Extended Multipoint Relays to Determine Connected Dominating Sets in MANETs

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***Abstract*— MPR (multipoint relays) [1] provides a localized and optimized way of broadcasting messages in a mobile ad hoc network (MANET). Using 2-hop neighborhood information, each node determines a small set of forward neighbors to relay messages. Selected forward nodes form a connected dominating set (CDS) to ensure full coverage. Adjih, Jacquet, and Viennot [2] later proposed a novel localized algorithm to construct a small CDS based on the original MPR without any broadcast information. Such an approach is called source-independent or broadcast-independent. In this paper, we provide several extensions of the source-independent MPR to generate a smaller CDS using 3-hop neighborhood information to cover each node's 2-hop neighbor set. In addition, we extend the notion of coverage in the original MPR. We show that the extended MPR has a constant local approximation ratio compared with a logarithmic local ratio in the original MPR. The effectiveness of our approach is confirmed through a simulation study.**

Keywords: Broadcasting, connected dominating set (CDS), mobile ad hoc networks (MANETs), multipoint relays.

I. Introduction

Wireless interfaces pose a unique challenge in designing efficient broadcasting in mobile ad hoc networks (MANETs). When a node sends a message, the message can reach all adjacent nodes and, therefore, only a subset of nodes is needed to relay a broadcast message in MANETs.

Efficient broadcasting in MANETs can be formulated by identifying a small *connected dominating set* (CDS) in the network where nodes in the set and only nodes in the set relay the message. A *dominating set* (DS) is a subset of nodes in the network where every node is either in the subset or a neighbor of a node in the subset. A DS is called a CDS if the subgraph induced by the DS is connected. Many existing works on finding a small CDS are not suitable for MANETs, since they rely on either global information (such as a global network topology) or global infrastructure (such as a spanning tree). In a MANET, network topology changes frequently and, hence, a global information/infrastructure approach may not be *combinatorially stable*. In a combinatorially stable system, the propagation of all topology updates is sufficiently fast to reflect the topology changes.

The k-hop localized approach is a solution to ensure that the combinatorially stable property in MANETs works for small k. In this approach, each node determines its status and/or the status of neighbors (forward or non-forward) based on its k-hop neighborhood information (such as local network topology within k hops). In general, k-hop neighbor set of node v, represented as $N_k(v)$, is a set of nodes that are at most k hops away from node v. If the neighborhood information is collected via periodically exchanging "Hello" messages, it takes k rounds for each node to collect its k-hop neighbor set. It is clearly impossible to collect up-to-date network topology information for large k; therefore, k is usually a small integer such as 2 or 3 in MANETs. A generic broadcast scheme based on different ways of using neighborhood information is given in [3]. MPR (multipoint relays) [1] is a special 2-hop localized approach, where each forward node determines the status of its neighbors based on its 2-hop neighbor set through node coverage. It should be stressed that in the MPR each node does not determine its forward status. Instead, each forward node (selected by its neighbors following certain rules discussed later) determines forward status for each of its neighbors. Specifically, each forward node selects a subset of its 1-hop neighbors to cover its 2-hop neighbor set. That is, each 2-hop neighbor is a neighbor of the selected subset of 1-hop neighbors.

The original MPR is source-dependent (also called broadcast-dependent), that is, the forward node set is determined during a broadcast process and is dependent on the source of the broadcast and on communication latency. Adjih, Jacquet, and Viennot [2] later proposed a novel source-independent (also called broadcast-independent) MPR. Specifically, the forward node set is determined before any broadcast process and is constructed based on the MPR following two simple rules. In [4], Wu enhanced the source-independent MPR through several modifications. In this paper, we provide several extensions of the source-independent MPR to generate a smaller forward node set using 3-hop neighborhood information to cover each node's 2-hop neighbor set. In addition, we extend the notion of coverage in the original MPR. We show that the extended MPR has a constant local approximation ratio compared with a logarithmic local ratio in the original MPR. The effectiveness of our approach is confirmed through

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a simulation study.

The rest of the paper is organized as follows. Section 2 provides preliminaries on general broadcasting in MANETs. Also, the MPR algorithm and its extensions are briefly reviewed. Section 3 proposes the enhanced MPR. In Section 4, we prove the upper bound of the proposed algorithm. Section 5 provides some simulation results. The related work is discussed in Section 6 and the conclusion is drawn in Section 7.

II. Preliminaries

The simplest way to perform a broadcasting is based on the following rule:

- **Blind flooding rule**: a node re-transmits the message once and only once.

The blind flooding may cause excessive redundancy and results in channel contention and message collision (also called *broadcast storm problem* [5]). In Figure 1 (a), when node u broadcasts, every other node relays once. In reality, either w or x is sufficient.

Broadcasting can also be fulfilled by requiring only the source node and nodes in the CDS (i.e. forward node set) to transmit the message. Therefore, limited broadcast relay is based on the following rule:

- **CDS rule**: a node retransmits the message once and only once if it belongs to the CDS.

In Figure 1 (a), node w forms a CDS and, hence, only w forwards the message (except for the source). The problem is now reduced to finding a small CDS in a localized way.

A. MPR (Multipoint Relays)

A MANET is represented by a unit disk graph $G = (V, E)$, where the node set V represents a set of wireless mobile nodes and the edge set E represents a set of bi-directional links between the neighboring nodes. Each node has a distinct ID. Two nodes are considered neighbors if and only if their geographic distance is no more than a given transmission range r.

In general, the k-hop subgraph $G_k(v)$, induced from k-hop information of v, is $(N_k(v), E_k(v))$. $N_k(v)$ denotes the k-hop neighbor set of node v which includes all nodes within k hops from v (also includes v itself). $H_k(v)$ denotes the k-hop node set of v which includes all nodes that are exactly k hops away from v; that is, $N_0(v) = H_0(v) = \{v\}$, $N_k(v) = N_{k-1}(v) \cup H_k(v)$, $H_k(v) = N_k(v) - N_{k-1}(v)$, for $k \geq 1$. For convenience, 1-hop neighbor set $N_1(v)$ and 1-hop node set $H_1(v)$ are represented as $N(v)$ and $H(v)$, respectively. $E_k(v)$ denotes the set of links between $N_k(v)$, excluding those links between $H_k(v)$. That is, $E_k(v) = N_{k-1}(v) \times N_k(v)$. For example, if v has 1-hop neighbor information, then it knows all its neighbors, but not the links between these neighbors. If V is a node set, $N(V)$ is the union of the neighbor sets of every node in V, that is, $N(V) = \cup_{\forall w \in V} N(w)$. *$V$ covers U* if $U \subset N_1(V)$.

In the MPR (multipoint relays) [1], each node v maintains 2-hop subgraph $G_2(v) = (N_2(v), E_2(v))$. Node v selects a small forward node set, $C(v)$, from its 1-hop neighbor set $N_1(v)$ to cover its 2-hop neighbor set $N_2(v)$; that is, $C(v) \cup v$ is a CDS for $N_2(v)$. $C(v)$ is also called the *coverage set* for v. When u is selected by v as a forward node, v is called the *selector* of u. Note that several selectors may exist for a particular node. A forward node may or may not actually retransmit the message; its actual status is determined by the following MPR rule [1]:

- **MPR rule**: a node retransmits the message once and only once if the first message received is from a selector.

The collection of nodes that have retransmitted the message plus the source node of the broadcasting form a CDS. The original MPR is also called source-dependent MPR. The source-dependent (or broadcast-dependent) approach depends on the source of a specific broadcast operation. When a specific broadcast starts, after receiving a broadcast packet, the node determines both its own and/or some of its neighbors' forward/non-forward statuses under a local view of its neighbor set. The local view of its neighbor set can be updated by the neighborhood information contained in the "Hello" message or by the broadcast history information piggybacked in the broadcast packet. As the broadcast packet traverses the network, the forward nodes eventually form a "dynamic" CDS of the given network.

A simple greedy algorithm for computing $C(v)$ (initially empty) at v in the MPR is shown as Algorithm 1 [1]. Note that in the MPR, when v transmits, $N(v)$ is covered; therefore, $H_2(v)$ $(= N_2(v) - N(v))$ is used instead of $N_2(v)$.

Algorithm 1 Greedy algorithm at node v

1. Add $u \in H_1(v)$ to $C(v)$, if there is a node in $H_2(v)$ covered only by u. Any node in $H_2(v)$ that is not covered by $C(v)$ is called an uncovered node.
2. Add $u \in H_1(v)$ to $C(v)$, if u covers the largest number of uncovered nodes in $H_2(v)$. Use node ID to break a tie when two nodes cover the same number of uncovered nodes.

In Figure 1 (b), suppose the following coverage sets are selected based on the above greedy algorithm: $C(u) = \{v, y\}$, $C(v) = \{x\}$, $C(w) = \{y\}$, $C(x) = \{v\}$, and $C(y) = \{w\}$. Collectively nodes v, w, x, and y form a CDS. As specified in the MPR, the actual set of forward nodes for a particular broadcast uses only a subset, and it depends on the location of the source and communication latency. For example, if v is the source and node x receives the first message from v, then x is a forward node. Also, if nodes w and y receive their first message from x and v, respectively, none of them will forward the message. Therefore, $\{v, x\}$ forms a CDS for this case. However, if node y receives the first message from u, then $\{v, x, y\}$ forms a CDS.

B. Source-independent MPR

The original MPR is source-dependent. Adjih, Jacquet, and Viennot [2] later proposed a novel localized algorithm

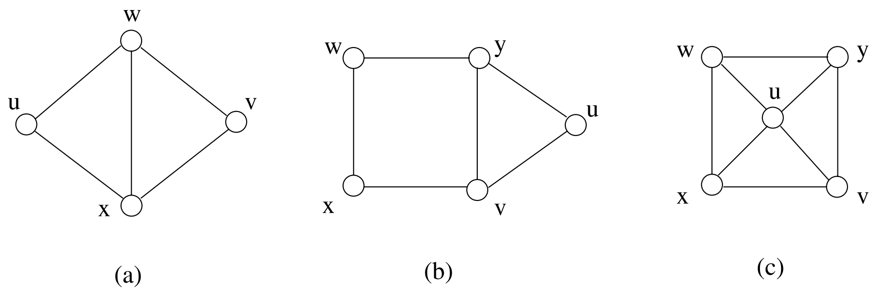


Fig. 1. Three sample networks.

to construct a CDS based on the MPR, and it is source-independent. The source-dependent approach depends on a particular broadcast. Therefore, the resultant forward node set depends on many factors, such as the location information of neighbors, node priority, message propagation delay, back-off delay, etc. The source-independent approach does not depend on a particular broadcast, and therefore, the resultant forward node set forms a "static" CDS of the network that depends only on local topology and node priority. In addition, the forward node set is generic and can be used for any broadcast.

A node belongs to a CDS if

- **Rule 1**: the node has a smaller ID than all its neighbors.
- **Rule 2**: the node is a forward node selected by its neighbor with the smallest ID.

Applying Rule 1 and Rule 2 to Figure 1 (b), $\{x, y, v, u\}$ forms a CDS. Compared with the set derived from the original MPR, node w is not in the final CDS since it is selected by y (which does not have the smallest ID among w's neighbors). In addition, node u is included since it has a smaller ID than all its neighbors. The correctness of source-independent MPR is given in [2].

C. Existing extensions

Wu [4] observed two potential drawbacks in the source-independent MPR:

1. Rule 1 is "useless" in many instances; that is, the node selected based on Rule 1 is not essential for a CDS.
2. The original MPR forward node selection (Algorithm 1) does not take advantage of Rule 2.

In Figure 1(a), u and v are selected based on Rule 1; however, they are useless. In fact, node w alone is sufficient for a CDS. Similarly, u selected by Rule 1 (in Figure 1(b)) is useless. On the other hand, we might have to include some smallest ID nodes even if they are not selected by any of their neighbors as forward nodes. In Figure 1(c), suppose node u is not selected by any of its neighbors. u has to be included (as it is selected by Rule 1), because any forward node selected by a node other than u will be ignored based on Rule 2.

In Figure 1(b), we assume that v selects x as its forward node. Based on Rule 2, since v is the smallest ID neighbor of x, x cannot ignore v's choice. On the other hand, if v chooses y, since v is not the smallest ID neighbor of y, v's choice will be ignored by y. Therefore, forward node y comes for "free" for v. That is, the inclusion of y does not increase the size of the forward node set.

Wu [4] then proposed two extensions to the source-independent MPR: one is on Rule 1 and the other is on the greedy algorithm (Algorithm 1).

- **Enhanced Rule 1**: the node has a smaller ID than all its neighbors, and it has two unconnected neighbors.

The Enhanced Rule 1 together with the original Rule 2 will generate a CDS under all cases except complete graphs. Note that when the network is complete, there is no need of a CDS, because each source forms a CDS. Wu [4] showed that the Enhanced Rule 1 is effective when the network is dense.

Wu [4] also introduced the notion of *free neighbor*. Node u is a free neighbor of v if v is not the smallest ID neighbor of u. In the enhanced forward node selection, we first include all free neighbors, then nodes with higher degrees (i.e., covering more uncovered 2-hop neighbors) are selected and use node ID to break a tie if needed until $H_2(v)$ is covered. The modified greedy algorithm is shown in Algorithm 2. Simulation results in [4] show that this extension is effective when the network is sparse. Combining the Enhanced Rule 1 and the modified greedy algorithm (Algorithm 2) at each node v, the result is effective for both sparse and dense networks.

Algorithm 2 Modified greedy algorithm at node v

1. Add all free neighbors to $C(v)$.
2. Follow steps 1. and 2. of Algorithm 1.

III. Proposed Approach

The proposed source-independent approach is motivated by the case of Figure 1 (b). Suppose the current node is u. In the original MPR or its extensions, both y and v need to be selected to cover u's 2-hop neighbors w and x. However, w falls into the 2-hop neighbor set of v. That is, w can be covered by v via x when v calculates its forward node set. Motivated by this example, our proposed approach selects a pair of nodes at each step. We first give an extended notion of *coverage*:

Definition 1: *A node u is covered by v if it is a 1-hop neighbor in $H_1(v)$ (directly covered) or it is a 2-hop neighbor in $H_2(v)$ (indirectly covered).*

In the example of Figure 1 (b), among 2-hop neighbors of u, x is directly covered by v and w is indirectly covered by v via x. In this case, when u selects node pair (v, x), u is a *direct selector* for v (to cover x) and u is an *indirect selector* for x (to cover w).

In the proposed approach, each node u still covers its 2-hop neighbor set, but uses 3-hop information. In fact, the only additional information used is about connections between any two 2-hop neighbors. We have then the following Enhanced Rule 2:

- **Enhanced Rule 2**: node u is a forward node if u is
  1. directly selected by a node in $H_1(u)$ that has the smallest ID in $H_1(u)$.
  2. indirectly selected by a node in $H_2(u)$ that has a smaller ID than all nodes in $H_1(u)$.

With the Enhanced Rule 2, we extend the notion of free neighbor to *1-hop free neighbor* and *2-hop free neighbor* as follows:

Definition 2: *Node u is a 1-hop free neighbor of v if u is in $H_1(v)$ and v's ID is not the smallest ID in $H_1(u)$. Node u is a 2-hop free neighbor of v if u is in $H_2(v)$ and u's ID is larger than at least one node ID in $H_1(u)$.*

The greedy algorithm can then use these free neighbors for neighbor coverage without any "cost". In the extended greedy algorithm (Algorithm 3), two nodes, u and w, as a pair are selected at each selection operation performed by current node v, where u is a 1-hop neighbor of v and w is a 2-hop neighbor of v which is also a 1-hop neighbor of u. We introduce the concepts of "cost" and "yield" to measure the quality of each selection.

Definition 3: *A "cost" of a selection operation is the number of selected nodes that are not free neighbors in the selection. A "yield" of a selection operation is the total number of the uncovered nodes that are covered by the selection divided by the cost of the selection.*

Note that each node v knows its 1-hop and 2-hop free neighbors because v has 3-hop neighbor set information, which also includes the neighbor set of each of its 2-hop neighbors.

Algorithm 3 Extended greedy algorithm at node v

1. Add all pairs of 1-hop free neighbor u and 2-hop free neighbor w to $C(v)$ and remove all their covered nodes from $H_2(v)$.
2. Add a pair of nodes $u \in H_1(v)$ and $w \in H_1(u) \cap H_2(v)$ to $C(v)$ that gives the highest yield in $H_2(v)$. Use node ID to break a tie if two selections give the same yield.

The major modification here is that a 2-hop neighbor w of v can be indirectly selected to cover other 2-hop neighbors. That is, a 1-hop neighbor u directly covers $H_1(u) \cap H_2(v)$ and u indirectly covers $H_1(w) \cap H_2(v)$ via w. Also, w always exists as long as $H_2(v)$ is not empty and is included even if it does not "contribute" additional coverage beyond what v covers. The extended greedy algorithm weighs the following considerations when selecting node pair (u, w) at v:

1) Both 1-hop free neighbor u and 2-hop free neighbor w can contribute additional coverage without any cost. Therefore, a pair of free neighbors should be included first.
2) Either 1-hop free neighbor u or 2-hop free neighbor w can decrease the total cost by half which leads to a higher yield.
3) Nodes u and w have equal cost and their contributions (in terms of coverage) are treated equally. Therefore, whichever covers a larger of number of uncovered nodes will give a higher yield.

The following theorem guarantees that the extended greedy algorithm generates a CDS for a given connected graph.

Theorem 1: *If the given connected graph is not a complete graph, the set of forward nodes selected by the Enhanced Rule 1 and Enhanced Rule 2 forms a CDS.*

Proof: Assume that the graph is not a complete graph; we first show that there exists at least one node in the forward node set. Let c be the node with the smallest ID in the network. If all other nodes are neighbors, at least two neighbors are not directly connected. Based on the Enhanced Rule 1, c is selected. If there exists another node that is not a neighbor of c, c will designate a neighbor $c^{'}$ for relaying. Since c is the smallest ID node, $c^{'}$ is selected based on the Enhanced Rule 2. Let C be the connected component in the forward node set that contains the smallest ID node c and/or its designated neighbor $c^{'}$. We prove that C itself is a dominating set (DS).

We prove by contradiction. If C is not a DS, there must exist some nodes that are not in $N(C)$, i.e., $\overline{N(C)}$ is not empty. Let V be the set of nodes that have at least one neighbor in C and at least one neighbor in $\overline{N(C)}$. V cannot be empty, since the network is connected. Also, $V \cap C = \phi$. Consider the smallest ID node s in $N(V)$.

- Assume s is in $\overline{N(C)}$ (which implies $s \notin V$). Since $s \in N(V)$ and $s \notin V$, there exists a neighbor v of s in V. Note that in general when $s \in N(V)$, s may not have a neighbor in V. Let u be a neighbor of v in C. Consider now the relay set for s. As u is a 2-hop neighbor of s, based on Algorithm 3, s has the following three choices to cover u:
  1) $s \to v(\in V) \to u$
  2) $s \to v(\in V) \to u^{'}(\in N(V)) \to u$
  3) $s \to s^{'}(\in N(V)) \to v(\in V) \to u$

  In the first case, s covers $v \in V$ directly; in the second case, s covers $v \in V$ directly; and in the third case, s covers $v \in V$ indirectly (via s'). In all these cases, s has the smallest ID among $N(V)$ which includes $N(v)$. Next we show that the second and third cases include all possible 3-hop paths connecting $s \in \overline{N(C)}$ and $u \in C$. Suppose the path is (s, x, y, u), clearly y connects to a

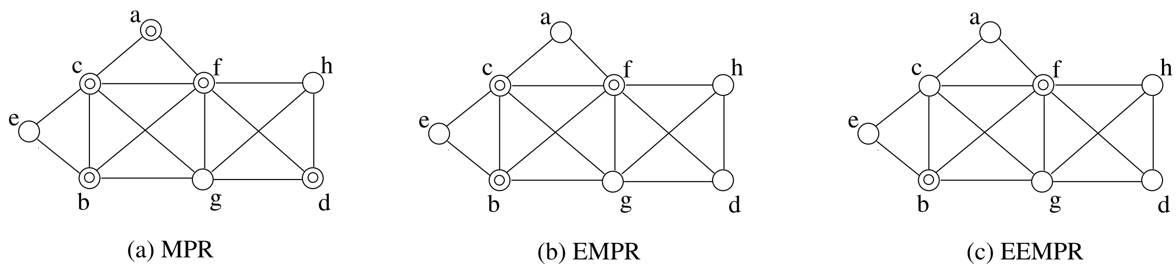


Fig. 2. A sample network with 8 nodes. The double-circled nodes are selected as forward nodes by (a) the MPR, (b) the EMPR, and (c) the EEMPR.

node in C and s connects to a node $\overline{N(C)}$. If x also connects to a node in C, then x belongs to V; otherwise, x belongs to $\overline{N(C)}$ which makes $y \in V$. It is also possible that both x and y belong to V. This case is included in both second and third cases since $V \subset N(V)$. In all cases, v is selected which contradicts $V \cap C = \phi$.

- Assume s is in $N(C)$ which can be partitioned into V and $N(C) - V$. (a) Suppose s is in V, based on the Enhanced Rule 1, s is selected since its ID is smaller than that of all its neighbors. In addition, s has two unconnected neighbors, one in $\overline{N(C)}$ and one in C. (b) Suppose s is in $N(C) - V$. Let v be a neighbor of s in V, and let u be a neighbor of v in $\overline{N(C)}$. Consider now the relay set for s. As u is a 2-hop neighbor of s, s has the following three choices to cover u:
  1) $s \to v(\in V) \to u$
  2) $s \to v(\in V) \to u^{'}(\in N(V)) \to u$
  3) $s \to s^{'}(\in N(V)) \to v(\in V) \to u$

  The rest of the proof is similar to the previous case.

In all cases, we reach a contradiction. Therefore, C has to be a DS. □

Figure 2 shows a sample network with 8 nodes. The double-circled nodes are selected as forward nodes by the source-independent MPR [2] (MPR), the enhanced source-independent MPR [4] (EMPR), and the proposed extended source-independent MPR (EEMPR). In Figure 2 (a), nodes a, b and d are the nodes with the smallest ID within their corresponding 1-hop neighbors; they are included in the CDS by Rule 1. Nodes c and f are selected as forward nodes by node a, which is the node with the smallest ID within c and f's 1-hop neighbors (Rule 2). Also, it is assured that node b, the smallest ID neighbor of node g, selects $\{c, f\}$ to cover $H_2(b)$. Therefore, $\{a, b, c, d, f\}$ are in the CDS for the MPR. In Figure 2 (b), nodes a and d are removed from the CDS by the Enhanced Rule 1 because node a's 1-hop neighbors (c and f) are connected and d's 1-hop neighbors (f, g, and h) are pairwise connected. Therefore, $\{b, c, f\}$ are in the CDS for the EMPR. In Figure 2 (c), node c is removed from the CDS by the Enhanced Rule 2 because c's 1-hop neighbor with the smallest ID, a, selects f and b to indirectly cover e. Thus, only $\{b, f\}$ are in the CDS for the EEMPR.

Figure 3 (a) shows a sample network with 80 nodes. Figures 3 (b - e) show the results with the MPR (Figure 3 (b)), the EMPR (Figure 3 (c)), the EEMPR (Figure 3 (d)), and the MCDS (Figure 3 (e)). In these figures, only nodes in the CDS and their induced subgraphs are shown. The MCDS is a global method based on [6] which can be used as the lower bound. The size of the CDS's for the MPR, EMPR, EEMPR and MCDS are 32, 29, 27 and 19, respectively.

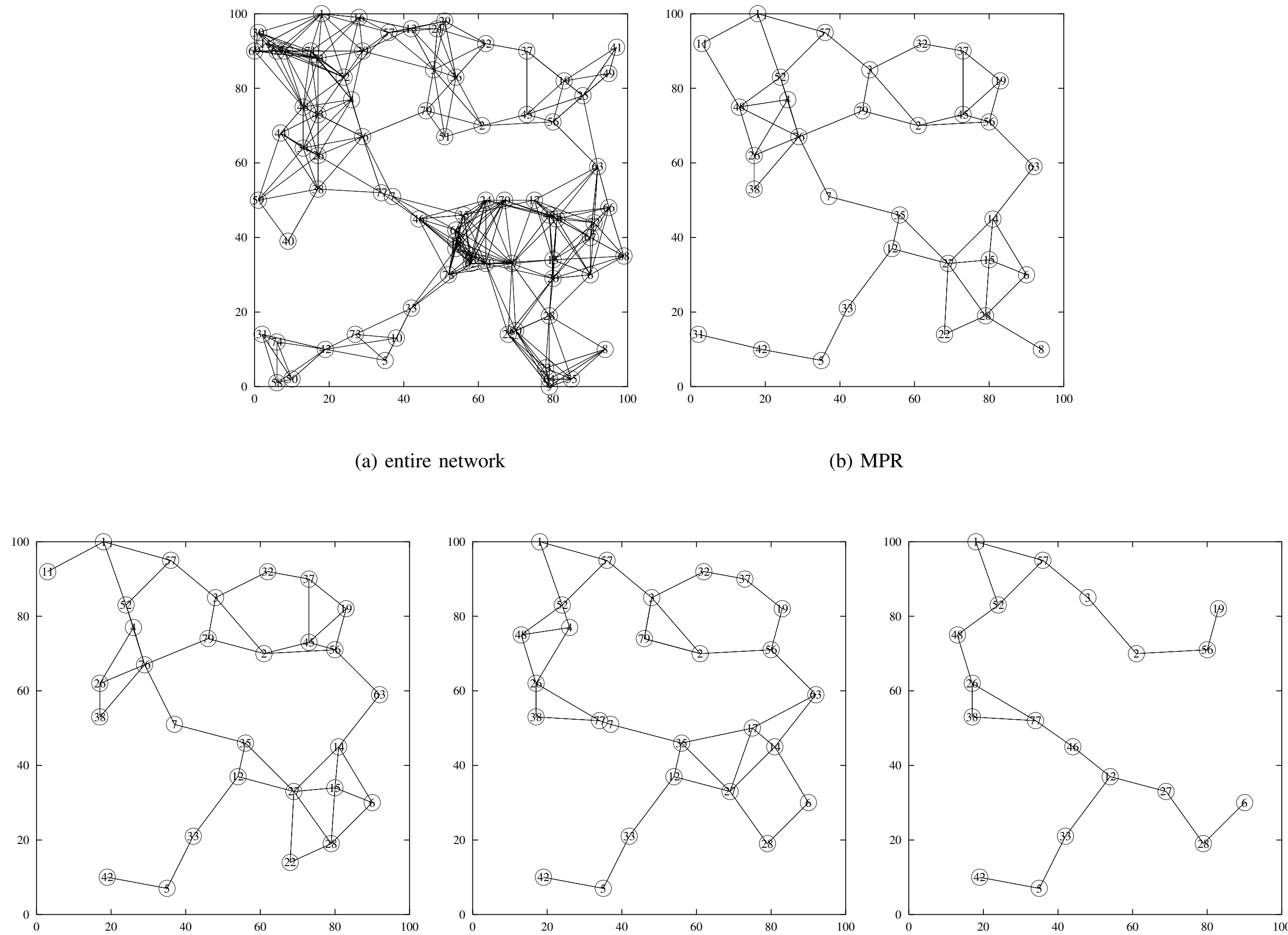
IV. The Upper Bound of the Proposed Extended Greedy Algorithm

In [1], Qayyum, Viennot, and Laouiti proved that the local upper bound of the ratio of the size of their proposed heuristic to that of the optimal MPR is $O(\log n^{'})$, where $n^{'}$ is the maximum size of the 2-hop neighbor set. Note that this ratio is with respect to the MPR methods only (i.e., methods where 2-hop nodes are covered by selected 1-hop nodes). In fact, the approximation ratio is $O(n^{'})$ among all algorithms that cover 2-hop neighbor sets locally. Consider the example in Figure 4 (a) where all 1-hop neighbors of v are on the circle of C (with radius r from center v) and all 2-hop neighbors of v are on the circle of C' (with radius $2r$ from center v). r is the uniform transmission range of each node. Clearly, when u computes its forward nodes, each 2-hop neighbor of v, say w, on the circle of C' can only be covered by exactly one 1-hop neighbor of v, say u, on the circle of C with position exactly on the line connecting v and w (that is, there is a one-to-one relation between v and w). When the number of nodes on C' increases, the number of selected forward nodes on C also increases with the same rate. In fact, as indicated in Figure 4(a), a constant number of nodes (9 double-circled nodes) are sufficient to cover all 1-hop and 2-hop neighbors of v. Therefore, the approximation ratio is $O(n^{'})$.

Next, we prove that for each single node v, the extended greedy algorithm (Algorithm 3) can provide a constant size of the forward node set $C(v)$.

Theorem 2: *The extended greedy algorithm has a constant local approximation ratio.*

Proof: Suppose v is the node that selects a forward node set $C(v)$ to cover $H_2(v)$. Based on the algorithm, v selects a pair of nodes u and w, where u is in $H_1(v)$ and w is an uncovered node in $H_1(u) \cap H_2(v)$; the pair covers the maximum number of uncovered nodes in $H_2(v)$. The selected nodes are put into $C(v)$ and the nodes covered by $C(v)$ in $H_2(v)$ are removed. Node v continues to select pairs u' and w', u'' and w'', ...,



(a) entire network (b) MPR

(c) EMPR (d) EEMPR (e) MCDS

Fig. 3. A sample network with 80 nodes: (a) entire network, (b) MPR, (c) EMPR, (d) EEMPR, and (e) MCDS.

and so on, until $H_2(v)$ becomes empty (see Figure 4(b)). For each selection, the newly selected 2-hop forward node, say w', is not adjacent to any already selected 2-hop forward node, say w, in $C(v)$. In other words, $\{w, w', w'', ...\}$ forms an independent set [1]. This suggests that, within a disk whose diameter is r (or radius $0.5r$), there exists at most one selected 2-hop forward node (of type w). In other words, such disks are non-overlapped. Notice that the possible location of v's 2-hop neighbor is only within the ring between r to $2r$. Thus, the disks with diameter r are confined within the ring between $0.5r$ to $2.5r$ (shaded area in Figure 4(b)). The maximum number of such disks is $\frac{\pi(2.5r)^2 - \pi(0.5r)^2}{\pi(0.5r)^2} = 24$. Therefore, the total number of $\{w, w', w'', ...\}$ is no larger than 24 and the total number of nodes in $C(v)$, which is twice the size of $\{w, w', w'', ...\}$, is no larger than 48. Note that the optimal number of forward nodes selected by each node to cover its 2-hop neighbor set is a constant. Therefore, the proposed approach has a constant local approximation ratio. □

In [7], a disk with radius kr is proved to have an upper-bounded constant number of nodes l_k in an IS, where $l_k \leq (2k+1)^2$. The extended greedy algorithm provides a special case when $k = 2$. Although the extended greedy algorithm provides each node a constant number of forward nodes, the upper bound of the CDS of the entire network is still $O(n)$ where n is the size of the network. The reason is that the collection of the independent sets that are selected locally does not correspond to a global IS. One worst case is shown in Figure 4 (c): all nodes sit along line AD with length of $3r$ and the nodes' IDs monotonously increase along the line from the left end to the right end. Each node determines its dominator, which has the smallest ID among its 1-hop neighbor set. Based on the algorithm, a node will finally become a forward node if it is selected by its dominator. When the density of the network becomes infinite, all $O(n)$ nodes on the segment BC

[1] An independent set is a set in which no two nodes are neighbors.

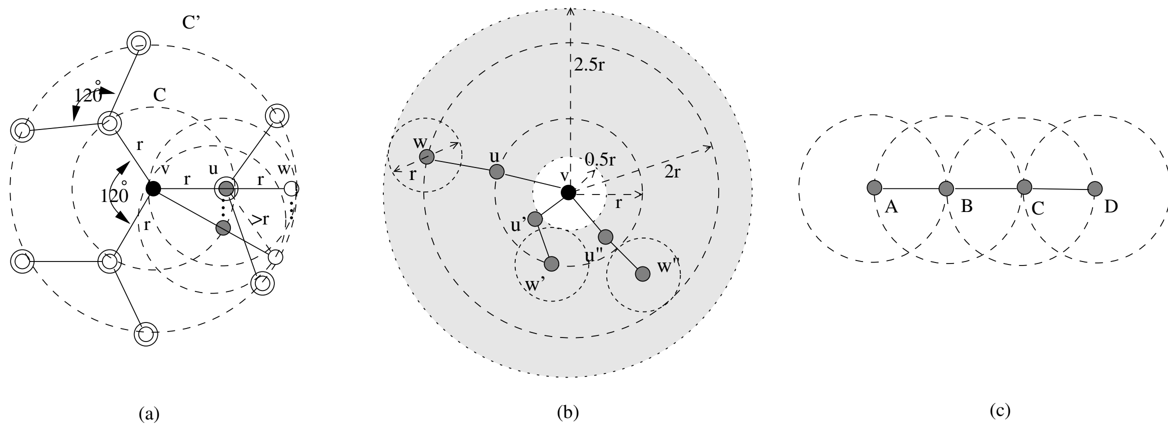


Fig. 4. (a) An example of the worst case where the number of node v's forward nodes is $O(n)$, (b) illustration of the extended greedy algorithm, and (c) the worst case where the CDS of the entire network is $O(n)$ for the extended greedy algorithm.

become forward nodes On the other hand, a CDS with only three nodes at positions A, B and C is sufficient to cover the entire network. However, this situation corresponds to the worst case which rarely occurs. The next section will show the competitive average performance through simulations.

V. Simulation

We compare the number of nodes in the CDS for the proposed extended source-independent MPR (EEMPR), the source-independent MPR [2] (MPR), and enhanced source-independent MPR [4] (EMPR) under three scenarios.

In the first scenario, a given number of nodes (ranging from 20 to 100 with a step of 10 and from 100 to 1,000 with a step of 100, respectively) were randomly distributed in a 100×100 2-D space. Each node has a fixed uniform transmission range r (r is 25 and 50, respectively). There is no consideration of node movement and channel collision. Thus, a pair of nodes are neighbors when their distance is smaller than r. If the generated network is not connected, it is discarded. For each fixed number of nodes, the results of a sufficient number of experiments are averaged to make 90% confidence interval within $\pm$ 5%.

Figures 5(a) and 5(b) show the simulation results when the node's transmission range is 25. Figure 5(a) shows the trend when the number of nodes in the network ranges from 20 to 100 (the corresponding graph is sparse), whereas Figure 5(b) shows the trend when the number of nodes in the network is from 100 to 1000 (the corresponding graph is dense). We find that all three curves have a rising trend as the number of nodes in the network increases. The number of nodes in the CDS increases because, when more nodes join in the network, the network density increases and a node may select more 1-hop neighbors as forward nodes, which increases the size of the CDS. From the figure, we also notice that the rising trend is more sensitive to node numbers in the range from 20 to 100 (relatively sparse) than to node numbers in the range from 100 to 1000 (relatively dense). The effect is more remarkable when the network is sparse because the greedy algorithm is a node coverage algorithm, that is, it selects 1-hop forward nodes to cover 2-hop neighbors. When the network is sparse, the collective coverage of the forward nodes may still leave some blank areas (i.e. areas with no nodes) within the 2-hop neighborhood. As more nodes join in, new nodes may appear in these blank areas thus resulting in the selection of more forward nodes. As the network density increases, the number of blank areas decreases as does the number of newly selected forward nodes. Therefore, the rising trend slows down as the number of node increases. Among these three algorithms, the performance of the MPR is the worst in all ranges. When the network is sparse (n is from 20 to 80), the curves of the EMPR and the EEMPR are almost the same. But as the number of nodes increases, the gap between the EMPR and the EEMPR becomes significant. When the number of nodes in the network is 1000, the number of nodes in the CDS determined by the EEMPR is only around 70% of that determined by the EMPR or MPR. The reason that the EEMPR shows great improvement in dense networks is that the selection of the forward nodes for one node has an upper bound that is irrelevant to the network density. Thus, the size of the CDS is less influenced by the network density.

Figures 6(a) and 6(b) show the results when the node's transmission range is 50 and the number of nodes in the network is from 20 to 100 and from 100 to 1000, respectively. When the transmission range increases, the graph becomes denser if the number of nodes is fixed. In this case, the size of the CDS increases only slightly as the size of the network increases. This is because, when the transmission range is 50, the corresponding graph is sufficiently dense so that the number of nodes has little effect on network density. Among these three algorithms, EEMPR outperforms the other two, followed by EMPR; MPR is the worst in all the ranges.

Comparing Figures 5(a) and 5(b) with Figures 6(a) and 6(b), we find that increasing the node's transmission range can increase the coverage area of each node and, therefore, reduce the diameter of the network, which leads to a smaller size of the CDS.

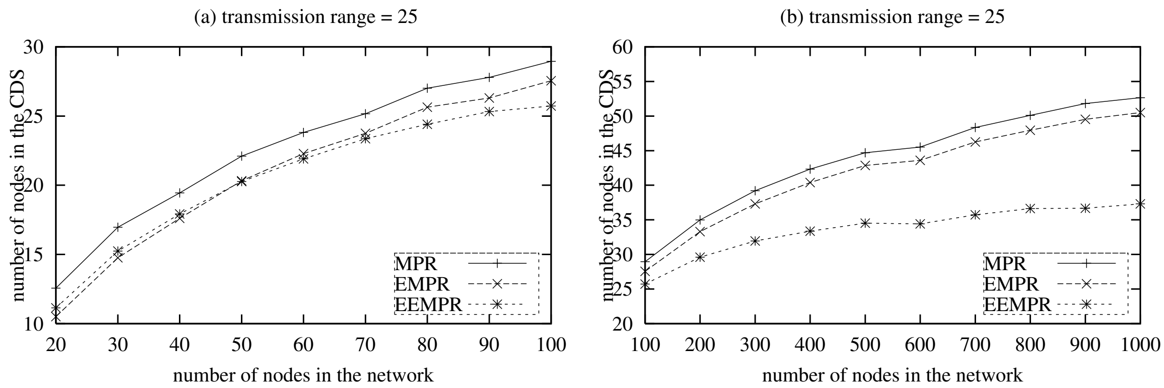


Fig. 5. The number of nodes in the CDS when r is 25: a) n ranges from 20 to 100, and (b) n ranges from 100 to 1000.

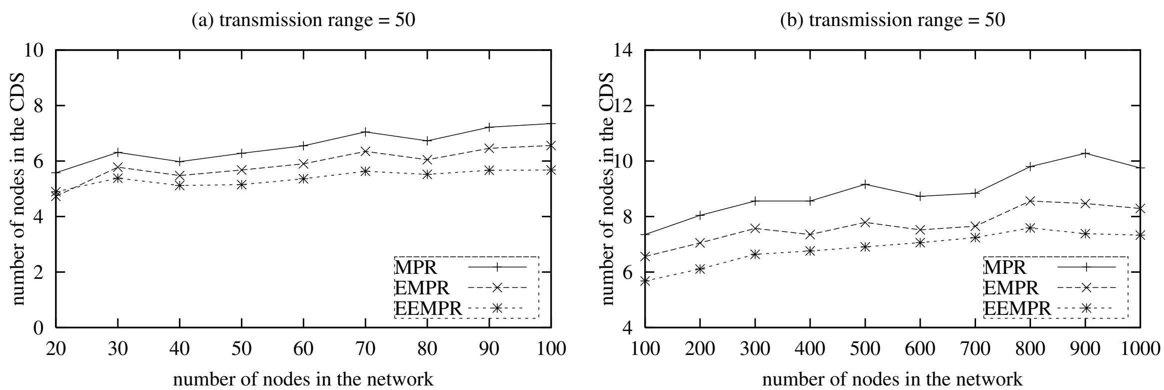


Fig. 6. The number of nodes in the CDS when r is 50: a) n ranges from 20 to 100, and (b) n ranges from 100 to 1000.

In the second scenario, a fixed number of nodes (n = 200 and 1000, respectively) is randomly distributed in the same 2-D space. The network density is determined by the node's transmission range r. For each fixed number of nodes, we run different experiments where the value of r changes from 20 to 75. The results of sufficient numbers of experiments for each fixed network density are averaged to guarantee the same confidence interval.

Figures 7(a) and 7(b) show the factor f versus the node's transmission range when the number of nodes is 200 and 1000, respectively. When the transmission range r increases, the factor decreases because the increase of r results in the decrease of the diameter of the network. Thus, less nodes are needed to cover the confined area.

From the above simulations, we conclude that the proposed EEMPR always outperforms the MPR and the EMPR regardless of the size of the network and the density of the network. Also, the factor of the number of nodes in the CDS to that in the network is more sensitive to the small size of the network than the large one. The results show localized approaches are scalable as the density of the network increases, especially for the EEMPR which has a constant size of local CDS.

VI. Related Work

Essentially, our work is to find a CDS that covers a unit disk graph with local information. The problem of finding a minimum CDS (MCDS) for a general network is proved to be NP-Complete [8]. Even for a unit disk graph, such a problem is also NP-Complete [9]. Therefore, only heuristic algorithms can be applied. Many algorithms that aim to construct CDS's are classified into four groups: global [6], [10], quasi-global [11], quasi-local [12], [13], and local [1], [2], [3], [4], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23].

Some earlier researchers proposed centralized greedy algorithms that use global information to provide *approximation ratio* $O(\ln \Delta)$ to the MCDS [10] for general networks, where Δ is the maximum node degree of the network. Quasi-global CDS algorithms [11] build shortest-path-tree-based CDS structures which provide constant approximation ratio for unit disk graphs. In contrast, quasi-local CDS algorithms construct a CDS by first electing clusterheads [12] or cores [13] and then using selected forward nodes to connect them.

Distributed broadcast algorithms that are based on local neighbor set information can also provide CDS's for a given network. In [3], a generic localized broadcast scheme

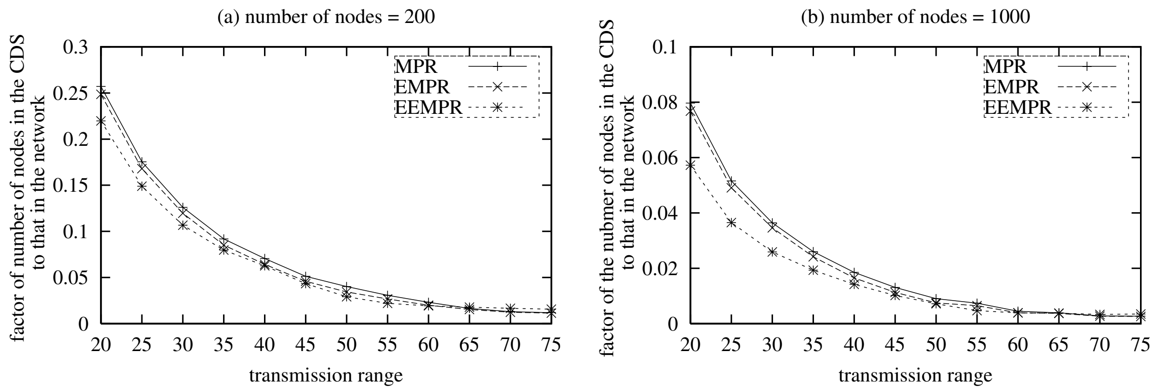


Fig. 7. The factor of the number of nodes in the CDS to that in the network when r is from 20 to 75: (a) n is 200 , and (b) n is 1000.

was proposed where source-independent and source-dependent approaches are uniformed. Many algorithms belong to the source-independent approach, such as MPR [2], EMPR [4], marking process with rules 1&2 [15] and its extensions [16], SPAN [17], and d-hop CDS [18]. Algorithms that belong to the source-dependent approach are the original MPR [1], dominant pruning [14] and its extensions [19], [20], LENWB [21], SBA [22], and neighbor-elimination-based broadcasting [23].

In [3], the distributed broadcast algorithms are also classified into *self-pruning*, *neighbor-designating*, and *hybrid* broadcasting approaches. In self-pruning approaches [15], [16], [17], [18], [21], [22], [23], each node determines its own status and is in the forward status by default. A node resigns its role of forward status by "itself" if a path from the source can be found for each of its neighbors. Nodes in such a path can be either already forwarded nodes or nodes that deem to forward. In the neighbor-designating broadcasting approaches [1], [2], [4], [14], [19], [20], a node determines its neighbor's forward/non-forward status, that is, a node selected by its neighbor updates its local view of neighbor set when it receives a broadcast packet and determines its neighbors' forward/non-forward statuses consequently. The hybrid approaches [3] combine both self-pruning and neighbor-designating methods.

The three algorithms (MPR [2], EMPR [4], and EEMPR) discussed in this paper belong to the source-independent approach; also they are all in the category of neighbor-designating approach.

VII. Conclusions

In this paper, we have proposed an enhanced source-independent MPR based on the recently proposed source-independent MPR. The enhancement is done by using 3-hop neighborhood information to cover each node's 2-hop neighbor set and by extending the notion of coverage in the original MPR. The effectiveness of the enhancement is confirmed through a simulation study on both sparse and dense networks. In this paper, we did not consider energy-aware multiple relays selection. One straightforward extension is to use residue energy level as the selection criteria instead of using node ID. That is, the smallest ID node is replaced by the node with the highest residue energy level. In this case, a node with the highest residue energy in its 1-hop neighborhood has a better chance to become a forward node based on the Enhanced Rule 1. In this way, we can conduct an energy-aware broadcasting [23].

VIII. Acknowledgement

This work was supported in part by NSF grants CCR 0329741, ANI 0073736, and EIA 0130806.

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