CISS 2015

Baltimore, Maryland

March 18th, 2015

TEMPLE UNIVERSITY®

Node-based Scheduling with Provable Evacuation Time

Bo Ji

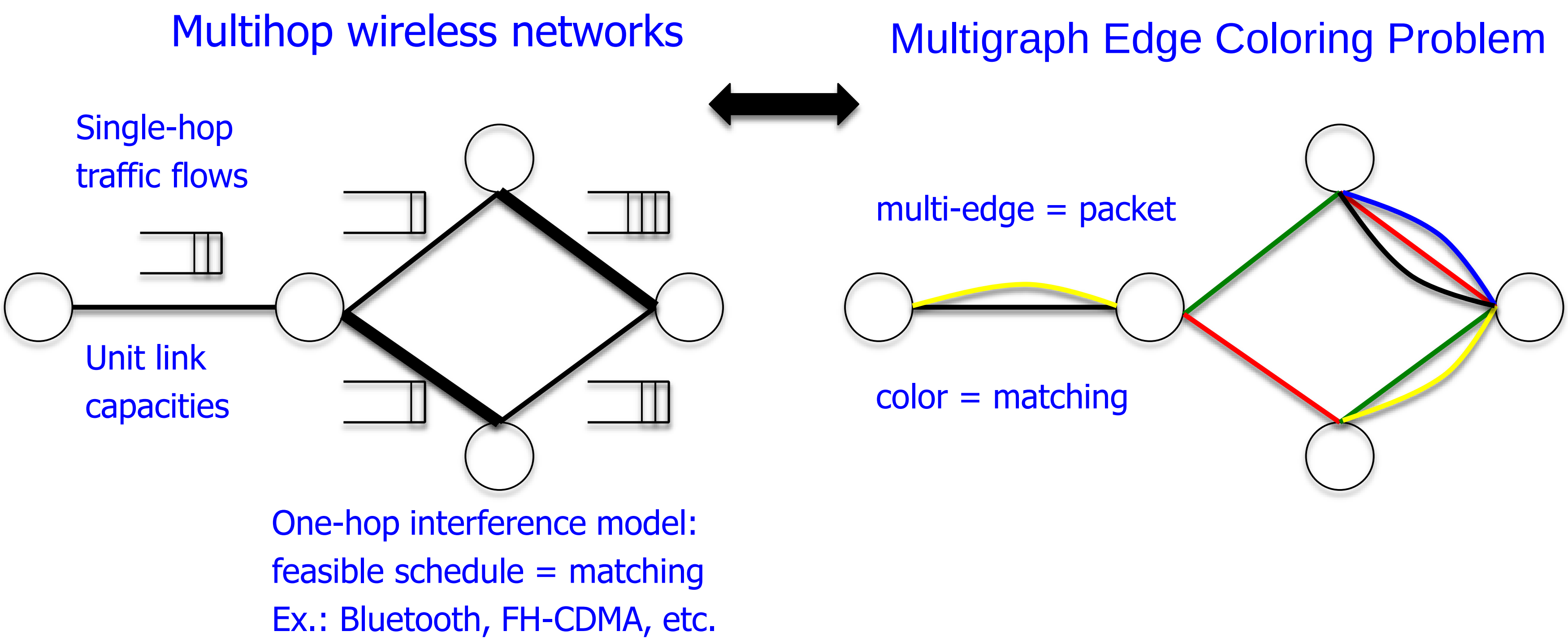
Dept. of Computer & Information Sciences

Temple University

E-mail: boji@temple.edu

Joint work with Jie Wu@Temple

Link Scheduling for Minimum Evacuation Time



- Evacuation time: time needed for draining all the existing packets
  - A critical metric in settings without future arrivals
    - Goal: minimize the evacuation time
  - In settings with arrivals, a good measure of short-term throughput & closely related to the delay performance

Multigraph Edge Coloring Problem

- The problem is generally NP-hard [Holyer'81]
- Approximations
  - Shannon's theorem [Shanon'49], Vizing's theorem [Vizing'64], …
  - Any constant-factor approximation ratio better than 4/3 is NP-hard [Holyer'81]
  - If a small additive term is allowed, much better approximations (exact or asymptotic) [Sanders & Steurer'08,…]
- A survey book on graph edge coloring [Stiebitz et al.'12]

- Limitations
  - All rely on recoloring-based techniques
  - The colors (or schedules) are computed all at once
  - The complexity depends on # of multi-edges (or # of packets)
    - Could be impractically high
    - Unsuitable for link scheduling and packet evacuation
  - More limited applications to settings with arrivals

Online Algorithms

- Quickly compute one color (or schedule) at a time
  - Complexity is only dependent on network size
    - Link count and node count
  - High complexity is distributed over time
  - Desirable for applications such as link scheduling
  - Functional even if packet arrivals are considered

- Example algorithms
  - Maximum Weighted Matching (MWM) algorithm
  - MWM-α algorithm
  - Greedy Maximal Matching (GMM) algorithm
  - Randomized Maximal Matching (RMM) algorithm

Edge-based (MWM, MWM-α, GMM)

Load-agnostic (RMM)

- Existing online algorithms all have an approximation ratio no better than 2! [Gupta et al.'09]

Node-based Approach

- Input-queued switches
  - Modeled as bipartite graphs
  - A class of Lazy Heaviest Port First (LHPF) algorithms [Gupta et al.'09]
    - Maximum Vertex-weighted Matching (MVM), also known as Longest Port First algorithm [Mekkittikul & McKeown'98]
    - Maximum Node Containing Matching algorithm [Tabatabaee & Tassiulas'09]
  - LHPF is both evacuation-time-optimal and throughput-optimal

- Multihop wireless networks
  - Modeled as general graphs
  - Evacuation-time performance is largely unknown
  - Our focus: develop and analyze node-based scheduling algorithms with provable evacuation time and lower complexity

Our Contributions

- Prove that MVM has an approximation ratio no greater than 3/2 in multihop wireless networks

- Propose a new node-based algorithm – Critical Node Matching (CNM) algorithm
  - CNM guarantees an approximation ratio no greater than 3/2 as well
  - CNM has a lower complexity of $O(m\sqrt{n})$ than $O(m\sqrt{n}\log n)$ of MVM, where m and n are the link count and the node count, respectively

- As a byproduct, these algorithms serve as an alternative for achieving Shannon's bound of $3/2\,\Delta$, where Δ is the maximum node degree

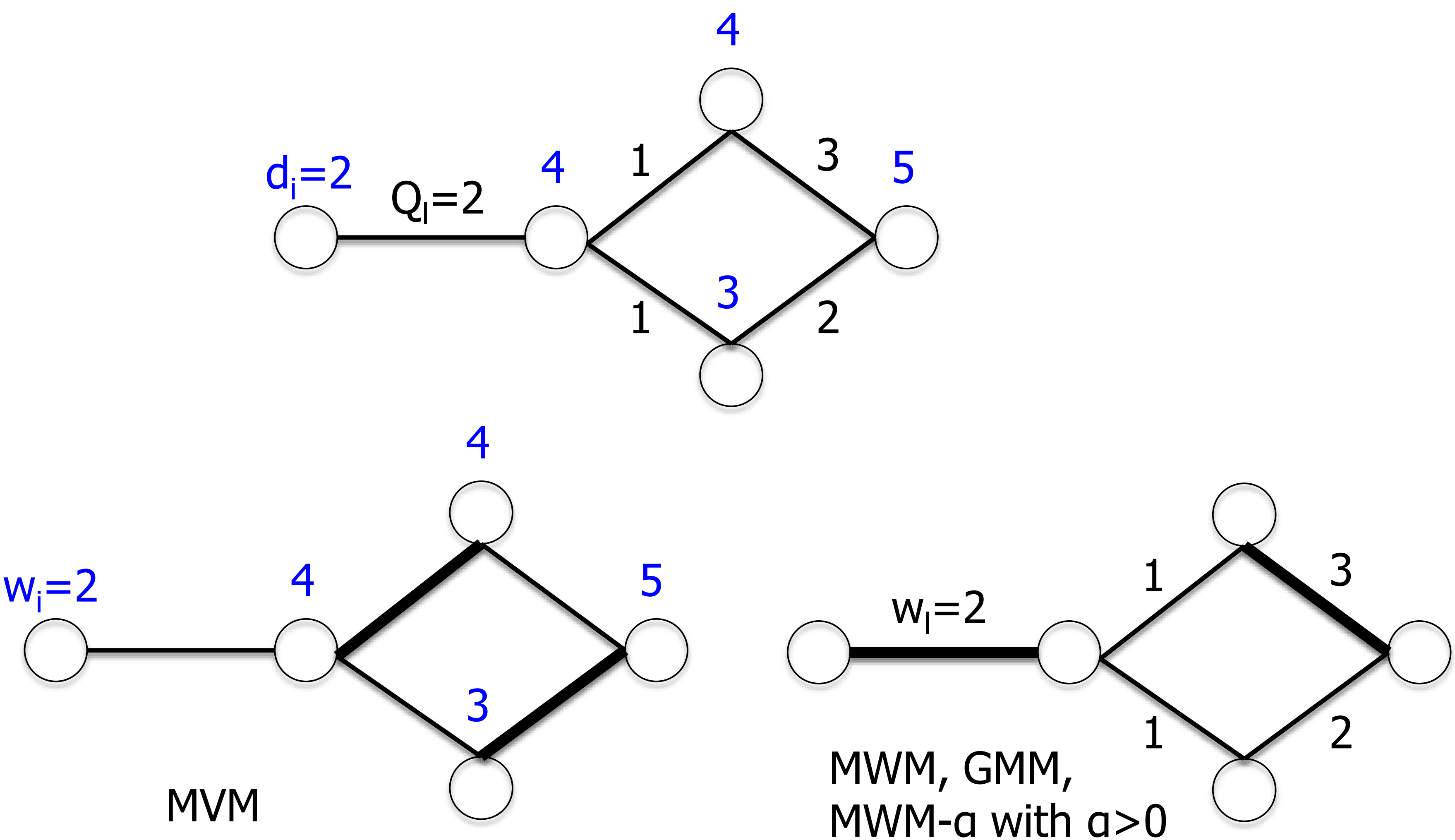
MVM

- $Q_l(t)$: # of packets waiting to be transmitted over link l
- $L(i)$: set of links incident to node i
- $d_i(t) = \sum_{l \in L(i)} Q_l(t)$: degree of node i
- M : matching
- Γ: set of all the matchings

MVM:

- $w_i(t) = d_i(t)$: weight of node i
- $w(M) = \sum_{i:L(i)\cap M \neq \emptyset} w_i(t)$: weight of matching M
- $MVM \in \arg\max_{M \in \Gamma} w(M)$: Maximum Vertex-weighted Matching
- The MVM algorithm finds an MVM in each time slot
- MVM has a complexity of O(m √n logn)

MVM - Example



Main Result

Theorem 1: MVM has an approximation ratio no greater than 3/2.

Proof Sketch:

- Minimum evacuation time ≥ maximum node degree = Δ
- MVM achieves Shannon's bound
  - Evacuation time of MVM ≤ 3/2 Δ (Proposition 1)

Proposition 1: Suppose the maximum node degree is no smaller than two. Under the MVM algorithm, the maximum node degree decreases by at least two within every three consecutive time-slots.

Proof Sketch of Proposition 1

Proposition 1: Suppose the maximum node degree is no smaller than two. Under the MVM algorithm, the maximum node degree decreases by at least two within every three consecutive time-slots.

Proof Sketch:

- If the maximum node degree does not decrease in a time-slot, it will decrease in both of the following two time-slots
  - **Critical node**: Node having a maximum degree
  - **Lemma 1**: If the subgraph induced by all the critical nodes is bipartite, then there exists a matching that matches all the critical nodes [Anstee & Griggs'96]
  - **Lemma 2**: If there exists a matching that matches all the critical nodes, then MVM will match all of them as well
  - In both of the following two time-slots, the subgraph included by all the critical nodes is indeed bipartite

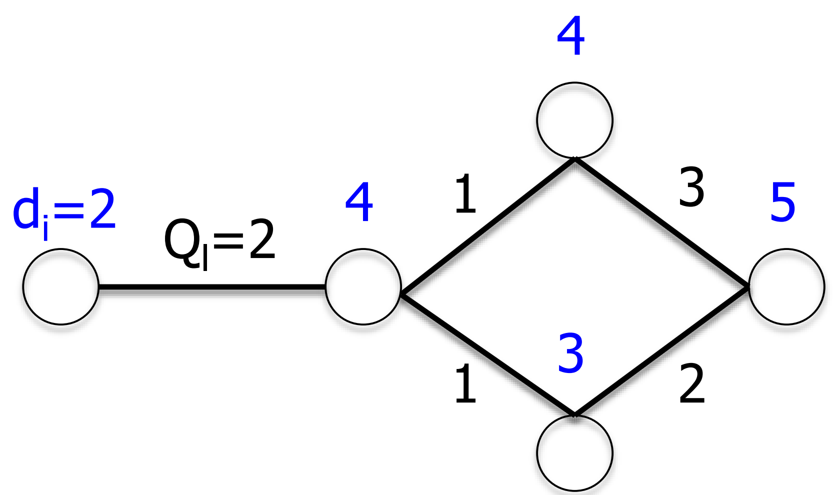
Observation: in order to achieve 3/2, it is sufficient to focus on scheduling the critical nodes

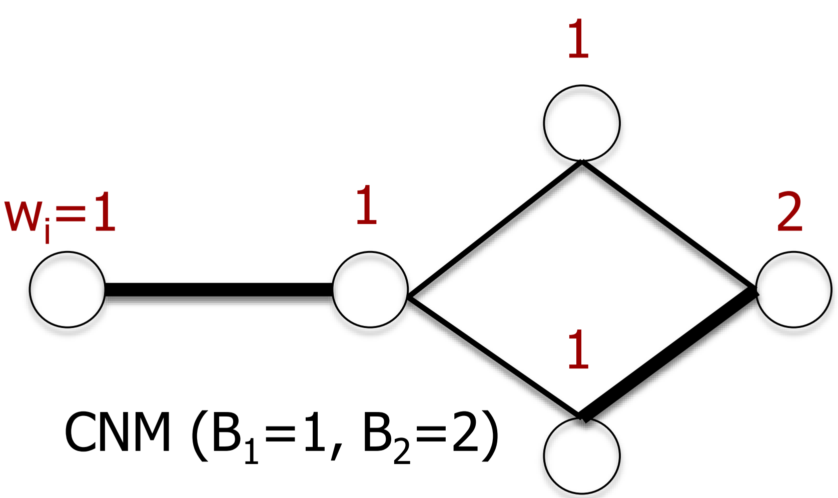
CNM – Lower Complexity MVM

- Critical Node Matching (CNM) algorithm
  - Motivated by the key observation, focus on scheduling the critical nodes
  - Assign node weights as follows:
    - $w_i(t) = B_2$, if i is a critical node
    - $w_i(t) = B_1$, otherwise
    - $0 < B_1 < B_2$ £ B, both B_1 and B_2 are bounded positive integer
  - Find an MVM based on the new weights in each time-slot
- An implementation with O(m √n) complexity for bounded integer weights [Huang & Kavitha’12, Pettie’12]

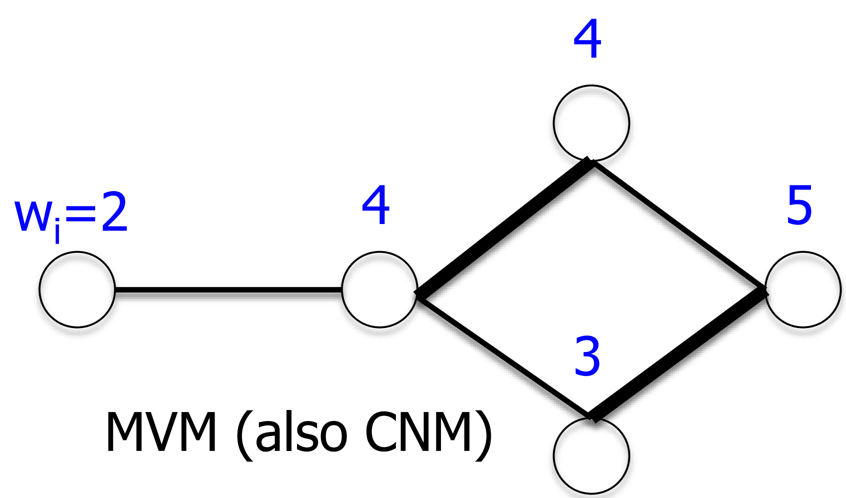
Theorem 2: CNM has an approximation ratio no greater than 3/2.

CNM - Example



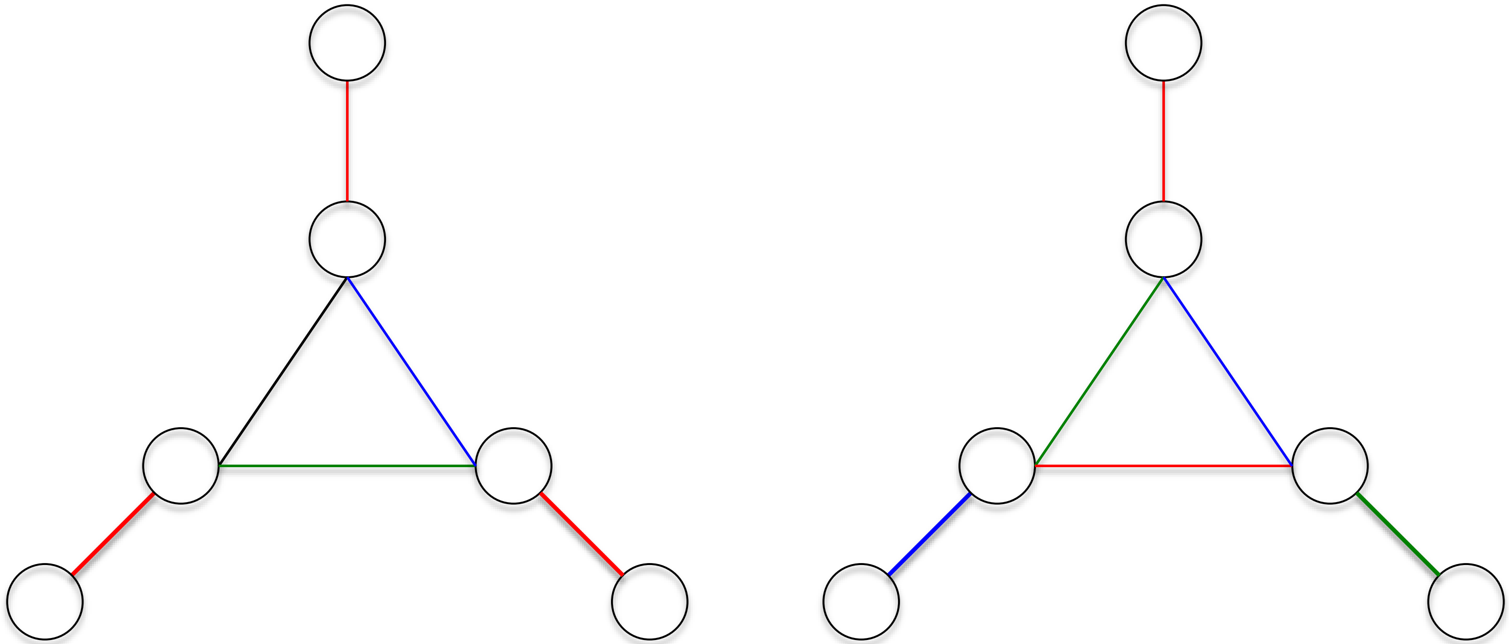


CNM ($B_1=1$, $B_2=2$)



MVM (also CNM)

Lower Bound of 4/3

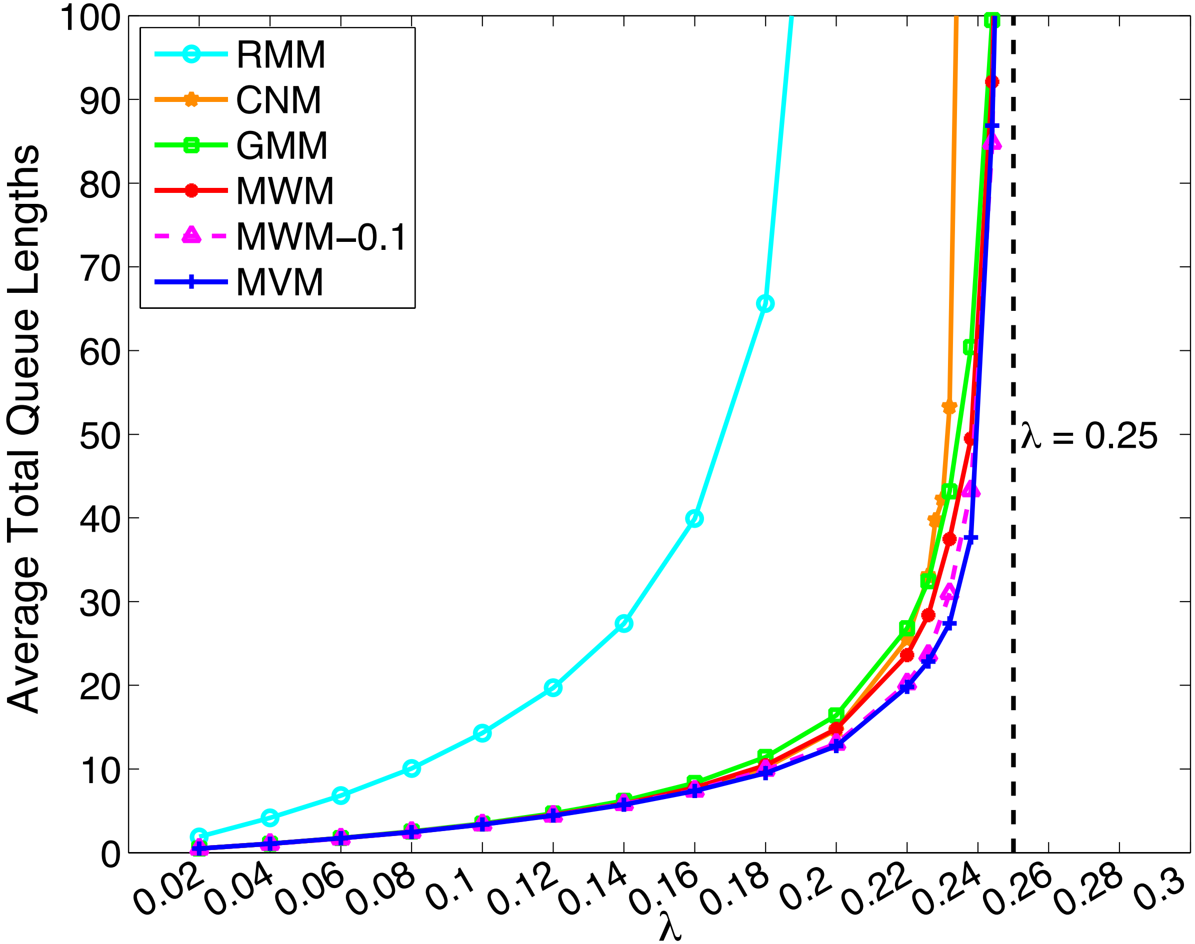


MVM & CNM: **4 slots**

Optimal: **3 slots**

First time-slot, second, third, and fourth.

Throughput & Delay Performance



Simulation settings

- 4X4 grid network
- unit link capacity
- A flow with arrival rate λ at each link

Observations

- MVM & CNM both empirically achieve good throughput performance
- MVM empirically achieves best delay performance

Conclusion

- Proved that MVM achieves an approximation ratio no greater than 3/2 for the minimum evacuation time problem

- By making a key observation that it is sufficient to focus on scheduling the critical nodes for achieving an approximation ratio no greater than 3/2, we proposed a lower-complexity algorithm – CNM – with a same performance guarantee

- These algorithms serve as an alternative for achieving Shannon's bound

- Node-based approach is less studied
  - Performance limits of the node-based algorithms?
  - Conjecture: 4/3 is tight for MVM (and CNM) – much more challenging
  - If an additive term is allowed, can we develop node-based algorithms with better approximations (exact or asymptotic)?
  - Throughput performance in settings with arrivals?

Thank You !

Questions?

E-mail: boji@temple.edu