Dublin (Seattle) carries 100 (200) passengers per day, on average

Results are averaged over 1,000 times for smoothness

## 6. Experiments

- Comparison algorithms
- MaxCardinality: ranks intersections by \# of bus routes and places RAPs at the top- $k$ intersections
- MaxVehicles: ranks intersections by \# of passing buses and places the RAPs at the top- $k$ intersections
- MaxCustomers: ranks the intersections by the \# of attracted passengers (flows) and places RAPs at the top- $k$ intersections.
- Random: places RAPs uniform-randomly among all the intersections


## 6. Experiments

## The impact of utility function (Dublin trace)

Shop in the city with $D=20,000$



$$
f(d)=\left\{\begin{array}{c}
0.001 \quad d \leq D \\
0 \quad \text { otheriwse }
\end{array}\right.
$$

$$
f(d)=\left\{\begin{array}{cc}
0.001 \times(1-d / D) & d \leq D \\
0 & \text { otheriwse }
\end{array}\right.
$$

Threshold utility function brings more passengers to the shop

## 6. Experiments

- The impact of shop location (Dublin trace)
$D=20,000$ feet with decreasing utility function


A better shop location can attract more passengers

## 6. Experiments

- The impact of threshold $D$ (Dublin trace)

Decreasing utility function with shop in the city


A larger threshold D can attract more passengers

## 6. Experiments

## Manhattan scenario (Seattle trace)

Threshold utility function with $D=25,000$ feet and shop in the


Algorithm 3 is the twostage solution for the Manhattan street only

Algorithm 1 is the greedy solution under the Manhattan street

More passengers can be attracted through utilizing the geographical property of the Manhattan streets

## 6. Experiment Summary

- Threshold utility function brings more passengers to the shop than decreasing utility function
- Better shop location can attract more passengers
city center > city > suburb
- A larger threshold D can attract more passengers
- More passengers can be attracted through utilizing the geographical property of the Manhattan street


## 7. Conclusion

- Roadside Advertisement Dissemination

A unique coverage problem: passengers, shopkeeper, Roadside Access Point (RAP)

Optimizing RAP placement to maximally attract passengers
Impact of traffic density and detour distance
Composite greedy algorithm with a ratio of $1-1 / \sqrt{e}$ to the optimal solution

Manhattan streets with a better approximation result

- Future directions: multiple shops at different locations


## Q \& A

