Supplemental Material for “On Data Center Network Architectures for Interconnecting Dual-Port Servers”
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1 Proof of Theorem 1
For \( c = 0 \), the lengths of a server-to-server-direct hop and a server-to-server-via-a-switch hop are equal. We consider the maximum number of other servers that a server \( S \) can reach within distance \( d \). Within distance 1, \( S \) has two choices to reach other servers: the first one is to connect two other servers directly, and the second one is to connect two switches, each of which connects \( n - 1 \) other servers, resulting in a total of \( 2(n-1) \) servers. Obviously, the second choice is better because \( S \) reaches more other servers, and \( S \) has one port remaining for further expansion. Within distance 2 of \( S \), based on the second choice, the \( 2(n-1) \) servers connect to \( 2(n-1) \) switches, each of which connects \( n - 1 \) other servers, resulting in another \( 2(n-1)^2 \). Extending to distance \( d \), \( S \) can reach at most \( 2(n-1) + 2(n-1)^2 + \cdots + 2(n-1)^d \) other servers. Plus the original server \( S \) itself, the maximum number of dual-port servers that any network can accommodate is: \( N_v \leq 1 + 2(n-1) + 2(n-1)^2 + \cdots + 2(n-1)^d = (2(n-1)^{d+1} - n)/(n-2) = N_v^{ub} \).

2 Proof of Theorem 2
Consider the maximal number of servers that a server \( S \) in a DCN can reach within distance \( d \). For \( 1 \leq d < 1 + c \), \( S \) can reach at most 2 other servers through server-to-server-direct hops; \( [d/(1+c)] = 1 \); the theorem holds.
For \( d \geq 1 + c \), we consider three choices of \( S \) to reach as many other servers as possible within two hops (server-to-server-direct hop(s) and/or server-to-server-via-a-switch hop(s)).

The first one is to reach other servers only by server-to-server-direct hops; in this case, it can reach at most 4 other servers (if possible), 2 of which have one port remaining for further outreaching, and \( S \)’s remaining outreaching distance is \( d - 2 \).

The second choice is connecting \( S \)’s two ports to two switches; by doing this, it can reach \( 2(n-1) > 4 \) other servers, all of which have one port remaining for further outreaching, and \( S \)’s remaining outreaching distance is \( d - (1+c) \geq d - 2 \). Thus, compared with the first choice, the second one is always better.

The third choice is to connect \( S \)’s two ports to two other servers first; next, the two new servers connect to two switches, each of which connecting \( n - 1 \) other servers, if \( d \geq 1 + (1+c) \). By the third choice, \( S \) can reach at most \( 2 + 2(n-1) = 2n \) other servers, of which \( 2(n-1) \) have one port remaining. However, if the next step of the third choice is possible, i.e. \( d \geq (1+c) + 1 \), in the second choice, the \( 2(n-1) \) servers can also connect to \( 2(n-1) \) other servers within distance \( 1 + (1+c) \). The second choice results in \( 4(n-1) > 2n \) new servers; there are also \( 2(n-1) \) servers with one port remaining. Thus, the second choice is also better than the third one.

Based on the analysis of these three choices, we can see that \( S \) should always try to reach other servers via server-to-server-via-a-switch hops, if the remaining outreaching distance allows it to do so. Within \( [d/(1+c)] \) server-to-server-via-a-switch outreaching hops, \( S \) can reach at most \( 2(n-1) + 2(n-1)^2 + \cdots + 2(n-1)^{\lfloor d/(1+c) \rfloor} \) other servers. Exploiting the remaining outreaching distance \( d - (1+c) \lfloor d/(1+c) \rfloor \), \( S \) can reach at most another \( 2(n-1)^{\lfloor d/(1+c) \rfloor} \) servers, if possible. Thus, the maximal number of servers in any network with diameter less than or equal to \( d \) is \( N_v \leq 2(n-1)^{\lfloor d/(1+c) \rfloor + 1} - n)/(n-2) + 2(n-1)^{\lfloor d/(1+c) \rfloor - 1} - n)/(n-2) = N_v^{ub} \).

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