Wu J. Collaborative mobile charging and coverage. JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY 29(4): 550–561 July 2014. DOI 10.1007/s11390-014-1449-2

Collaborative Mobile Charging and Coverage

Jie Wu (吴 杰), *Fellow, IEEE*

Department of Computer and Information Sciences, Temple University, Philadelphia, PA 19122, U.S.A.

E-mail: jiewu@temple.edu

Received March 3, 2014; revised April 30, 2014.

Abstract Wireless energy charging using mobile vehicles has been a viable research topic recently in the area of wireless networks and mobile computing. This paper gives a short survey of recent research conducted in our research group in the area of collaborative mobile charging. In collaborative mobile charging, multiple mobile chargers work together to accomplish a given set of objectives. These objectives include charging sensors at different frequencies with a minimum number of mobile chargers and reaching the farthest sensor for a given set of mobile chargers, subject to various constraints, including speed and energy limits of mobile chargers. Through the process of problem formulation, solution construction, and future work extension for problems related to collaborative mobile charging and coverage, we present three principles for good practice in conducting research. These principles can potentially be used for assisting graduate students in selecting a research problem for a term project, which can eventually be expanded to a thesis/dissertation topic.

Keywords Internet of Things, mobile charger, optimal solution, sensor, wireless networks mobile computing

1 Introduction

Recent breakthroughs in rechargeable batteries, which support the wireless energy transfer, provide several applications of mobile vehicles in the field of wireless networks and mobile computing. These mobile vehicles act as either mobile sinks, mobile chargers, or combinations of both, to collect data from the wireless devices (simply called sensors) and/or wirelessly transfer energy to the sensors. Therefore, the use of mobile vehicles can assist in the sustainability and applicability of sensors that are widely used in the general field of Internet of Things (IoTs)[1].

In this paper, we focus on mobile vehicles taking the role of chargers to provide energy to sensors. Hence, we simply call these vehicles mobile chargers (MCs). There have been many research results reported in the related field of mobile coverage in the wireless sensor network (WSN) community (under the term *mobile sink*[2]) and in the delay tolerant network (DTN) community (under the term *ferry*[3]). Recently, some work has been done on various optimization problems in mobile charging, with a focus on the scheduling of individual charges. Limited work has been done in the subarea of collaborative mobile charging in applications where individual mobile chargers cannot solve individually or efficiently. Collaborative mobile charging and coverage focus on the planning and scheduling of MCs collectively to solve given sensor charging and coverage problems. Sensors, located at different geographical locations, may require different recharge frequencies. MCs are subject to moving speed limits or even energy capacity limit that restricts MCs moving distance and recharge capacity. Here, mobile charging is not limited to MCs charging sensors, but base station (BS) charging MCs and MCs charging MCs themselves.

We consider the following two hypothetical military-related problems to motivate our study:

• *Problem* 1. There is a large number of small villages that require protection via military patrol cars at different patrolling frequencies (e.g., every 1-hour, 5-hour, 24-hour). How should we deploy a minimum number of patrol cars for such a coverage mission?

• *Problem* 2. For a country with no navy carrier, is it possible for jet fighters to reach an enemy target far away from a given land base which is out of reach from any single jet fighter? If so, how should we schedule a minimum number of jet fighters to carry out such a mission (i.e., one jet fighter can eventually reach the target)? In addition, all jet fighters have to come back safely to their land base.

In the above two questions, both patrol cars and jet fighters can be considered as MCs in collaborative mobile charging and coverage. In problem 1, a trivial solu-

---

Survey

The work was supported in part by the National Science Foundation of USA under Grant Nos. CCF 1301774, ECCS 1231461, CNS 1156574, CNS 1065444, and ECCS 1128209.

tion is to assign one petrol car to each village. To reduce the number of cars, a car may have to cover multiple villages (separated by given distances). The challenge is to schedule cars to meet petrol frequencies under the maximum speed limit. In problem 2, a single jet fighter cannot reach its target. Jet fighters have to mutually charge fuel in the air or get charged directly from the land base. One more challenge is that all jet fighters should have sufficient fuel to come back to the same land base. We will discuss collaborative mobile charging and coverage problems, and their solutions that lead to the answer to the above two problems.

Through the process of problem formulation, solution construction, and future work extension for problems related to collaborative mobile charging and coverage, we present the following three principles for good practice in research:

- Principle 1: select a *simple* problem;
- Principle 2: find an *elegant* solution;
- Principle 3: use *imagination* for extensions.

These principles can potentially be used for assisting graduate students in selecting a research problem for a term project, which can eventually be expanded to a thesis/dissertation topic.

These principles will be elaborated on in Section 2. Section 3 focuses on a problem that eventually leads to the solution of problem 1. Here, we focus on the challenge of the speed limitation of MCs, while trying to meet the frequency requirement in coverage. We assume that each MC itself does not need to be recharged. Section 4 considers a problem where MCs need to be recharged at the BS or among themselves. The solution to this problem also solves problem 2. A quick review of other recent work in the field is given in Section 5, before a conclusion in Section 6.

2 Principles for Conducting Research

The following are three principles for conducting research that may lead to good results, especially for beginners. These principles were gathered by the way of experiences throughout the author's academic life, including many years of graduate student guidance and two years working on a project related to collaborative mobile charging and coverage. These experiences, however, are highly personal and may not be generalized to all cases.

The first principle is to select a *simple* problem by asking a *right* question through proper abstraction. A simple problem is not a trivial problem with an easy solution. A simple problem usually is easy to describe to a layman, but its solution requires some careful thought. The selected problem usually has some utility values in terms of intellectual merit and broader impacts. There are two extreme views on utility values of research. One view believes all research should have values in practice. One such school is utilitarianism[4]. The opposite view believes the best research is not measured by its utility. One school of pure mathematics[5] firmly takes this view. In the discipline of computer science and engineering, finding the right problem is extremely important, as there are usually many of possible problem formulations. Many problem formulations are either too trival or too complex. In searching for a research problem, some researchers tend to be too quick in formulating an intractable problem in a complex setting that tries to achieve multiple objectives. A complex problem is usually important in the long term, but not before its simple versions have been mastered.

The second principle is to find an *elegant* solution. An elegant solution is not a straightforward solution. It is usually short, but requires some deep insight. Some researchers tend to overly use mathematics in both presentation and analysis, partially to make them feel better and to make the papers look better. The general rule should be to avoid mathematics unless a concept or solution cannot be explained in a precise and concise way. A classic example of an elegant solution appears in Dijkstra's self-stabilization paper[6]. Edsgar W. Dijkstra, an accomplished computer scientist with a deep skill in mathematics, presented a quite profound theory in this 2-page paper, hardly using any mathematical symbols. In seeking a solution, some researchers use existing solutions or their trivial extensions without offering new techniques or insights to solve a given problem. We should obviously avoid this trend, as it does not add much to the body of knowledge in the corresponding field.

Note that the first two principles are related. In many cases, when a right question is asked, the corresponding solution is partially made. Sometimes, it may take several rounds of problem formation followed by a quick trial of solutions to meet both principles. Therefore, practicing these principles is more art than science, and requires proper disciplines of practitioners.

Once the core problem has been defined and solved, we can then apply the third principle of using *imagination* for extensions. Why is imagination needed? This is because, usually, there are many possible ways of extending a core problem. Some are straightforward while others are complex. We should strive to push to the limit in terms of technical depth. Sometimes, imagination requires intuition and understanding the core problem and broader knowledge of other fields. An extension can be detailed or sketched. In the latter case, the first two principles apply again. In some cases, sketched solutions and ideas are recommended so that researchers can have more time to think deeper or switch to other extensions. In the art world, what makes Picasso and Matisse famous is not their attention to detail in painting, but their expressiveness of ideas through a few unique shapes and colors.

In the following two sections, we discuss two problems related to collaborative mobile charging and coverage. We start with a subsection of applying the first principle to problem formation. After seeking various solutions in the second subsection, we present an optimal solution that illustrates the essence of the second principle. Each section ends with a subsection of extensions. The third principle is used by providing several extensions with either detailed or sketched solutions.

3 Collaborative Mobile Charging and Coverage Without a Capacity Limit

3.1 Problem Formation

Consider a system of IoTs, where each sensor needs to be recharged at a different frequency. A mobile charger (MC) can charge a sensor after it moves to the location of the sensor. We assume that the MC has an unlimited charging capability, moves at a speed subject to a given limit, and that the charging time is negligible. An optimization problem can be presented on a time-space coverage of sensors so that none of them will run out of battery energy. We consider the following two problems, simply stated as follows: 1) What is the minimum number of MCs needed? 2) Given the minimum number of MCs, how should MCs be scheduled in terms of trajectory planning?

3.2 Seeking Solutions

Consider a circle track (or 1-D ring) with circumference 3.75 that is densely covered with sensors having frequency 1, as shown in Fig.1. In addition, there are 1) a sensor with frequency 2 at position 0, 2) a sensor with frequency 4 at position 0.25, and 3) a sensor with frequency 2 at position 0.5. To simplify our discussion, we assume the maximum speed to be one unit distance per unit time for each MC (also shown as a car in the following figures).

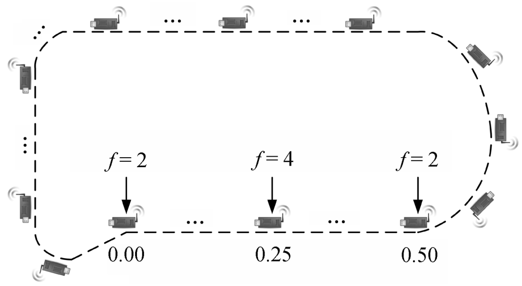

Fig.1. Example of a 1-D ring of sensors of different recharge frequencies.

Fig.2 shows several heuristic solutions, mainly to illustrate the complexity of problem solving space. Solution (a) uses seven MCs. Three fixed MCs are assigned, with one to each of the distinct sensors that require a recharge frequency of over 1. Then, four equally-spaced MCs are assigned to the circle track to cover the rest of the sensors. These four MCs are moved at a full speed in the same direction (clockwise or counter-clockwise).

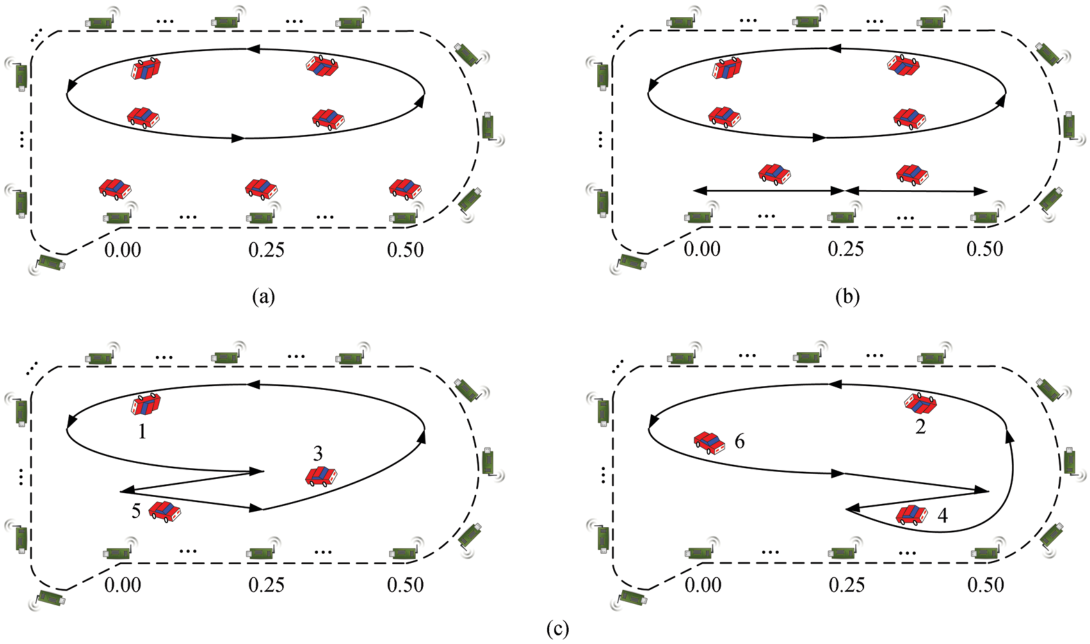

Fig.2. Three possible solutions. (a) Solution with 7 MCs. (b) Solution with 6 MCs. (c) Solution with 6 MCs: the left subfigure is for odd nodes and the right subfigure is for even nodes.

Solution (b) uses six MCs. The difference is that only two MCs are used to cover three distinct MCs. One moves between the sensors at 0.00 and 0.25 and the other one moves between the sensors at 0.25 and 0.50. Solution (c) also uses six MCs, and all sensors are covered by all MCs collectively (i.e., MCs move in a circular fashion to collaboratively cover all sensors). However, MCs do not completely share the same circle. The MCs can be scheduled in such a way that MCs are placed with equal distance (say, a sensor at 1.00 in the shared part of trajectory is visited by an MC at a fixed frequency of 6/4.25). In region [0, 0.5], all even nodes follow one trajectory, and all odd nodes follow another one, as shown in Fig.2 (c). Both trajectories have the same circumference of $3.75 + 0.25 + 0.25 = 4.25$.

It turns out that five MCs are sufficient to ensure the coverage of all sensors of required frequencies. However, the optimal scheduling is more intriguing, as shown in Fig.3, as we have to select proper speeds for MCs. Let us define 0.25 as a mini-unit of a time step (or simply a mini-unit). One MC enters location 0 at unit 0, and one more MC enters the same location for every one additional time unit. Once having entered the region of [0, 0.5], each MC in mini-units performs mini-steps as follows: (enters at mini-unit 0) at position 0, 1) 0.25, 2) 0, 3) 0.25, 4) 0.25, 5) 0.5, 6) 0.25, and (exits at 7) 0.5. The trajectory between 0 and 0.25 and between 0.25 and 0.5 is repeated by one mini-cycle each. In addition, there is a 0.25-time-unit stop at 0.25, which is equivalent to 0.25 distance at a full speed. Hence, the circumference of a virtual ring in Fig.3 is 3.75 + 0.5 + 0.5 + 0.25 = 5. Therefore. we only need five MCs, which are equally spaced on the virtual ring.

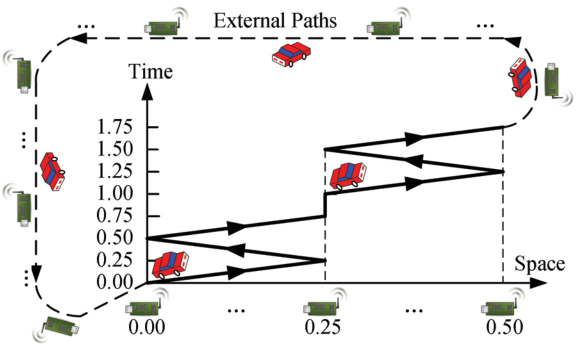

Fig.3. Solution of the example in Fig.1.

The above process shows the complexity of the problem in terms of scheduling MCs. This scheduling spans across three dimensions: time, space, and speed. In the sample example, the optimal schedule does not require all MCs to move at a full speed (in fact, zero speed at a particular point in time and space). The difficulty mainly lies in dealing with the requirements of different frequencies. Note that determining the minimum number of MCs is another challenge. To have a better handle on the core problem, we start with cases of uniform frequency.

3.3 Optimal Solution

To put the problem more formally, we consider a k-dimensional (k-D) space with two types of nodes: $S = \{s_i\}$ where s_i, called sensors, have fixed locations in k-D space; $MC = \{MC_j\}$, where MC_j, called mobile chargers, are mobile with a given moving speed limit. Each s_i is required to be visited by MCs at frequency f_i. That is, the time duration between two adjacent visits to s_i (it can be visited by different MCs) is no more than $\frac{1}{f_i}$.

Our study begins with homogeneous WSNs on a 1-D ring with a circumference of L, where $f_i = 1$ and the moving speed is limited by 1 without loss of generality. In [7], an optimal solution, called Global-or-Local, is given by the following.

Global-or-Local[7]

- Global method: there are $m_1 = \lceil L \rceil$ equally-spaced MCs moving continuously around the circle.
- Local method: there are m_2 MCs moving inside fixed intervals of length, $\frac{1}{2}$, so that all sensors are covered.

In Global-or-Local, the optimal result is either the global or local method, whichever generates a smaller number of MCs, i.e., $\min\{m_1, m_2\}$. The global method corresponds to a global collaborative coverage while the local method uses a local coverage. In both methods, all MCs are moving at a full speed of 1. The optimality proof is non-trivial and uses a special way of proof by contradiction, and can be found in [7]. Fig.4 shows these two methods.

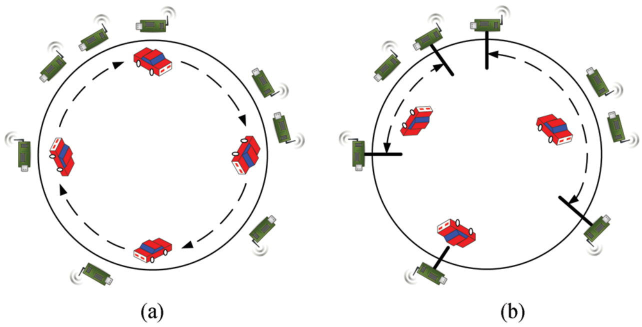

Fig.4. Optimal solution for the 1-D ring of sensors with uniform recharge frequencies. (a) Overlapped global trajectory. (b) Non-overlapped local intervals.

The local method requires more discussion as it starts with a *cut*, that converts a 1-D ring to a 1-D line. The process can start with the left end to partition the line into intervals. Such an interval starts and ends with sensors. Each interval should be the longest for an MC to serve back and forth, while still meeting the frequency requirement. For a system with n sensors, there are n possible starting points (or rounds). Each sensor can be used as an end of an interval once. If the sensor is used as the end of interval for a second time in search of another cut, then the result of the remaining partition used in the previous round can be applied directly. Therefore, such a partition process is linear with respect to n.

3.4 Extensions

Now that we have the optimal result for the core problem, let us consider several extensions. One obvious extension is to change the topology from a 1-D ring to a 1-D line. In this case, only one round partition is needed from the left end to the right end. We can also extend the approach to include energy charge time by converting such a time period as an extra distance.

Our first extension considers the case where each sensor s_i has a distinct frequency f_i, as in the first military-related problem in the introduction. We can use an approximation for solving this problem. In the example of Fig.3, only two kinds of speed are used: zero or full. We can easily show that these two kinds of speed can emulate any optimal solution using a different speed. Suppose s_i and s_j are two arbitrarily-selected adjacent sensors in the trajectory of an optimal solution. s_i and s_j are visited at time t_i and t_j, respectively. At time t_i, an MC will move from s_i to s_j at a full speed. If the MC arrives at s_j at time t_j, it charges and moves ahead to the next sensor. If the MC arrives at s_j before t_j, it stays there (zero speed) until time t_j before taking the next move.

Back to the solution of sensors with different frequencies, it turns out that with an arbitrary cut, the same partition algorithm will give a factor of 2 of the optimal solution for sensors with different recharge frequencies on a 1-D ring and a 1-D line. Again, we assume that one MC is assigned to an interval, moving from the leftmost to the rightmost at a full speed to cover the interval. When we say that all sensors in an interval are covered, it means that their visit frequencies are met. Here, we do not try to emulate any optimal solution, but directly give a solution that can meet recharge frequencies for all sensors using only one speed: full speed.

To prove the approximation of 2, we have the following detailed analysis. First, we assume that the optimal solution uses m MCs in total. Without loss of generality, we assume that MCs do not meet or pass each other; otherwise, switching the velocity (both speed and direction) of the crossed MCs will lead to the same or a better solution. The trajectory of each MC covers an interval. These intervals cannot be nested within each other, as MCs do not meet. Note that each car in its interval may go back and forth at a different speed and may visit sensors at different frequencies. If we use the left end and right end of each interval as cuts to partition the 1-D line into another set of intervals, there are at most $2m-1$ new intervals, as shown in Fig.5.

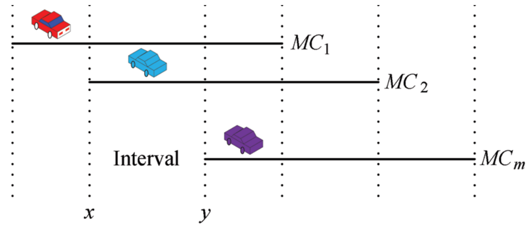

Fig.5. Partition of a line into $2m-1$ segments of different colors.

Next, we show that each new interval can be served by a single MC moving back and forth between the leftmost point and the rightmost point of the interval at a full speed to meet the frequency requirement for all sensors in that interval. Suppose $[x, y]$ is a new interval, i is the location of an arbitrary sensor s_i in the interval; the leftmost MC within the interval will pass i to reach y (at least once based on the interval definition) before any other MC can serve s_i (as MCs never meet). Therefore, $2(y-i)f_i \leqslant 1$, where f_i is the visit frequency for sensor s_i. A similar argument also applies to $2(i-x)f_i \leqslant 1$ for the rightmost MC. Therefore, an MC at a full speed can serve all sensors in the interval. For a 1-D ring, one extra MC is needed for the conversion of a 1-D ring to a 1-D line. More details of the proof are shown in [7].

To view the average performance of this approximation, we conducted a simple proof-of-concept simulation on a 1-D line with 10 sensors by varying frequency and distance. The selection of a small-size problem in simulation is primarily due to the implementation complexity of the optimal solution. In our simulation, the frequencies of sensors (f) follow normal distribution, with μ and σ for mean and variance, respectively. Meanwhile, the distances between adjacent sensors (d) also follow normal distribution. The speeds of MCs are either zero or one unit (i.e., the maximum speed). The average value of the frequencies and distances are represented by μ_f and μ_d, respectively, while σ_f and σ_d indicate their fluctuation. In Table 1, we fix three parameters at a time among μ_f, μ_d, σ_f, σ_d to be 0.5, and tune the remaining one parameter to observe its influence. Each simulation is repeated until the confidence interval of the average result is sufficiently small ($\pm 1\%$ percent for 90% probability). Comparing with greedy

Table 1. Simulation Results

Tunable Input		Output		Tunable Input		Output	
Parameter	Value	Greedy	Optimal	Parameter	Value	Greedy	Optimal
μ_f	0.1	5.1	3.2	μ_d	0.1	5.0	3.5
(frequency	0.2	5.3	3.3	(frequency	0.2	5.2	3.6
mean)	0.3	5.7	3.6	mean)	0.3	5.5	3.8
	0.4	6.0	4.1		0.4	5.9	4.1
	0.5	6.3	4.6		0.5	6.3	4.6
	0.6	6.6	5.1		0.6	6.5	4.1
	0.7	6.8	5.5		0.7	6.8	5.4
	0.8	7.0	5.8		0.8	7.2	6.1
	0.9	7.1	6.0		0.9	7.7	6.9
	1.0	7.2	6.1		1.0	8.6	8.1
σ_f	0.1	5.2	4.2	σ_d	0.1	5.8	3.5
(frequency	0.2	5.5	4.3	(frequency	0.2	5.9	3.7
mean)	0.3	5.8	4.4	mean)	0.3	6.0	3.9
	0.4	6.1	4.5		0.4	6.1	4.2
	0.5	6.3	4.6		0.5	6.3	4.6
	0.6	6.4	4.8		0.6	6.5	4.9
	0.7	6.5	5.0		0.7	6.7	5.4
	0.8	6.7	5.3		0.8	6.7	5.4
	0.9	7.1	5.7		0.9	7.4	6.6
	1.0	7.5	6.4		1.0	7.9	7.4

and optimal, the average ratio is between 1.1 to 1.7, which is much lower than the worst case bound of 2. In Table 1, when distance mean is 0.9, the average number of cars for our greedy approach (listed as Greedy) and the optimal solution (listed as Optimal) are 7.7 cars and 6.9 cars, respectively. This corresponds to a ratio of $7.7/6.9 = 1.116$.

For an extension to a k-D space, we sketch a possible solution using the Hilbert curve[①] to perform line fitting between k-D and 1-D space to maximally preserve locality. Fig.6 shows using the Hilbert curve for a 2-D space to fit in a 1-D space. In general, a level-k Hilbert curve is constructed from four level-$(k-1)$ curves, as shown in Figs. 6(b) and 6(c). This figure also illustrates the locality property (i.e., the closeness of two sensors is tightly related in terms of distance in both spaces).

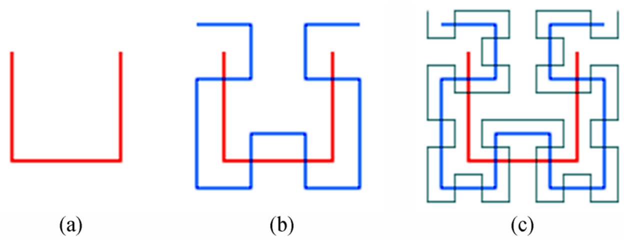

Fig.6. Hilbert curves in a 2-D space. (a) Level-1. (b) Level-2. (c) Level-3.

Note that this model also addresses the first military-related problem discussed in the introduction. While we have a simple approximation solution for all general frequencies, it is still an open problem on the existence of an efficient optimal solution for sensors with different frequencies in a 1-D space, let alone in the general k-D space.

The model in this section assumes unlimited charging capacities for MCs. In the next section, we will consider another model with limited charging capacities for MCs.

4 Collaborative Mobile Charging and Coverage with a Capacity Limit

4.1 Problem Formation

Consider again a system of IoTs, where each sensor needs to be recharged as in the previous section. However, an MC has a limited charging capability. We assume that there is a base station (BS) with unlimited charging capacity. If there is only one BS, we can iteratively apply three types of charges: BS-to-MC, MC-to-MC, and MC-to-S (S for sensors). Given a BS and sensors in a 1-D line, what is the maximum distance MCs can reach for charging sensors and coming back for their recharge at BS? Note that all MCs should stay "alive" (i.e., their energy cannot be depleted, otherwise, an MC as a jet fighter will fall and crash).

In order to focus on important issues such as the recharge and movement schedule, we use the first prin-

[①] http://en.wikipedia.org/wiki/Hilbert_curve, May 2014.

ciple on simplicity to abstract the following core model: MC's capacity is B. There are two types of energy consumption for an MC: moving cost c per unit distance (in meters) and sensor recharge cost b per sensor. We do not specify the sensor recharge frequency at this stage. There is no overhead in charging. All sensors are laid out in a 1-D line with unit distance apart, including BS to the first sensor, as shown in Fig.7.

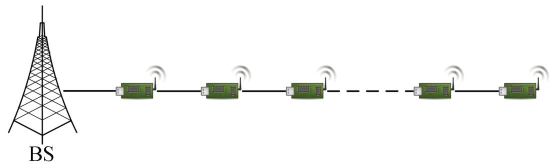

Fig.7. l-D line with a BS.

4.2 Seeking Solutions

Suppose m MCs are used in a 1-D line. A simple approach is that all MCs follow the same trajectory and move as far as possible before coming back for recharging at the BS. Each sensor is jointly charged by m MCs, with their moving trajectory shown in Fig.8.

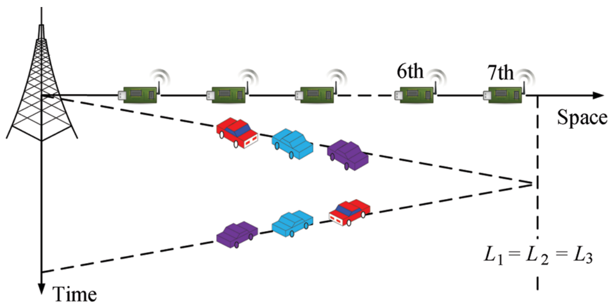

Fig.8. Scheme 1: equal-charge.

Scheme 1 (Equal-Charge). *Each sensor is jointly charged by m MCs, i.e., each MC charges a sensor b/m J (J for Joule).*

Let us consider an example with $B = 100$ J, $b = 4$ J, $c = 6$ J/m, and $m = 3$ MCs. Scheme 1 can cover seven sensors. In general, if we ignore the density of sensors and remove all costs associated with sensor recharge, the maximum distance an MC can go back and forth is bounded $\frac{B}{2c}$. To derive a closed form expression, we assume that sensors are uniformly and densely deployed. Sensor charge is still b per unit distance. Let L_i be the farthest distance that the i-th MC moves away from the BS. Without loss of generality, we label them in descending order of their respective maximum traveling distances away from the BS. That is, MC_1 reaches the farthest point. For Scheme 1, all MCs follow the same trajectory with both moving and sensor charge cost. We have the following result for Scheme 1.

$$L_1 = \cdots = L_m = \frac{B}{2c + \dfrac{b}{m}},$$

and $L_1 = \frac{B}{2c}$ when m reaches infinity.

Next we consider another scheme where a 1-D line is partitioned into m intervals, one for each MC for sensor charging. The farthest interval from the BS is shorter, as it requires more moving cost. In Scheme 2, as shown in Fig.9, each MC has a different trajectory. Intervals are defined in such a way that all MCs use up their battery energy upon returning to the BS.

Scheme 2 (One-to-One Charge). *The 1-D line is partitioned into disjoint intervals. Each interval is assigned to one distinct MC for sensor charging.*

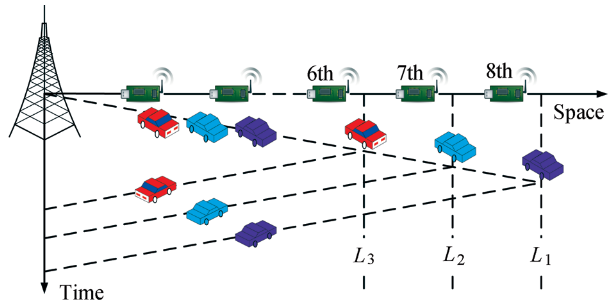

Fig.9. Scheme 2: one-to-one charge.

Let us consider the same example of Fig.7. Scheme 2 can cover eight sensors. In general, as the MC that covers the farthest sensor still needs a round trip, the maximum distance is still bounded for $\frac{B}{2c}$. Compared with Scheme 1, Scheme 2 can reach farther. For each MC, charging costs at all intervals except the last one are avoided. In the continuous model, for MC_m, its total energy (B) translates into two parts: charging sensors from BS to L_m, and moving from BS to L_m and from L_m to BS, so we have

$$B = bL_m + 2cL_m.$$

For MC_i, its total energy translates into two parts: charging sensors from L_{i+1} to L_i, and moving from BS to L_i and from L_i to BS. So we have

$$B = b(L_i - L_{i+1}) + 2cL_i.$$

Therefore, for Scheme 2, we have the following:

$$L_m = \frac{B}{2c + b},$$
$$L_i = \frac{B + bL_{i+1}}{2c + b},$$
$$L_1 = \frac{B + bL_2}{2c + b},$$

and
$$L_1 = \frac{B}{2c}\Big[1 - \Big(\frac{b}{b+2c}\Big)^m\Big],$$
as shown in Appendix A.1. Clearly, just as Scheme 1, $L_1 = \frac{B}{2c}$ in Scheme 2 as m approaches infinity. However, in Appendix A.2, we will prove that Scheme 2 always beats Scheme 1.

Our third approach uses collaborative recharge among MCs, i.e., it uses MC-to-MC charge. If we label the location of the furthest sensor L_1, intersections between intervals starting from the BS ($= L_{m+1}$) as $L_m, L_{m-1}, \dots, L_1$. Scheme 3 as shown in Fig.10, is the same as Scheme 2, where each MC_i turns around at L_i. The difference is that MC_i transfers battery energy to $MC_{i-1}, MC_{i-2}, \dots, MC_1$ to their full capacity at L_i. Each MC_i has just enough battery energy to return to the BS. As in Scheme 2, MC_i charges all sensors in the interval between L_{i+1} and L_i. We call each L_i a *rendezvous point*, represented as a small box in the figure, which is a place for MC-to-MC charge.

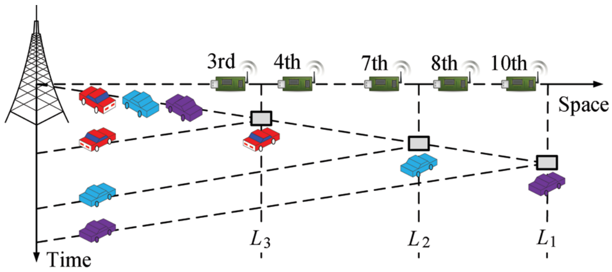

Fig.10. Scheme 3: single MC-to-MC charge.

Scheme 3 (Single MC-to-MC Charge). *Same as Scheme 2, except each MC transfers battery energy to all MCs with smaller indices to full.*

Clearly, Scheme 3 can reach ten sensors, which is further than Scheme 1 and Scheme 2. However, the maximum distance is still limited to $\frac{B}{c}$ as the last MC still needs a return trip without any further charge. In the continuous model, we have the following: For MC_m, its energy translates into three parts: 1) charging sensors from BS to L_m, 2) moving from BS to L_m and from L_m to BS, and 3) charging the other $m-1$ mobile chargers to their full batteries at L_m. So we have

$$B = bL_m + 2cL_m + (m-1)cL_m.$$

For the general MC_i, its energy translates into three parts: 1) charging sensors from L_{i+1} to L_i, 2) moving from L_{i+1} to L_i and from L_i back to BS, and 3) charging the other $i-1$ chargers to their full batteries at L_i. So we have

$$B = b(L_i - L_{i+1}) + 2c(L_i - L_{i+1}) + cL_{i+1} + (i-1)c(L_i - L_{i+1}).$$

Therefore, for Scheme 3, we have the following:

$$L_m = \frac{B}{(m+1)c+b},$$
$$L_i = \frac{(m-i+1)B}{(m+1)c+b},$$
$$L_1 = \frac{mB}{(m+1)c+b},$$

and $L_1 = \frac{B}{c}$ as m approaches infinity.

4.3 Optimal Solution

We now consider a charging scheme for MCs to cover unlimited distance. The scheme called Push-and-Wait[8] uses two small, but elegant, ideas: 1) "Push": it limits as few MCs as possible to go forward in order to save energy on moving cost; 2) "Wait": it efficiently uses the battery capacity of each sensor through two rounds of charges. Again, the 1-D line is partitioned into m intervals, as shown in Fig.11.

Push-and-Wait (Double MC-to-MC Charge)[8]

1) MC_i charges sensors between L_{i+1} and L_i.
2) MC_i transfers battery energy to $MC_{i-1}, MC_{i-2}, \dots, MC_1$ to their full capacity at L_i.
3) MC_i waits at L_i, while all MCs with smaller indices keep moving forward.
4) After $MC_{i-1}, MC_{i-2}, \dots, MC_1$ return to L_i, MC_i evenly balances battery energy among them (including itself).
5) Each $MC_i, MC_{i-1}, \dots, MC_1$ has just enough battery energy to return to L_{i+1}.

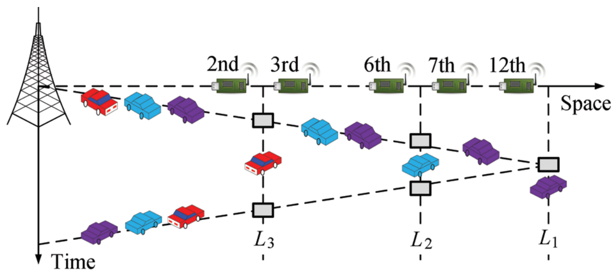

Fig.11. Push-and-Wait: double MC-to-MC charge.

Again, Push-and-Wait can reach 12 sensors with three MCs for the example of Fig.7. Now let us show that Push-and-Wait can reach any distance as long as there are sufficient MCs. Note that each MC_i uses b for sensor coverage per unit and consumes two rounds of its moving and charging to all MCs with smaller indices: $2ic$. Therefore, the length of the interval from L_{i+1} to L_i is $\frac{B}{2ic+b}$. Summation of these segments gets the following:

$$L_m = \frac{B}{2mc + b},$$
$$L_i = \sum_{j=i}^{m} \frac{B}{2jc + b},$$
$$L_1 = \sum_{j=1}^{m} \frac{B}{2jc + b},$$

and L_1 reaches infinity as m approaches infinity. This is because this sequence corresponds to a harmonic sequence with a simple modification as shown in Appendix A.3. Push-and-Wait is clearly the best, and can reach any distance given a sufficient number of MCs. Push-and-Wait is also optimal in terms of maximizing the ratio of energy used in charging and energy used in moving[8].

4.4 Extensions

We start with several extensions while still maintaining optimality. Push-and-Wait applies to a 1-D line with non-uniform distance between adjacent sensors. If there is a smaller uniform recharge cycle for all sensors, a simple pipeline extension of Push-and-Wait can be found. The problem becomes complex when the problem is extended to a k-D space or non-uniform recharging frequency for sensors. Several heuristic solutions have been discussed in [8], together with some simulation results.

We discuss another possible solution called Local-Charge-Only. The main idea is still based on the 1-D line partition into intervals. The difference is that each MC stays inside its own interval. Instead of going back to the BS for recharging, each MC gets its battery energy exclusively from the MC covering the adjacent interval closer to the BS at their rendezvous point. It has been shown in [8] in detail that both approaches are "equivalent" in terms of coverage distance, but not in terms of speed. Push-and-Wait is faster, but requires that the BS have sufficient "bandwidth" for charging multiple MCs simultaneously.

Push-and-Wait gives an answer to the second military-related problem in the introduction, with an assumption that the jet fighter does not consume energy during the waiting period. If such an energy consumption rate r is relatively small, jet fighters with speed s can reach far to an enemy target at distance L, with extra energy linearly proportional to $r(L/s)$.

A more general problem is to consider multiple enemy targets. In this case, a special minimum cost multicast tree in terms of the number of MCs needs to be constructed from the land base to multiple enemy targets. This is a general type of Steiner tree②. The basic idea can be sketched as follows. Under this model, the intermediate nodes of the tree, where a route is split into multiple routes, can be at any location within a given 2-D or 3-D Euclidean space. The cost associated with each branch, a summation of edge costs, varies depending on its distance to the land base.

5 Related Work

In this section, we review some related work in classic coverage problems in graph theory and data collection and communication in wireless network and mobile computing. Most of them focus on individual mobile charging and coverage.

Mobile charging can be modeled as the *travelling salesman problem* (TSP)③, where an MC constructs a tour of all sensors once and only once. In some cases, when an MC recharges energy to a node, it can also charge nodes in its neighborhood. This problem can be modeled as a *coverage salesman problem* (CSP)[9] to identify the least-cost tour of a subset of given cities (i.e., sensors in this paper) such that every city not on the tour is within some predetermined covering distance of a city that is on the tour. Usually, a predetermined distance corresponds to a 1-hop neighborhood, as used in CSP[9]. When neighborhood distance does not matter, CSP is similar to a connected-dominating-set-based tour construction[10]. Note that an MC does not have to be at a sensor for charging, and this corresponds to an extension of CSP in Qi-ferry[11].

The notion of MCs evolves from mobile sinks in wireless sensor networks (WSNs), including data mules[2] and multiple mobile base stations[12], and from message ferries[3] and its extensions[13] in delay tolerant networks (DTNs) for data collection and routing. Another evolution comes from the recent wireless energy transfer technology (e.g., electromagnetic radiation[14], magnetic resonant coupling[15] using MCs, wireless power transfer using multiple transmitters/receivers[16]). MCs offer energy to sensors, and also consume energy due to their own movement. More recently, there are some exciting works on energy transfer through ambient RF signals, such as using existing TV and cellular transmissions as in [17].

Xie *et al.*[18] and Guo *et al.*[19] proposed several optimization models by considering an MC as both a data collector and an energy charger. Their focus is primarily on energy minimization using optimization and

② http://en.wikipedia.org/wiki/Steiner_tree_problem, May 2014.

③ http://en.wikipedia.org/wiki/Travelling_salesman_problem, May 2014.

approximation on different scenarios of data collection and energy recharge. These approaches focus less on scheduling of MCs, as only one MC or individual MCs are used in scheduling. Fig.12 shows one model where several subtrees are constructed among sensors. The roots of these subtrees collect data from respective subtrees. MCs move around to recharge all sensors (including roots) at or near the location of these charging targets and collect data from these roots. Various optimization problems can be formulated, including the maximization of vacation time for MCs to stay in the BS. Additional constraints can be added, including the timelessness of the data to be collected at the BS.

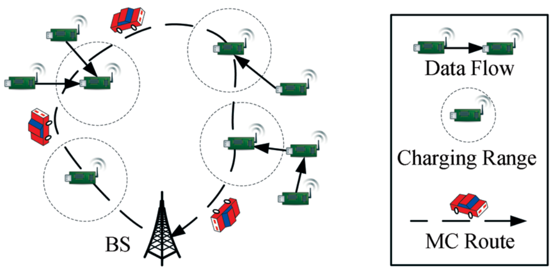

Fig.12. Data collection from roots using a mobile sink with a BS.

With the help of some imagination, the collaborative mobile charging and coverage resembles, to a limited extent, the *banana-eating camel problem*[4]: a farmer grows 3 000 bananas to sell at market 1 000 miles away. He can get there only by means of a camel. This camel can carry a maximum of 1 000 bananas at a time, but needs to eat a banana to refuel for every mile that he walks. What is the maximum number of bananas that the farmer can get to market? This problem is somewhat similar to our second model, where a banana can be considered as energy. There are several major differences. In the camel problem, the energy can be stored (bananas placed on the ground). There is only one camel involved. So there is no collaboration. Fig.13 gives a sketch of an optimal solution in time and space. Eventually, the camel reaches the market with 1 600/3 bananas for sale. Note that the above solution resembles a single MC emulation of the Local-Charge-Only solution discussed in [8].

6 Conclusions

This paper gives a short survey of some recent work on collaborative mobile charging and coverage. The general model is usually rather complex, as it deals with

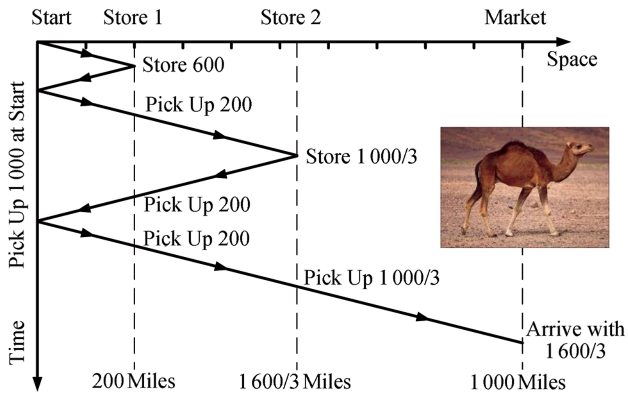

Fig.13. Solution to the banana-eating camel problem.

optimization through careful scheduling mobile charges across three dimensions: time, space, and speed. This field is still relatively new. Research results in this area can be potentially used in several mobile applications, including DARPA flying robots[5] and Google WiFi balloon[6]. We also advocate the following three principles: simplicity, elegance, and imagination as good practice to derive some good (and hopefully beautiful) research results. These principles are used and illustrated when we discuss models, solutions, and extensions of two collaborative mobile charging and coverage problems studied in the paper.

Acknowledgment Thanks are given to my collaborators of the past two years, Richard Beigel, Sheng Zhang, and Huanyang Zheng, with whom large amounts of time are spent on stimulus discussions.

References

[1] Atzori L, Iera A, Morabito G. The Internet of Things: A survey. *Computer Networks: Int. J. Computer and Telecommunications Networking*, 2010, 54(15): 2787-2805.

[2] Shah R C, Roy S, Jain S, Brunette W. Data MULEs: Modeling a three-tier architecture for sparse sensor networks. In *Proc. the 1st IEEE SNPA Workshop*, May 2003, pp.30-41.

[3] Zhao W, Ammar M, Zegura E. A message ferrying approach for data delivery in sparse mobile ad hoc networks. In *Proc. the 5th ACM MobiHoc*, May 2014, pp.187-198.

[4] Bentham J. An Introduction to the Principles of Morals and Legislation. Clarendon Press, 1879.

[5] Hardy G H. A Mathematician's Apology. Cambridge University Press, 1992.

[6] Dijkstra E W. Self-stabilizing systems in spite of distributed control. *Commun. ACM*, 1974, 17(11): 643-644.

[7] Beigel R, Wu J, Zheng H. On optimal scheduling of multiple mobile chargers in wireless sensor networks. In *Proc. the 1st International Workshop on Mobile Sensing, Computing and Communication*. (to be appeared)

[8] Zhang S, Wu J, Lu S. Collaborative mobile charging for sensor networks. In *Proc. the 9th IEEE MASS*, Oct. 2012, pp.84-92.

[4] http://www.easycalculation.com/puzzles/hard/camel.php, May 2014.

[5] http://robohub.org/eurathlon-and-the-darpa-robot-challenge-a-difference-of-approach/, May 2014.

[6] http://googleblog.blogspot.com/2013/06/introducing-project-loon.html, May 2014.

[9] Current J R, Schilling D A. The covering salesman problem. *Transportation Science*, 1989, 23(3): 208-213.

[10] Srinivasan A, Wu J. TRACK: A novel connected dominating set based sink mobility model for WSNs. In *Proc. the 17th IEEE ICCCN*, Aug. 2008, pp.1-8.

[11] Li K, Luan H, Shen C C. Qi-ferry: Energy-constrained wireless charging in wireless sensor networks. In *Proc. IEEE WCNC*, April 2012, pp.2515-2520.

[12] Gandham S, Dawande M, Prakash R *et al.* Energy efficient schemes for wireless sensor networks with multiple mobile base stations. In *Proc. IEEE GLOBECOM*, Dec. 2003, pp.377-381.

[13] Wu J, Yang S, Dai F. Logarithmic store-carry-forward routing in mobile ad hoc networks. *IEEE Transactions on Parallel and Distributed Systems*, 2007, 18(6): 735-748.

[14] He S, Chen J, Jiang F, Yau D K, Xing G, Sun Y. Energy provisioning in wireless rechargeable sensor networks. In *Proc. IEEE INFOCOM*, April 2011, pp.2006-2014.

[15] Kurs A, Karalis A, Moffatt R, Joannopoulos J D, Fisher P, Soljačić M. Wireless power transfer via strongly coupled magnetic resonances. *Science*, 2007, 317 (5834): 83-86.

[16] Ahn D, Hong S. Effect of coupling between multiple transmitters or multiple receivers on wireless power transfer. *IEEE Trans. Industrial Electronics*, 2013, 60(7): 2602-2613.

[17] Liu V, Parks A, Talla V *et al.* Ambient backscatter: Wireless communication out of thin air. *SIGCOMM Comput. Commun. Rev.*, 2013, 43(4): 39-50.

[18] Xie L, Shi Y, Hou Y T, Lou W, Sherali H D, Midkiff S F. Bundling mobile base station and wireless energy transfer modeling and optimization. In *Proc. IEEE INFOCOM*, April 2013, pp.1636-1644.

[19] Guo S, Wang C, Yang Y. Mobile data gathering with wireless energy replenishment in rechargeable sensor networks. In *Proc. IEEE INFOCOM*, April 2013, pp.1932-1940.

Jie Wu is chair and Laura H. Carnell Professor in the Department of Computer and Information Sciences at Temple University. Prior to joining Temple University, he was a program director at the National Science Foundation and Distinguished Professor at Florida Atlantic University. His current research interests include mobile computing and wireless networks, routing protocols, cloud and green computing, network trust and security, and social network applications. Dr. Wu regularly publishes in scholarly journals, conference proceedings, and books. He serves on several editorial boards, including IEEE Transactions on Service Computing and Journal of Parallel and Distributed Computing. Dr. Wu was general co-chair/chair for IEEE MASS 2006, IEEE IPDPS 2008 and IEEE ICDCS 2013, as well as program co-chair for IEEE INFOCOM 2011 and China Computer Federation (CCF) CNCC 2013. Currently, he is serving as general chair for ACM MobiHoc 2014. He was an IEEE Computer Society Distinguished Visitor, ACM Distinguished Speaker, and chair for the IEEE Technical Committee on Distributed Processing (TCDP). Dr. Wu is a CCF Distinguished Speaker and a Fellow of the IEEE. He is the recipient of the 2011 CCF Overseas Outstanding Achievement Award.

Appendix

A.1 Derivation of Scheme 2

As previously analyzed, we have

$$L_i = \frac{B + bL_{i+1}}{b+2c},$$

where $L_{m+1} = 0$ (i.e., BS) and

$$L_m = \frac{B}{b+2c}.$$

Then, we rewrite the above as follows:

$$\Big(L_i - \frac{B}{2c}\Big) = \frac{b}{b+2c}\Big(L_{i+1} - \frac{B}{2c}\Big).$$

Performing the recursion on the above equation, we have

$$\begin{aligned} L_i &= \frac{B}{2c} + \Big(\frac{b}{b+2c}\Big)^{m+1-i}\Big(L_{m+1} - \frac{B}{2c}\Big) \\ &= \frac{B}{2c}\Big[1 - \Big(\frac{b}{b+2c}\Big)^{m+1-i}\Big]. \end{aligned}$$

Therefore, when $i = 1$, we will have

$$L_1 = \frac{B}{2c}\Big[1 - \Big(\frac{b}{b+2c}\Big)^{m}\Big].$$

A.2 Comparisons Between Scheme 1 and Scheme 2

In this subsection, we prove that Scheme 2 is always better than Scheme 1, in terms of the distance covered under the same number of MCs (denoted by m). As previously analyzed, the distance covered by Scheme 1 is

$$\frac{B}{2c + b/m},$$

while the distance covered by Scheme 2 is

$$\frac{B}{2c}\Big[1 - \Big(\frac{b}{b+2c}\Big)^{m}\Big],$$

as shown in Appendix A.1.

For presentation simplicity, we introduce a ratio k denoted as

$$k = \frac{b}{2c} \quad (k > 0).$$

Then, the comparison between $\frac{B}{2c+b/m}$ and $\frac{B}{2c}[1 - (\frac{b}{b+2c})^m]$ can be simplified to the comparison between $\frac{m}{k+m}$ and $1 - (\frac{k}{k+1})^m$.

Now, let us prove the following by induction:

$$\frac{m}{k+m} \leqslant 1 - \Big(\frac{k}{k+1}\Big)^{m}.$$

When $m = 1$, the inequality is true since the left side is equal to the right side. Assume this inequality holds when $m = i$, then we have

$$\frac{i}{k+i} \leqslant 1 - \Big(\frac{k}{k+1}\Big)^i,$$

or

$$\Big(\frac{k}{k+1}\Big)^i \leqslant \frac{k}{k+i}.$$

In addition, since $i \geqslant 1$, we have

$$\frac{k}{k+1} \leqslant \frac{k+i}{k+i+1}.$$

Combining $(\frac{k}{k+1})^i \leqslant \frac{k}{k+i}$ and $\frac{k}{k+1} \leqslant \frac{k+i}{k+i+1}$, we can obtain

$$\Big(\frac{k}{k+1}\Big)^{i+1} \leqslant \frac{k}{k+i+1},$$

or

$$\frac{i+1}{k+i+1} \leqslant 1 - \Big(\frac{k}{k+1}\Big)^{i+1}.$$

Therefore, this inequality holds when $m = i + 1$. According to this induction, the statement

$$\frac{m}{k+m} \leqslant 1 - (\frac{k}{k+1})^m$$

is true.

Since $k = \frac{b}{2c}$ and B is a positive constant, we have

$$\frac{B}{2c + b/m} \leqslant \frac{B}{2c}\Big[1 - \Big(\frac{b}{b+2c}\Big)^m\Big],$$

meaning that Scheme 2 is always better than Scheme 1, in terms of the covered distance under the same number of MCs.

A.3 Derivation of Push-and-Wait

For Push-and-Wait, we have

$$\lim_{m\to\infty} L_1 = \lim_{m\to\infty} \sum_{j=1}^{m} \frac{B}{2jc+b} >$$

$$\lim_{m\to\infty} \sum_{j=j'}^{m} \frac{B}{2jc+b} (\text{let } 2j'c \geqslant b,$$

$$\text{and such } j' \text{ always exists}) >$$

$$\lim_{m\to\infty} \sum_{j=j'}^{m} \frac{B}{2jc+2jc} > \lim_{m\to\infty} \frac{B}{4c} \sum_{j=j'}^{m} \frac{1}{j} = \infty.$$

Therefore, L_1 reaches infinity as m approaches infinity.