Node-based Scheduling with Provable Evacuation Time

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Link Scheduling for Minimum Evacuation Time

Evacuation time: time needed for draining all the existing packets

- A critical metric in settings without future arrivals
  - Goal: minimize the evacuation time
- In settings with arrivals, a good measure of short-term throughput & closely related to the delay performance
Multigraph Edge Coloring Problem

- The problem is generally NP-hard \cite{Holyer'81}
- Approximations
  - Shannon’s theorem \cite{Shannon'49}, Vizing’s theorem \cite{Vizing'64}, …
  - Any constant-factor approximation ratio better than $4/3$ is NP-hard \cite{Holyer'81}
  - If a small additive term is allowed, much better approximations (exact or asymptotic) \cite{Sanders & Steurer'08,…]
- A survey book on graph edge coloring \cite{Stiebitz et al.'12}

- Limitations
  - All rely on recoloring-based techniques
  - The colors (or schedules) are computed all at once
  - The complexity depends on # of multi-edges (or # of packets)
    - Could be impractically high
    - Unsuitable for link scheduling and packet evacuation
  - More limited applications to settings with arrivals
Online Algorithms

• Quickly compute one color (or schedule) at a time
  • Complexity is only dependent on network size
    • Link count and node count
  • High complexity is distributed over time
  • Desirable for applications such as link scheduling
  • Functional even if packet arrivals are considered

• Example algorithms
  • Maximum Weighted Matching (MWM) algorithm
  • MWM-α algorithm
  • Greedy Maximal Matching (GMM) algorithm
  • Randomized Maximal Matching (RMM) algorithm

• Existing online algorithms all have an approximation ratio no better than 2! [Gupta et al.’09]
Node-based Approach

- Input-queued switches
  - Modeled as bipartite graphs
  - A class of Lazy Heaviest Port First (LHPF) algorithms [Gupta et al.’09]
    - Maximum Vertex-weighted Matching (MVM), also known as Longest Port First algorithm [Mekkittikul & McKeown’98]
    - Maximum Node Containing Matching algorithm [Tabatabaee & Tassiulas’09]
  - LHPF is both evacuation-time-optimal and throughput-optimal

- Multihop wireless networks
  - Modeled as general graphs
  - Evacuation-time performance is largely unknown
  - Our focus: develop and analyze node-based scheduling algorithms with provable evacuation time and lower complexity
Our Contributions

- Prove that MVM has an approximation ratio no greater than 3/2 in multihop wireless networks

- Propose a new node-based algorithm – Critical Node Matching (CNM) algorithm
  - CNM guarantees an approximation ratio no greater than 3/2 as well
  - CNM has a lower complexity of $O(m \sqrt{n})$ than $O(m \sqrt{n \log n})$ of MVM, where $m$ and $n$ are the link count and the node count, respectively

- As a byproduct, these algorithms serve as an alternative for achieving Shannon’s bound of $3/2 \Delta$, where $\Delta$ is the maximum node degree
MVM

- $Q_l(t)$: # of packets waiting to be transmitted over link $l$
- $L(i)$: set of links incident to node $i$
- $d_i(t) = \sum_{l \in L(i)} Q_l(t)$: degree of node $i$
- $M$: matching
- $\mathcal{M}$: set of all the matchings

MVM:
- $w_i(t) = d_i(t)$: weight of node $i$
- $w(M) = \sum_{i: L(i) \cap M \neq \emptyset} w_i(t)$: weight of matching $M$
- $\text{MVM} = \arg\max_M w(M)$: Maximum Vertex-weighted Matching
- The MVM algorithm finds an MVM in each time slot
- MVM has a complexity of $O(m \sqrt{n \log n})$
MVM - Example

\[ d_i = 2, \quad Q_i = 2 \]

\[ w_i = 2 \]

MVM, GMM, MWM-\( \alpha \) with \( \alpha > 0 \)
Main Result

**Theorem 1:** MVM has an approximation ratio no greater than $3/2$.

Proof Sketch:
- Minimum evacuation time $\geq$ maximum node degree $= \Delta$
- MVM achieves Shannon’s bound
  - Evacuation time of MVM $\leq 3/2 \Delta$ (Proposition 1)

**Proposition 1:** Suppose the maximum node degree is no smaller than two. Under the MVM algorithm, the maximum node degree decreases by at least two within every three consecutive time-slots.
Proof Sketch of Proposition 1

**Proposition 1:** Suppose the maximum node degree is no smaller than two. Under the MVM algorithm, the maximum node degree decreases by at least two within every three consecutive time-slots.

Proof Sketch:
- If the maximum node degree does not decrease in a time-slot, it will decrease in both of the following two time-slots
  - **Critical node**: Node having a maximum degree
  - **Lemma 1**: If the subgraph induced by all the critical nodes is bipartite, then there exists a matching that matches all the critical nodes [Anstee & Griggs’96]
  - **Lemma 2**: If there exists a matching that matches all the critical nodes, then MVM will match all of them as well
  - In both of the following two time-slots, the subgraph included by all the critical nodes is indeed bipartite

Observation: in order to achieve 3/2, it is sufficient to focus on scheduling the critical nodes
Critical Node Matching (CNM) algorithm

- Motivated by the key observation, focus on scheduling the critical nodes
- Assign node weights as follows:
  - \( w_i(t) = B_2 \), if \( i \) is a critical node
  - \( w_i(t) = B_1 \), otherwise
  - \( 0 < B_1 < B_2 \leq B \), both \( B_1 \) and \( B_2 \) are bounded positive integer
- Find an MVM based on the new weights in each time-slot

An implementation with \( O(m \sqrt{n}) \) complexity for bounded integer weights [Huang & Kavitha’12, Pettie’12]

**Theorem 2**: CNM has an approximation ratio no greater than 3/2.
CNM - Example

d_i=2  
Q_i=2  

w_i=1  
1  
1  

w_i=2  
4  
4  

CNM (B_1=1, B_2=2)  
MVM (also CNM)
Lower Bound of 4/3

MVM & CNM: 4 slots

Optimal: 3 slots

First time-slot, second, third, and fourth.
Throughput & Delay Performance

Simulation settings
- 4X4 grid network
- unit link capacity
- A flow with arrival rate $\lambda$ at each link

Observations
- MVM & CNM both empirically achieve good throughput performance
- MVM empirically achieves best delay performance
Conclusion

- Proved that MVM achieves an approximation ratio no greater than 3/2 for the minimum evacuation time problem.

- By making a key observation that it is sufficient to focus on scheduling the critical nodes for achieving an approximation ratio no greater than 3/2, we proposed a lower-complexity algorithm – CNM – with a same performance guarantee.

- These algorithms serve as an alternative for achieving Shannon’s bound.

- Node-based approach is less studied:
  - Performance limits of the node-based algorithms?
  - Conjecture: 4/3 is tight for MVM (and CNM) – much more challenging.
  - If an additive term is allowed, can we develop node-based algorithms with better approximations (exact or asymptotic)?
  - Throughput performance in settings with arrivals?
Thank You!

Questions?

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