

ICCCN 2022

*A Defense-Attack Game  
under Multiple  
Preferences and Budget  
Constraints with  
Equilibrium*

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# ***Outline***

Attack and defense game

Contributions

(In)complete and (Im)perfect information

Evaluation of model

Conclusion

# *Attack - Defense Game*

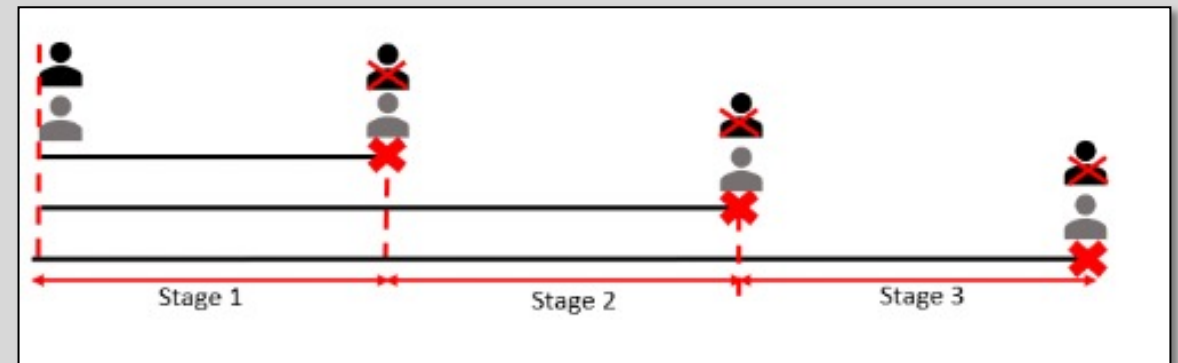
VS





# ***Attack Defense Game***

- *In the real world, players can choose to protect themselves as well as kill their opponents to maximize their gain.*
- *Gain for attack + gain for defend*
- *Sequential game*
- *Multiple round game*



## **Contributions:**

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*A new game with two players who have preferences of both protection and destruction.*

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*Based on their preferences and intentions, players allocate their resources to actions against their opponents.*

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*A new utility function that includes both gains from staying in the game and gains from killing the opponents.*

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*Modeling the behaviors of players under budget constraints when players have incomplete or imperfect information at the time of decision making.*

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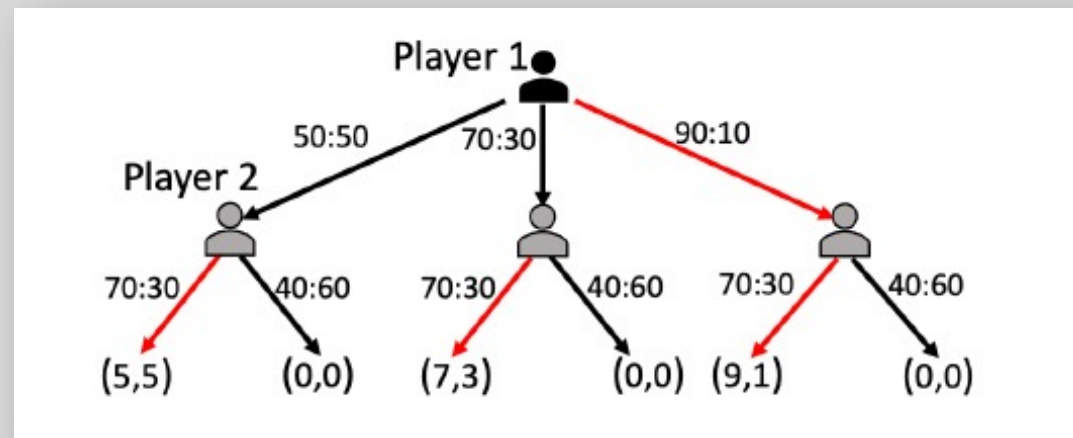
*Effects of the players' preferences, budget, and cost/expenditure on the model. Finding equilibrium involves balancing budget allocations with satisfying preferences.*

*Player allocates their budget for both defending and attacking actions.*

*Player decides how well to attack and how well to defend against others.*

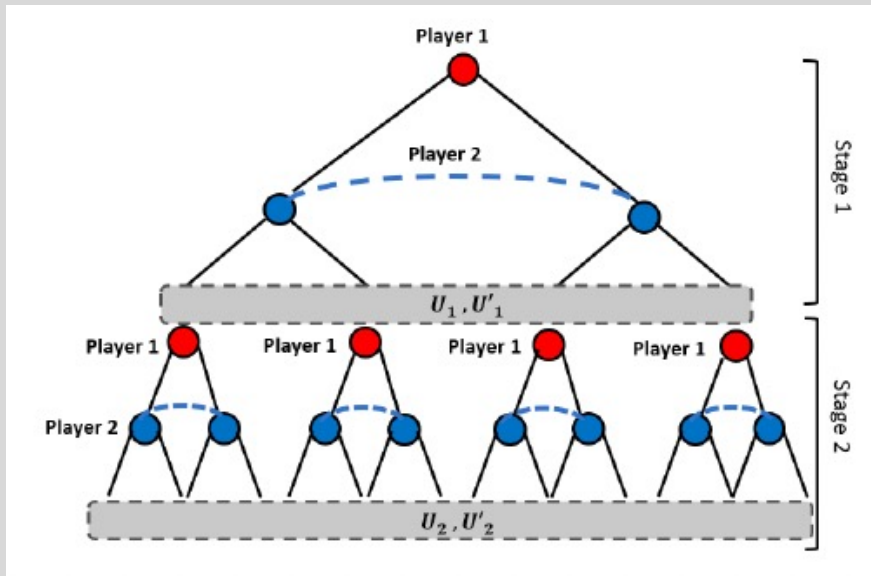
*Players should allocate their budgets appropriately for each action throughout multiple rounds when playing such a game.*

*The probabilities of surviving and killing in each round are determined by what happened in the previous rounds and the amount of the remaining budget.*



# Utility Function

- The value  $\alpha$  represents the amount of reward that Player 1 obtains by surviving for a round.
- The value  $G$  represents the amount of reward that Player 1 obtains by killing the apponnet for a round.
- Different values for  $G$  and  $\alpha$  .
- A  $D$ -minded player finds higher  $\alpha$  than an  $A$ -minded player for surviving in a given round.



Contest success function:  $k = \frac{A}{A + D}$

$$U_1 = \alpha + G \frac{A_1}{A_1 + D'_1}$$

Gain of surviving

Gain of killing

Probability of successful attack

Probability of failed attack at stage 1

Probability of successful attack at stage 1

Probability of successful attack at stage 2

$$U_1 = \alpha + G \frac{A_1}{A_1 + D'_1}$$

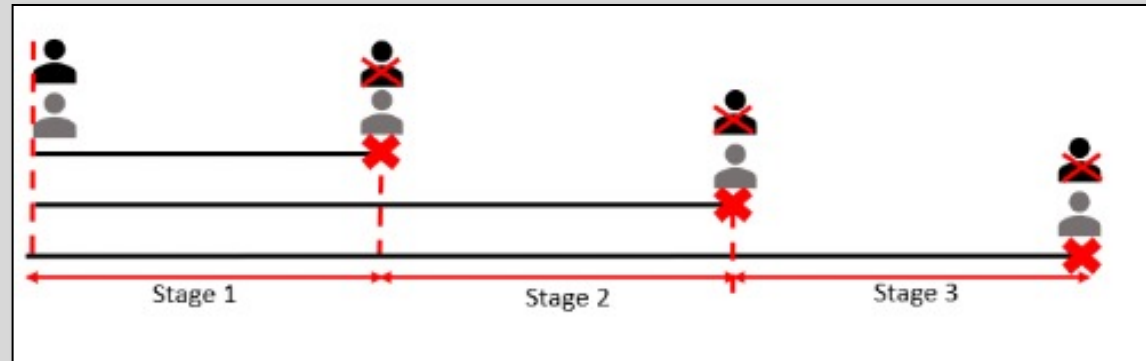
$$U_2 = 2\alpha + G \left( \frac{A_1}{A_1 + D'_1} + \left( 1 - \frac{A_1}{A_1 + D'_1} \right) \times \frac{A_2}{A_2 + D'_2} \right)$$

The probability of a successful attack at round 1 or at round 2

$$U_3 = 3\alpha + G(k_1 + (1 - k_1)k_2 + (1 - k_1)(1 - k_2)k_3)$$

$$k_i = \frac{A_i}{A_i + D'_i}$$

$$k'_i = \frac{A'_i}{A'_i + D_i}$$





Getting kill by  
opponent at stage  $i$ :

$$d_1 = k'_1, \quad d_2 = (1 - k'_1)k'_2, \quad d_3 = (1 - k'_1)(1 - k'_2)k'_3.$$

Gain of killing the opponent  
in the game

$$k_i = \frac{A_i}{A_i + D'_i}$$

$$k'_i = \frac{A'_i}{A'_i + D_i}$$

$$U(\mathbf{K}) = \alpha \sum_{x=1}^T ((x-1)d_x) + G \sum_{x=1}^T (k_x (\prod_{j=1}^{x-1} (1 - k'_j))),$$

Gain of staying in the game

Concave utility  
function:

$$\begin{aligned} & \max U(\mathbf{K}) \\ & \text{subject to } \sum_{1 \leq i \leq T} (A_i + D_i) \leq B \\ & D_i, D'_i \geq C \end{aligned}$$

B: total budget

C: base cost of  
remaining in the game

# *Player's Information*

In any repeated game, how the game proceed will be determined by the result of each round. Information of player about the game can be categorized as:

Complete

Incomplete

Imperfect

$$U(S, S'|\theta) = \sum_{\theta'} P(\theta'|\theta) \cdot U(S_{\theta}, S'_{\theta'})$$

$$S_{\theta} = \arg \max_S \sum_{\theta'} P(\theta'|\theta) \cdot U(S_{\theta}, S'_{\theta'}).$$

# Incomplete and Imperfect Information

- It is possible for the player not to be aware of everything his opponent does.
- Despite knowing that his opponent is playing according to one of the possible types, he cannot see which action exactly he is taking.
- player only sees the opponent's action, but does not know what its objective is. To meet this challenge, this paper develops a game where players decide how to allocate resources when they have partial information

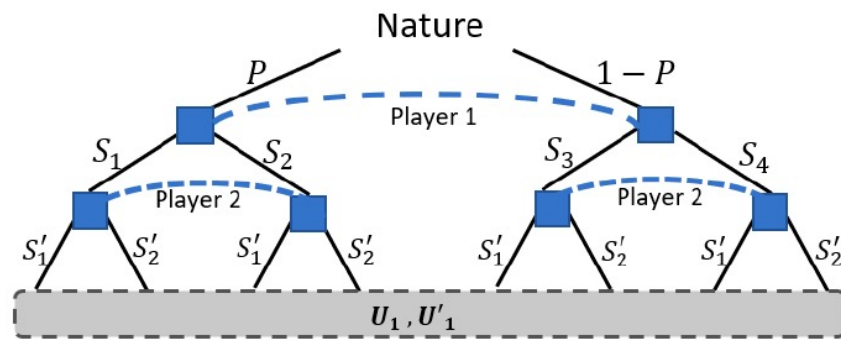


Fig. 5. Partial information for Player 1's preference.

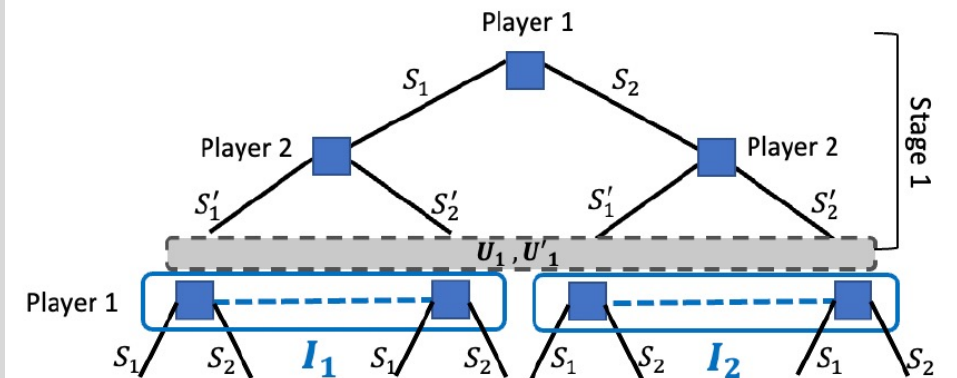


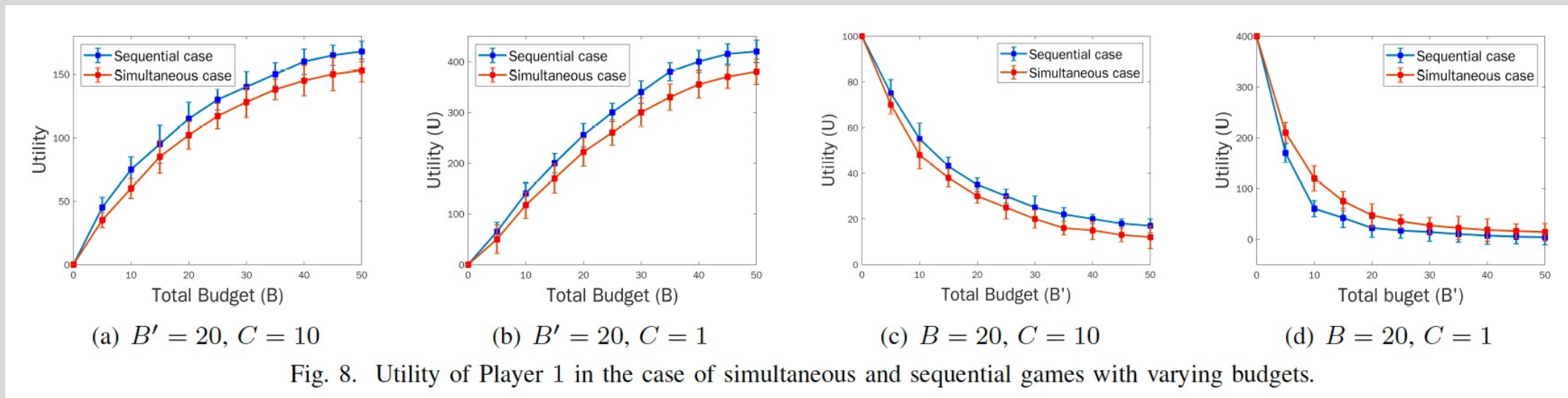
Fig. 6. Game with imperfect information for one of the players.<sup>11</sup>



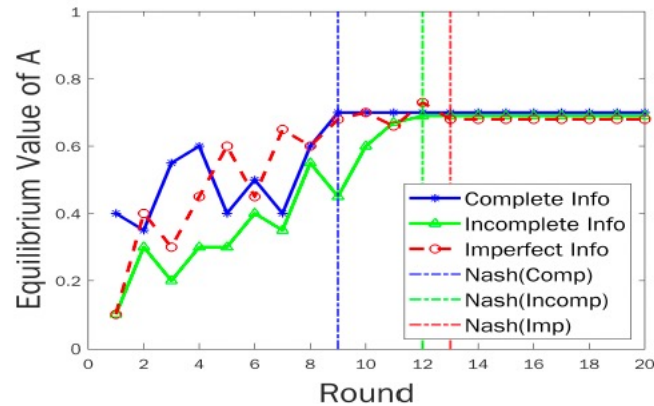
# *Evaluation*

- How utility changes based on the
  - Total budget
  - Belief
  - Partial information
  - Different values of cost and gain

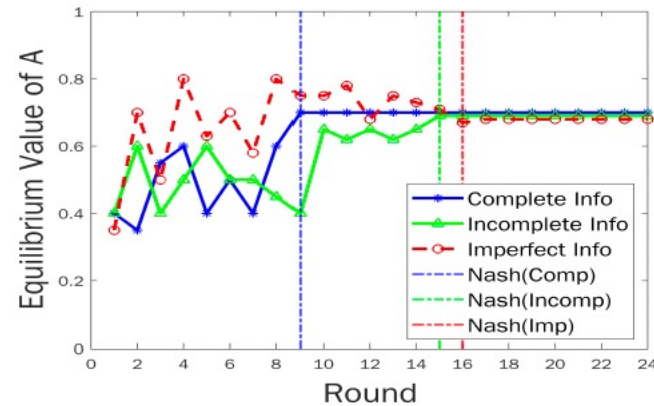
# Impact of Different Amounts of Budget and Different Costs



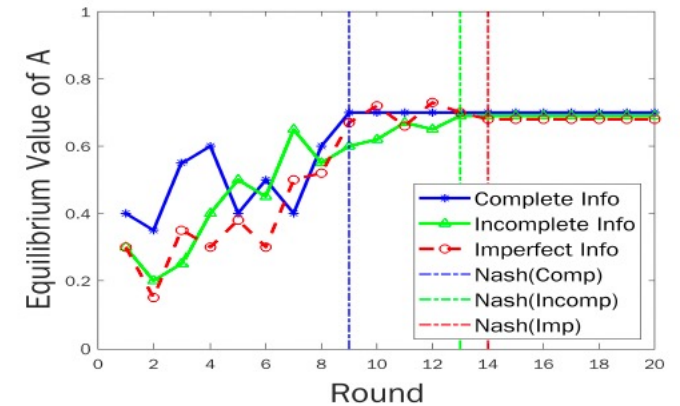
# Convergence in Different Information and Initial Beliefs



(a) Initial Belief:(10%:90%)



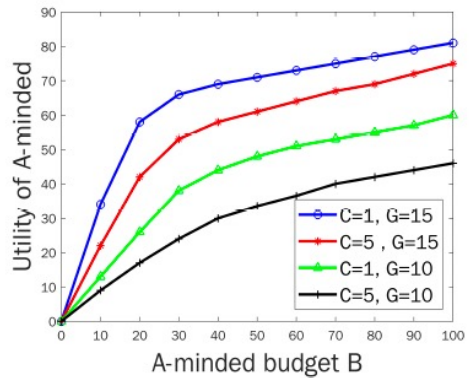
(b) Initial Belief:(50%:50%)



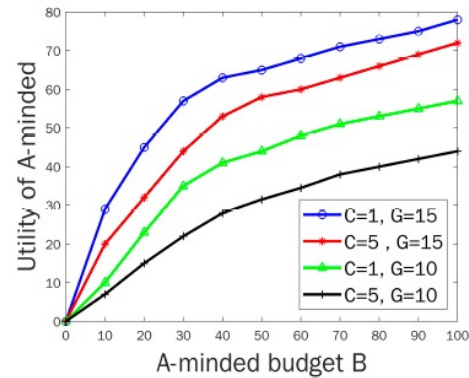
(c) Initial Belief:(30%:70%)

Fig. 9. Speed of convergence in the case of different belief. Dashed lines show the time of converging to the Nash equilibrium.

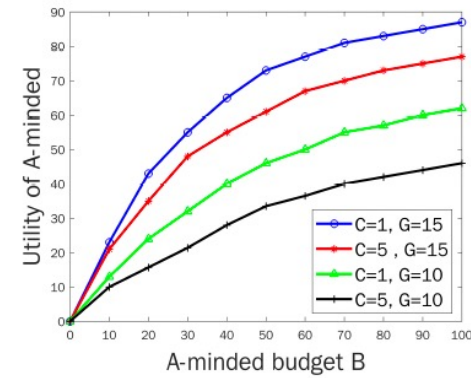
# Impact of Different Amounts of Gain and Cost



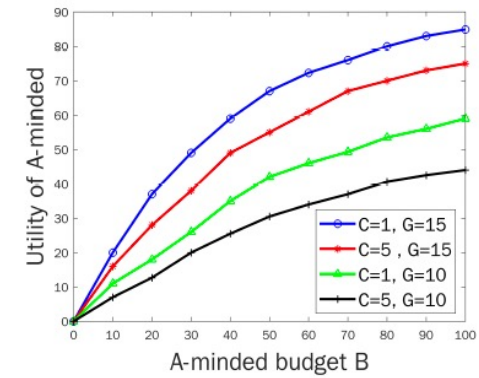
(a)  $B' = 20, \alpha = 1$



(b)  $B' = 40, \alpha = 1$



(c)  $B' = 20, \alpha = 5$



(d)  $B' = 40, \alpha = 5$

Fig. 10. The effect of different values of cost and gain on utility.



**Thanks for  
your  
attention!**