ICCCN 2022

# *A Defense-Attack Game under Multiple Preferences and Budget Constraints with Equilibrium*

Nadia Niknami, Abdalaziz Sawwan, Jie Wu
Temple university

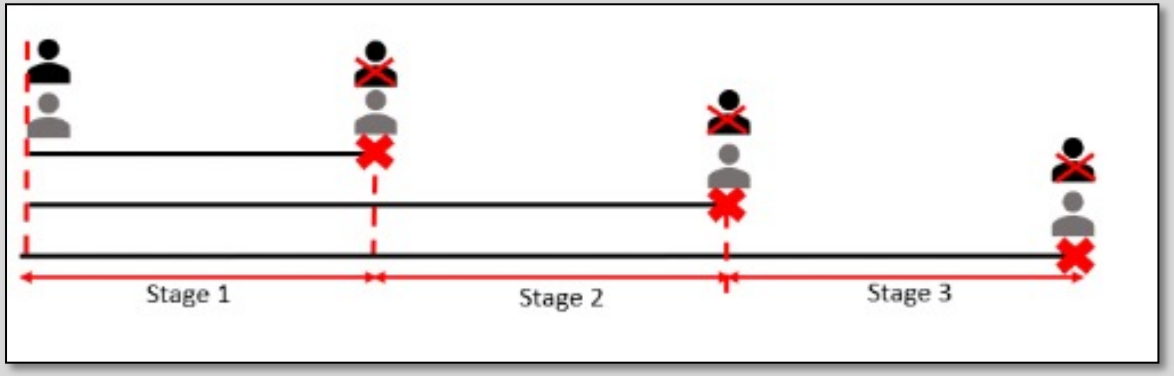
# Outline

- Attack and defense game
- Contributions
- (In)complete and (Im)perfect information
- Evaluation of model
- Conclusion

# Attack - Defense Game

# Attack Defense Game


- *In the real world, players can choose to protect themselves as as well as kill their opponents to maximize their gain.*
- *Gain for attack + gain for defend*
- *Sequential game*
- *Multiple round game*



# Contributions:

*A new game with two players who have preferences of both protection and destruction.*

*Based on their preferences and intentions, players allocate their resources to actions against their opponents.*

*A new utility function that includes both gains from staying in the game and gains from killing the opponents.*

*Modeling the behaviors of players under budget constraints when players have incomplete or imperfect information at the time of decision making.*

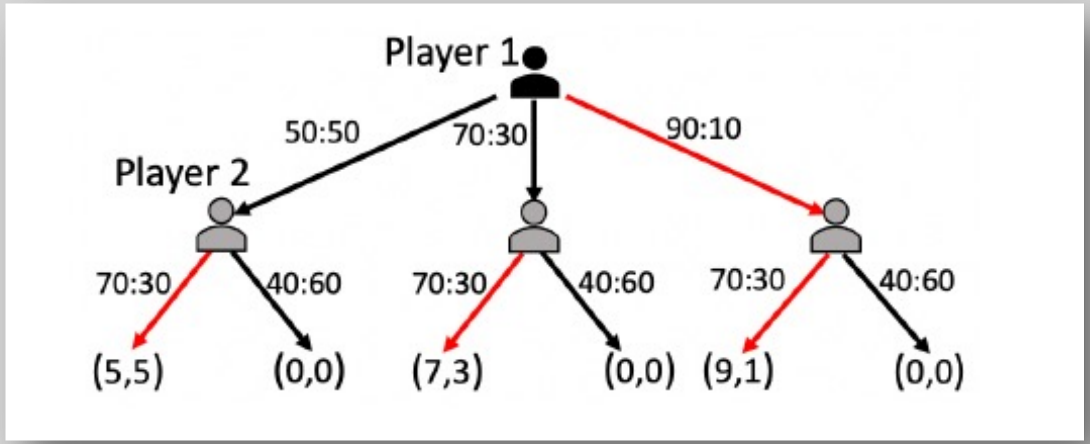
*Effects of the players' preferences, budget, and cost/expenditure on the model. Finding equilibrium involves balancing budget allocations with satisfying preferences.*

*Player allocates their budget for both defending and attacking actions.*

*Player decides how well to attack and how well to defend against others.*

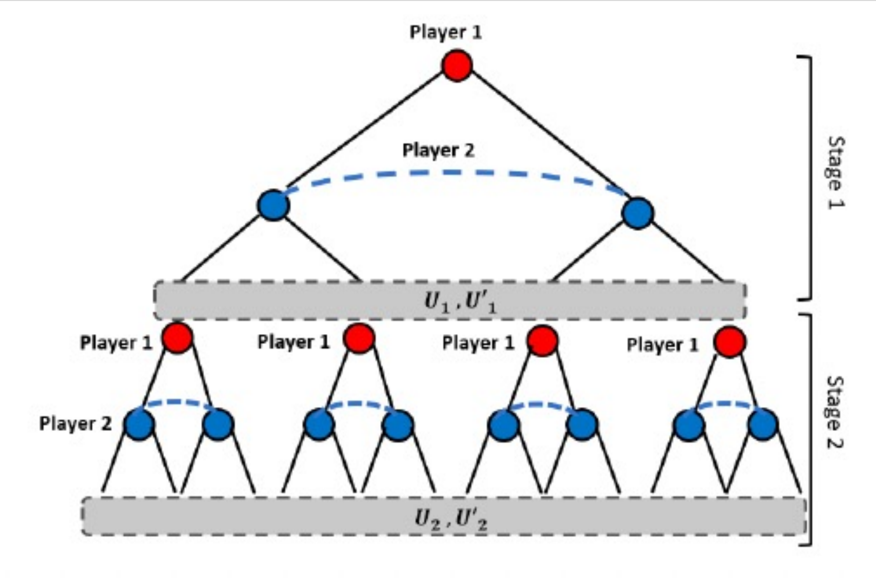
*Players should allocate their budgets appropriately for each action throughout multiple rounds when playing such a game.*

*The probabilities of surviving and killing in each round are determined by what happened in the previous rounds and the amount of the remaining budget.*



# ***Utility Function***

- *The value α  represents the amount of reward that Player 1 obtains by <u>surviving</u> for a round.*
- *The value G represents the amount of reward that Player 1 obtains by <u>killing the apponnet</u> for a round.*
- *Different values for G and α .*
- *A D-minded player finds higher α than an A-minded player for surviving in a given round.*




*Contest success function:* $k = \dfrac{A}{A + D}$

$$U_1 = \alpha + G \frac{A_1}{A_1 + D'_1}.$$

Gain of surviving

Gain of killing

Probability of successful attack

Probability of failed attack at stage 1

Probability of successful attack at stage 1

Probability of successful attack at stage 2

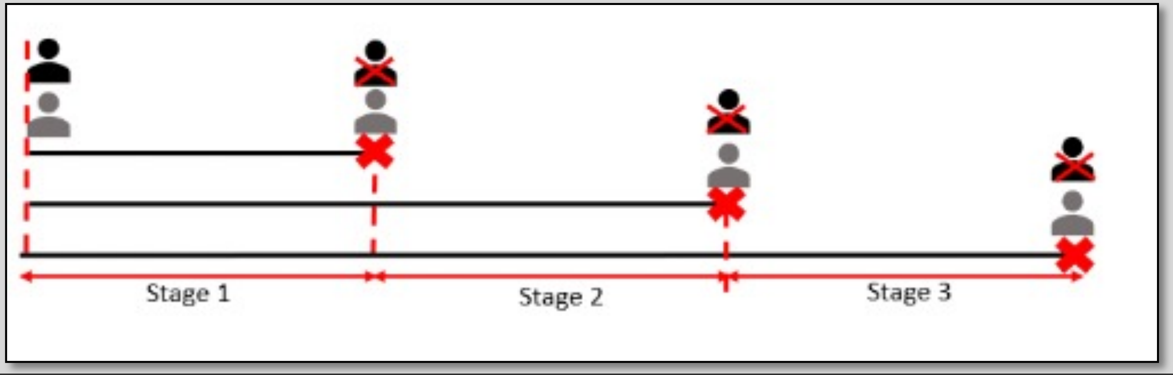
$$U_1 = \alpha + G\frac{A_1}{A_1 + D_1'}.$$

$$U_2 = 2\alpha + G\underbrace{\left(\frac{A_1}{A_1 + D_1'} + \left(1 - \frac{A_1}{A_1 + D_1'}\right) \times \frac{A_2}{A_2 + D_2'}\right)}_{\text{The probability of a successful attack at round 1 or at round 2}}.$$

$$k_i = \frac{A_i}{A_i + D_i'}.$$

$$k_i' = \frac{A_i'}{A_i' + D_i}$$

$$U_3 = 3\alpha + G(k_1 + (1 - k_1)k_2 + (1 - k_1)(1 - k_2)k_3).$$



Getting kill by opponent at stage i:

$$d_1 = k'_1, \quad d_2 = (1 - k'_1)k'_2, \quad d_3 = (1 - k'_1)(1 - k'_2)k_3.$$

$$k_i = \frac{A_i}{A_i + D'_i}$$

$$k'_i = \frac{A'_i}{A'_i + D_i}$$

$$U(\mathbf{K}) = \alpha \underbrace{\sum_{x=1}^{T}((x-1)d_x)}_{\text{Gain of staying in the game}} + G \overbrace{\sum_{x=1}^{T}\left(k_x\left(\prod_{j=1}^{x-1}(1-k'_j)\right)\right)}^{\text{Gain of killing the opponent in the game}},$$

Concave utility function:

$$\begin{aligned} \max \quad & U(\mathbf{K}) \\ \text{subject to} \quad & \sum_{1 \le i \le T} (A_i + D_i) \le B \\ & D_i, D'_i \ge C \end{aligned}$$

B: total budget

C: base cost of remaining in the game

# Player's Information

In any repeated game, how the game proceed will be determined by the result of each round. Information of player about the game can be categorized as:

- Complete
- Incomplete
- Imperfect

$$U(S, S'|\theta) = \sum_{\theta'} P(\theta'|\theta) \cdot U(S_\theta, S'_{\theta'})$$

$$S_\theta = \arg\max_S \sum_{\theta'} P(\theta'|\theta) \cdot U(S_\theta, S'_{\theta'}).$$

# Incomplete and Imperfect Information

- *It is possible for the player not to be aware of everything his opponent does.*
- *Despite knowing that his opponent is playing according to one of the possible types, he cannot see which action exactly he is taking.*
- *player only sees the opponent's action, but does not know what its objective is. To meet this challenge, this paper develops a game where players decide how to allocate resources when they have partial information*

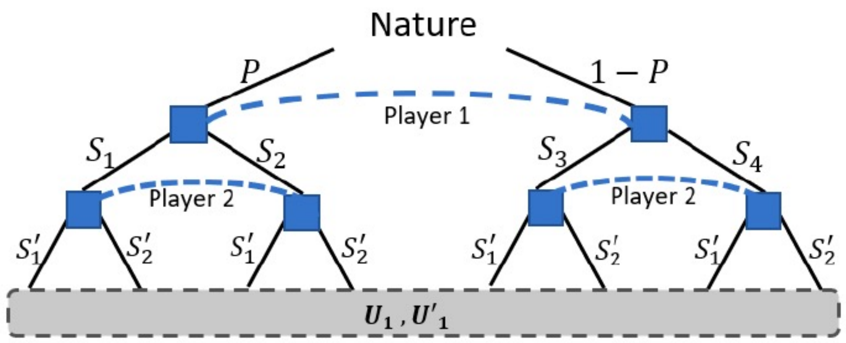



Fig. 5. Partial information for Player 1's preference.

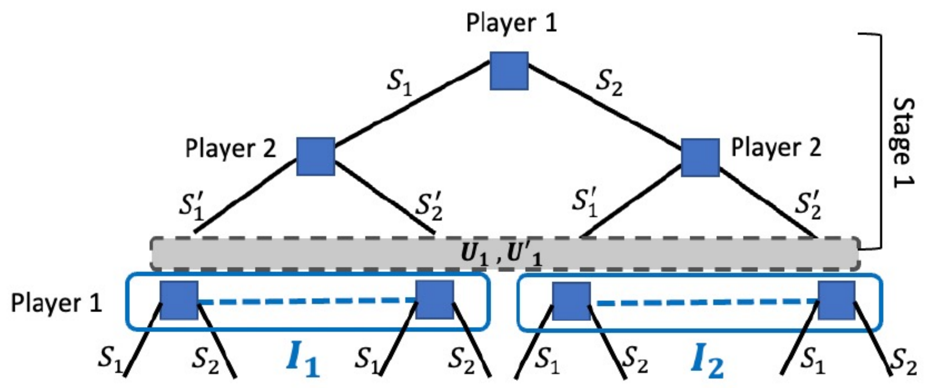
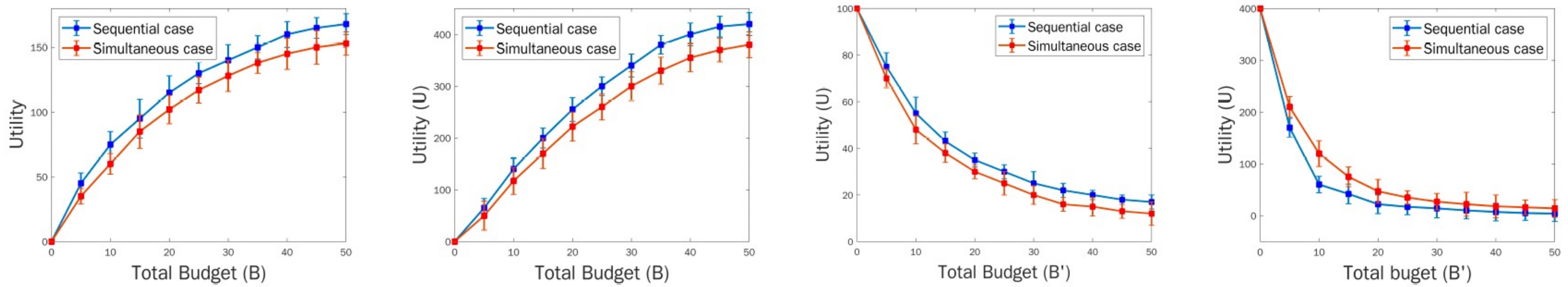



Fig. 6. Game with imperfect information for one of the players.

# Evaluation

- How utility changes based on the
  - Total budget
  - Belief
  - Partial information
  - Different values of cost and gain

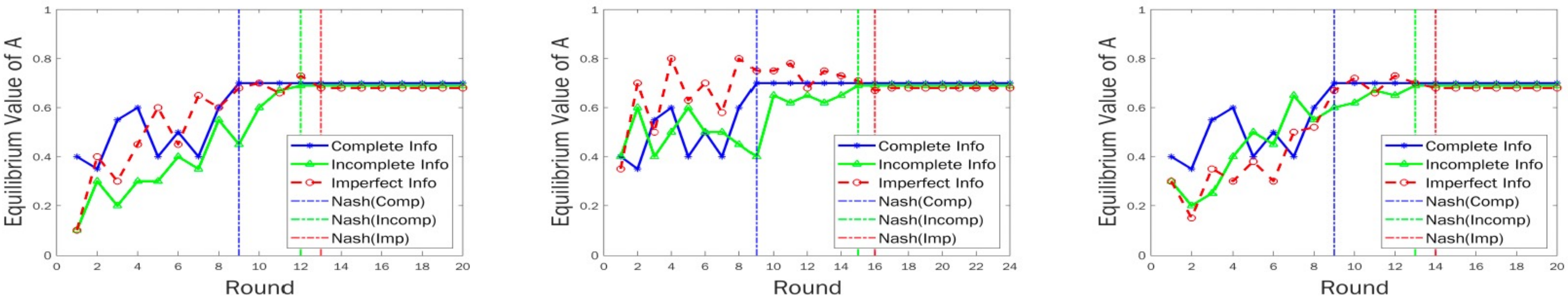
# Impact of Different Amounts of Budget and Different Costs




(a) $B' = 20$, $C = 10$ (b) $B' = 20$, $C = 1$ (c) $B = 20$, $C = 10$ (d) $B = 20$, $C = 1$

Fig. 8. Utility of Player 1 in the case of simultaneous and sequential games with varying budgets.

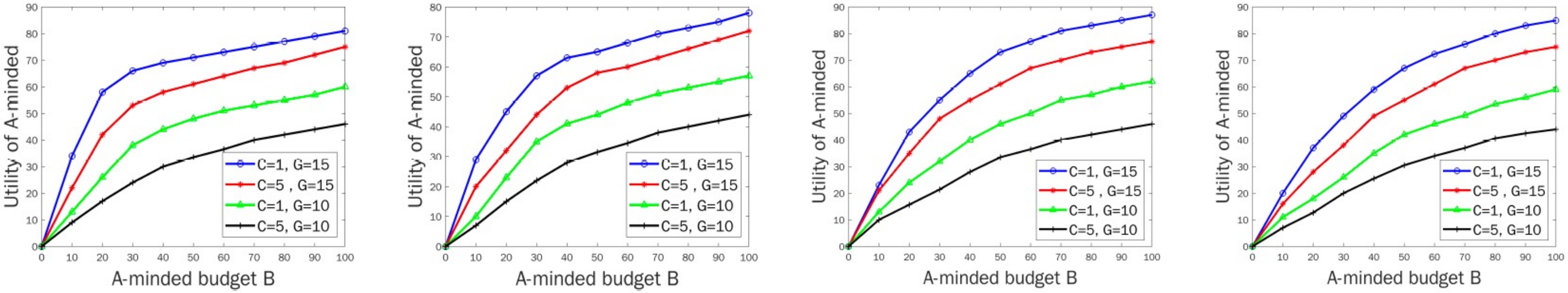
# Convergence in Different Information and Initial Beliefs




(a) Initial Belief:(10%:90%) (b) Initial Belief:(50%:50%) (c) Initial Belief:(30%:70%)

Fig. 9. Speed of convergence in the case of different belief. Dashed lines show the time of converging to the Nash equilibrium.

# Impact of Different Amounts of Gain and Cost




(a) $B' = 20, \alpha = 1$ (b) $B' = 40, \alpha = 1$ (c) $B' = 20, \alpha = 5$ (d) $B' = 40, \alpha = 5$

Fig. 10. The effect of different values of cost and gain on utility.


# Thanks for your attention!