# On Maintaining Sensor-Actor Connectivity in Wireless Sensor and Actor Networks 

Jie ${ }^{W}{ }^{\dagger} \dagger$, Shuhui Yang $\ddagger$, and Mihaela Cardei $\dagger$<br>$\dagger$ Department of Computer Science and Engineering<br>Florida Atlantic University, Boca Raton, FL 33431<br>$\ddagger$ Department of Computer Science<br>Rensselaer Polytechnic Institute, Troy, NY 12180


#### Abstract

In wireless sensor and actor networks (WSANs), a group of sensors and actors are connected by a wireless medium to perform distributed sensing and acting tasks. Sensors usually gather information in an event area and pass it on to actors, which are resource-rich devices that make decisions and perform necessary actions. Therefore, it is vital to maintain connections between sensors and actors for effective sensoractor coordination. In this paper, we first define several sensoractor connection requirements, including weak and strong actorconnectivity, and then propose several local solutions that put as many sensors as possible to sleep for energy saving purposes, while meeting different actor-connectivity requirements. We also prove the relationship between the proposed actor-connectivity and the connectivity in regular graphs, which helps with the implementation of the proposed solutions. Comprehensive performance analysis is conducted through simulations.


## I. Introduction

Recent technological advances have lead to the emergence of distributed hybrid sensor networks consisting of both resource-rich sensor devices (called actors) and resourceimpoverished sensor devices (called sensors). It is called a wireless sensor and actor network (WSAN) [1], [8], as shown in Figure 1. In this figure, actors are connected among themselves and to the sink through special channels.

When sensors detect a phenomenon, they either transmit data to actor nodes (also called actuators) which then initiate appropriate actions, or route data to the sink which then issues action commands to actors. We focus on the former approach, where actors are deployed to perform distributed actuation tasks upon the environment. For example, a smoke detector (sensor) reports a fire event to one or several nearest water sprinklers (actors) instead of the distant central control system (sink). The water sprinkler(s) then perform an action and report the event to the central system for further processing.

There are two types of coordinations: actor-actor and actorsensor. We focus on the latter one. The number of actors is relatively small and since they are resource-rich devices with a long transmission range, their connection to the sink can be treated in a relatively easy way [14]. For example, a separate wireless interface can be used to communicate with neighboring actors so they can perform long-range communication without any involvement from the sensors.

Existing actor-sensor coordination focuses on energyefficient connectivity from a sensor to a nearby actor. However,


Fig. 1. A sample data gathering process in a WSAN with circles representing sensors and triangles representing actors.
none of these existing approaches are localized. In a local solution unlike the traditional distributed solutions, a decision at each node is purely based on local information and there is no information propagation. In this paper, we use a different approach to construct a self-organizing framework for data routing from sensors to actors. We first give a formal graph model for WSANs. We propose several local solutions for maintaining different versions of sensor-actor connectivity by putting as many sensors to sleep as possible, while still considering area coverage and fault-tolerance. In these solutions, only neighborhood information (neighbor set) is required, and location/distance information is not used. In addition, other issues such as sensor energy efficiency and delay sensitivity of individual routing paths are discussed. More specifically, instead of finding efficient routes from sensors to actors in the entire network, we try to reduce the routing space by putting as many sensors to sleep as possible to limit the energy consumption subject to the following two requirements:

- Coverage: each sleeping sensor has at least one neighbor that is either an active sensor or an actor.
- Connectivity: each active sensor is still connected to the same set of actors as it was before sensors were put to sleep (called persistent actor-connectivity) or to at least one actor (called at-least-one actor-connectivity).
The coverage requirement is used to ensure the coverage of all the sensors which are discrete points. This point coverage can approximate the area coverage, especially when sensors are densely deployed [3]. The connectivity requirement ensures that information collected by any active sensor can be
delivered to at least one actor in at-least-one actor-connectivity. In certain situations, connection to one actor is not sufficient. Multiple actors should be informed in order to make decisions regarding the most appropriate way to perform actions.

A sufficient degree of connectivity is needed to protect against the loss of sensors and actors due to failure. To ensure a certain degree of fault-tolerance, the network should still meet the coverage and connectivity requirement after removing $k-1$ arbitrary nodes (sleep sensors, active sensors, or actors). This property is called $k$-actor-connectivity. In addition, more active sensors and a higher connectivity degree help to find a more efficient route in terms of delay.

We propose several local solutions that put as many sensors to sleep as possible while meeting different coverage and connection requirements. Note that we try to minimize the energy consumption in a single iteration. Network activity is organized as a sequence of iterations, where a new schedule is decided at the beginning of each iteration. Sensors can be scheduled to work in different iterations to balance energy consumption and to prolong network lifetime. In this paper, we do not deal with the actual routing protocol which can be designed on the active sensors derived from our methods.

In summary, we will focus on the following technical issues:

1) We give a formal graph model for WSANs and define several sensor-actor connections based on the coordination requirement.
2) We develop two local solutions for the different versions of sensor-actor connectivity.
3) We prove the relationship between the traditional connectivity in the graph and the newly-defined sensor-actor connectivity.
4) We extend the sensor-actor connectivity and the corresponding local solutions for fault-tolerant consideration.
5) We conduct performance analysis through simulations on the proposed algorithms.

## II. Model

A WSAN is represented as an undirected graph $G=(V, E)$. $V=S \cup A$, where $S$ is the sensor set and $A$ is the actor set. $E \subset(S \times S) \cup(S \times A)$ is the edge set for sensor-sensor and sensor-actor connections. There is no direct connection between any two actors. They are connected indirectly through other means (such as special channels).

Figure 2 shows several sample WSANs. Each sensor in $G_{1}$ is connected to one actor while each sensor in $G_{2}$ and $G_{3}$ is connected to two actors. A graph $G$ is actor-connected if each sensor is connected to an actor through nodes in $G$. Note that an actor-connected WSAN does not imply that the whole graph is connected. For example, $G=G_{2} \cup G_{3}$ in Figure 2 is not connected, although it is actor-connected. Now suppose a subset $S^{\prime}$ of $S$ is put to sleep (for energy saving). We denote $G^{\prime}=G\left[V-S^{\prime}\right]$, i.e., the network after removing $S^{\prime}$.

Definition 1: Given an actor-connected network $G$ :

- $G^{\prime}$ is persistent actor-connected if it maintains the same actor-connectivity as $G$, i.e., if a sensor, sleeping or active, is connected to an actor through nodes in $G$, then it is still connected to the actor through nodes in $G^{\prime}$.


Fig. 2. Sample WSANs.

- $G^{\prime}$ is at-least-one actor-connected if each sensor, sleeping or active in $G$, is connected to at least one actor through nodes in $G^{\prime}$.
Note that all of the above conditions imply coverage of sleeping nodes. That is, each sleeping node has at least one neighbor that is an active sensor or an actor. Suppose sensors are densely deployed such that the given area is fully covered by sensors and actors. We assume each actor also has sensing capability with the same sensing range as a sensor. Due to the coverage requirement, each sleeping node has at least one active neighbor to cover it. Therefore, the set of active sensors and actors still cover the whole area approximately. Connectivity varies depending on the required degree: persistent or at-least-one.

In Figure $2 G_{2}$, if $S^{\prime}=\left\{s_{4}\right\}, G_{2}^{\prime}$ is persistent actorconnected since all sensors are still connected (through active nodes) to both actors. If $S^{\prime}=\left\{s_{1}, s_{2}, s_{4}\right\}, G_{2}^{\prime}$ is no longer persistent but it is at-least-one actor-connected, since all sensors are connected to at least one actor (e.g., $s_{4}$ to $a_{2}$ via active sensor $s_{3}$ ). Note that traditional clustering approaches [10] can be used to meet the coverage requirement, where only clusterheads are active. Such approaches, however, are not localized (there is information propagation).

## III. Proposed Methods

A local algorithm [16] relies only on local information, i.e., properties of nodes within its vicinity. In addition, there is no sequential propagation of any partial computation result. The status of each node depends on its $h$-hop topology only for a small constant $h$, and is usually determined after $h$ rounds of information exchange among neighbors. Compared with a global algorithm, the local algorithm consumes less overhead, but may not achieve the optimal result. Let us assume that each node is equipped with its $h$-hop neighborhood information (for $h=2$ or 3 ). Also, each node $s$ has a priority $p(s)$ and such a priority is totally ordered within its $h$-hop neighborhood, which could be the node ID, node degree, or energy level based on different applications. Note that we can use node ID or node energy level or other parameters of the nodes as their priorities for different applications. In addition, all actors have the same priority, which is higher than any sensor priority. Let $p(a)$ be the actor priority. In Figure $2 G_{3}$, 1-hop neighborhood of $s_{1}$ includes $a_{1}, s_{2}$, and $s_{3}$, but no connections among 1-hop neighbors, the edge between $a_{1}$ and $s_{3}$ or the one between $s_{3}$ and $s_{2}$. 2-hop neighborhood of $s_{2}$ cover the whole network.

Let us now consider a WSAN that is initially actorconnected. With local information only, how can we remove
some sensors (i.e., put them to sleep) while ensuring that the resultant graph is still at-least-one or persistent actorconnected? We propose the following two rules:

Local rule for persistent actor-connectivity: The default status of a sensor is active. A sensor $u$ is in sleep mode if, for any two of its neighbors $w$ and $v, w$ and $v$ are connected by a path with all intermediate nodes (sensors or actors if any) having higher priorities than $u$.

The above path is called a replacement path for node $u$. The intuition behind this rule is that a sensor $u$ can be put to sleep if any two neighbors can be re-connected through nodes on a replacement path. Note that nodes on a replacement path can also be put to sleep. To avoid inconsistencies and a possible iterative process of putting sensors to sleep, a global priority is defined on each node. Note that if a sensor does not have two neighbors, then the replacement path condition is satisfied and the status of the sensor is sleeping. The neighbor could be a sensor or an actor. Suppose that in the Figure 2 the priority of sensors is the following: $p\left(a_{1}\right)=p\left(a_{2}\right)>p\left(s_{1}\right)>p\left(s_{2}\right)>$ $p\left(s_{3}\right)>p\left(s_{4}\right)$. Using 2-hop neighborhood information, $s_{1}$ and $s_{3}$ are put to sleep in $G_{1}$ in persistent actor-connectivity; $s_{4}$ is in sleeping in $G_{2}$ and $s_{3}$ and $s_{4}$ are in sleeping in $G_{3}$.

Suppose $S^{\prime}$ is the set of sleeping sensors and $G^{\prime}$ is the subgraph after removing $S^{\prime} . V^{\prime}$ is the vertex set of $G^{\prime}$.

Theorem 1: Suppose $S^{\prime}$ is the set of sleeping sensors after applying the local rule for persistent actor-connectivity.

- (Coverage) Each sensor in $S^{\prime}$ has a neighbor in $V^{\prime}$.
- (Connectivity) $G^{\prime}$ has the same actor-connectivity as $G$.

Proof: Suppose $S(a)$ is a subset of $S$ connected to actor $a$ in $G$. We show that $S(a)$ is still connected to $a$ through nodes in $G^{\prime}$. We prove this by contradiction. Suppose $W$ is a subset of $S(a)$ not connected to $a$. Note that nodes in $W$ can be sleeping or active. Let $U=N(W)-W$ be the sleeping neighbors of $W$ (see Figure 3) that are connected to $a . U \neq$ 0 since $W$ is connected to $a$ in $G$. Let $u$ be the node in $U$ with the highest priority. From the assumption, $u$ has two neighbors, $w$ and $v$, from $W$ and $V-U-W$, respectively. Any replacement path for $u$ must contain at least one node $u^{\prime} \in U$. That contradicts the assumption that $p(u)>p\left(u^{\prime}\right)$. Therefore, all nodes in $S(a)$ are still connected to $a$ and all sleeping nodes in $W$ have neighbors that are active sensors or actors.

To provide a local rule for at-least-one actor-connectivity, we define an extended replacement path as follows:

1) it is regular replacement path for $u$ connecting two neighbors $w$ and $v$, or
2) $w$ and $v$ are each connected to an actor. These two actors can be distinct and all intermediate nodes in these two connections have higher priorities than $u$.

Local rule for at-least-one actor-connectivity: The default status of a sensor is active. A sensor $u$ is in sleep mode if for any two of its neighbors $w$ and $v$, an extended replacement path for $u$ connecting $w$ and $v$ exists.


Fig. 3. Illustration for the proof of Theorems 1 and 2.

The intuition behind the above rule is that sensor $u$ can be put to sleep as long as any two neighbors can be either connected through a regular replacement path or each of them is connected to an actor. In Figure 2, using 2-hop neighborhood information, $s_{1}$ and $s_{3}$ are asleep in $G_{1}$ for at-least-one actorconnectivity; $s_{1}, s_{2}$, and $s_{4}$ are asleep in $G_{2}$ and all sensors are asleep in $G_{3}$.

Theorem 2: Suppose $S^{\prime}$ is the set of sleeping sensors after applying the local rule for at-least-one actor-connectivity.

- (Coverage) Each sensor in $S^{\prime}$ has a neighbor in $V^{\prime}$.
- (Connectivity) Each node in $G^{\prime}$ is connected to at least one actor.
Proof: We use a similar proof as in Theorem 1. In our model, each node in $S$ is connected to at least one actor. We show that each node in $S$ is still connected to an actor through nodes in $G^{\prime}$. We prove by contradiction. Suppose $W$ is the subset of $S$ not connected to any actor. Let $U=N(W)-W$ be the sleep neighbors of $W$ that are connected to an actor. Let $u$ be the node in $U$ with the highest priority. From the assumption, $u$ has two neighbors, $w$ and $v$, from $W$ and $V-$ $U-W$, respectively; Any replacement path for $u$ must contain at least one node $u^{\prime} \in U$. Such a replacement path connects $w$ via $u^{\prime}$ to either $v$ or an actor directly as shown in Figure 3. That contradicts the assumption that $p(u)>p\left(u^{\prime}\right)$. Therefore, all nodes in $S$ are connected to actors and all sleeping nodes in $W$ have neighbors that are active sensors or actors.


## IV. Extensions

In this section we introduce two new notions of connectivity.
Definition 2: A WSAN, G, is called weak $k$-actor-connected if each sensor is connected to $k$ actors. A WSAN, $G$, is called strong $k$-actor-connected if each sensor is connected to at least one actor after removing any $k-1$ nodes (sensors or actors) from $G$.

Based on Definition 2, strong $k$-actor-connectivity implies weak $k$-actor-connectivity, i.e., connection to $k$ actors. The $k$-actor-connectivity is used for reliability. Weak $k$-actorconnectivity can tolerate $k-1$ actor failures, while strong $k$-actor-connectivity can tolerate $k-1$ failures of any nodes, sensors and actors. Figure 2 shows several sample WSANs. $G_{1}$ is 1-actor-connected, although each node has two nodedisjoint paths to actor $a_{1} . G_{2}$ is weak 2 -actor-connected but


Fig. 4. A $k$-connected graph $G \cup \bar{G}$.
not strong 2-actor-connected. $G_{3}$ is strong 2-actor-connected. To simplify the notation, we use $k$-actor-connected for strong $k$-actor-connected.

We now consider maintaining $k$-actor-connectivity while putting some sensors to sleep. Let $G$ be a $k$-actor-connected network and $G^{\prime}=G\left[V-S^{\prime}\right]$ ( $S^{\prime}$ is a set of sleeping sensors).

Definition 3: Given a $k$-actor-connected network $G$,

- $G^{\prime}$ is persistent $k$-actor-connected if it maintains the same actor-connectivity as $G$ after removing any $k-1$ nodes (sensors or actors).
- $G^{\prime}$ is at-least-one $k$-actor-connected if each sensor, sleeping or active, in $G$ is connected to at least one actor through nodes in $G^{\prime}$ after removing any $k-1$ nodes.

Here, "the same actor-connectivity" means that if a sensor in $G$ is connected to an actor through nodes in $G$, then this sensor (which might be sleeping) is still connected to the actor through nodes in $G^{\prime}$. At-least-one $k$-actor-connected is the regular $k$-actor-connected (and hence it is simply called $k$ -actor-connected), while persistent $k$-actor-connected requires a stronger condition. Here we use the general case of removing $k-1$ nodes, which includes both sensors and actors. In this case, the persistent connectivity means the existence of a path from $G^{\prime}$ to a previously connected actor (before the removal of $k-1$ nodes) even if that actor has been removed. Next, we give local rules that ensure $k$-actor-connectivity. Two paths are called node-disjoint if they do not share any intermediate nodes.

Local rule for persistent $k$-actor-connectivity: The default status of a sensor is active. A sensor $u$ is in sleep mode if for any two of its neighbors $w$ and $v, k$ node-disjoint replacement paths for $u$ connecting $w$ and $v$ exist.

The at-least-one version of $k$-actor-connectivity uses the extended replacement path, a less restrictive condition than the regular replacement path.

Local rule for at-least-one $k$-actor-connectivity: The default status of a sensor is active. A sensor $u$ is in sleep mode if for any two of its neighbors $w$ and $v, k$ node-disjoint extended replacement paths for $u$ connecting $w$ and $v$ exist.

Note that both $w$ and $v$ can be actors and in this case, there is no intermediate node. Also, the actor cannot be shared in two extended replacement paths. In Figure $2 G_{3}, s_{4}$ is put to sleep based on local rules for persistent 2-actor-connectivity. That is, even if a node is removed arbitrarily from $G_{3}$, all sensors are still connected to both actors. For example, when $s_{3}$ is removed, $s_{4}$ is connected to $a_{1}$ via $a_{2}$. Sensors $s_{3}$ and $s_{4}$ are put to sleep from $G_{3}$ for 2-actor-connectivity. In this case, $s_{1}$ and $s_{2}$ in $G_{3}$ are both 2-actor-connected after making $s_{3}$ and $s_{4}$ sleep. $s_{3}$ is put to sleep by checking all neighbor pairs, for which each has 2 node-disjoint paths. For example, for neighbors $s_{1}$ and $s_{2}$ of $s_{3}$, one path is from $s_{1}$ to $s_{2}$ and the other is from $s_{1}$ to $a_{1}$ and from $s_{2}$ to $a_{2}$.

## V. Properties

We first construct a $k$-connected graph by treating all actors in $A$ as regular nodes. These actors are connected by a complete bipartite graph $\bar{G}$ with node set $A \cup A^{\prime}$, where $|A|=\left|A^{\prime}\right|$ and each node in $A$ is connected to each node in $A^{\prime}$. There is no direct connection among nodes in $A$ (and among nodes in $A^{\prime}$ ). Note that $A^{\prime}$ is a set of virtual nodes. Now we first show that $G \cup \bar{G}$ (shown in Figure 4) is $k$-connected.

Theorem 3: If $G$ is $k$-actor-connected, then $G \cup \bar{G}$ is $k$ connected.

Proof: Based on the definition of $k$-actor-connectivity, we can see that $|A| \geq k$. We arbitrarily select two nodes $s$ and $d$ from $G \cup \bar{G}$, and we have the following three cases:

1) If both $s$ and $d$ are in $A \cup A^{\prime}$, they are clearly connected after removing $k-1$ nodes, since $\bar{G}$ is a complete bipartite graph with $|A|=\left|A^{\prime}\right| \geq k$.
2) If one is in $S$ and the other in $A \cup A^{\prime}$, based on the definition of $k$-actor-connectivity, the one in $S$ is connected to at least one node in $A$ after removing $k-1$ nodes from $G \cup \bar{G}$, which in turn is connected to any node in $A \cup A^{\prime}$, including $d$.
3) If both are in $S$, suppose one is connected to a node $a$ in $A$ and the other is connected to a node $a^{\prime}$ in $A$ after removing any $k-1$ nodes from $G \cup \bar{G}$. Based on the construction of $\bar{G}$, nodes $a$ and $a^{\prime}$ are still connected.
Therefore, based on the definition, $G \cup \bar{G}$ is $k$-connected.

Now we show that the two local rules in the previous section preserve $k$-actor connectivity.

Theorem 4: Given a $k$-actor-connected graph $G$, the graph $G^{\prime}$ derived using the local rule for (persistent) $k$-actorconnectivity is (persistent) $k$-actor-connected.

Proof: Suppose $G$ is the original $k$-actor-connected graph. Now we arbitrarily remove $k-1$ nodes from $G$ and obtain $G^{F}$. By relating $k$-actor-connectivity to $k$-connectivity (Theorem 3) to generate $G^{T} \cup \bar{G}^{T}$ as in Figure 4, and then apply the Menger's theorem [7] on connectivity to $G^{T} \cup \bar{G}^{T}$, it is easy to derive that $G^{F}$ still preserves the same actor-connectivity as $G$. Based on these two rules for $k$-actor-connectivity, each sleeping node in $G$ has $k$ node-disjoint replacement or extended replacement paths for any pair of neighbors. Removing any
$k-1$ nodes will leave at least one replacement or extended replacement path. That is, a sleeping node using rules for $k$ -actor-connectivity in $G$ will still be a sleeping node using corresponding rules for actor-connectivity in $G^{F}$. That is, $G^{\prime}$ (obtained by applying local rules for $k$-actor-connectivity on $G$ ) has at least the same degree of actor-connectivity as $\left(G^{F}\right)^{\prime}$ (obtained by applying the corresponding local rules for actorconnectivity on $G^{F}$ ). The rest of the proof follows by applying Theorems 1 and 2 to $\left(G^{F}\right)^{\prime}$, which shows the relevant actorconnectivity.

The following are two more properties: one relates $k$-actorconnectivity to node-disjoint paths to $k$ actors, and the other to node-disjoint paths to actors after applying local rules.

Property 1: If $G$ is $k$-actor-connected, then each node in $S$ has node-disjoint paths to at least $k$ nodes in $A$.

We can use the following argument to prove this property. Suppose we have a sensor $s$ in $S$ and the other node $d$ in $A^{\prime}$. Menger's theorem states that in a $k$-connected graph there are $k$ node-disjoint paths between any two nodes. Using this property, we have $k$ node-disjoint paths between $s$ and $d$. Among these paths, all neighbors of $d$ are distinct actors in $A$. Therefore, any node in $S$ has node-disjoint paths to at least $k$ distinct actors.
Suppose $S^{\prime}$ is the subset of $S$ that is removed (put to sleep) after applying the local rule for (persistent) $k$-actorconnectivity and again $G^{\prime}=G\left[V-S^{\prime}\right]$. We have:

Property 2: $G^{\prime} \cup \bar{G}$ is still $k$-connected, and each node in $S-S^{\prime}$ has node-disjoint paths to at least $k$ nodes in $A$.

Based on Theorem 4, the local rule for (persistent) $k$-actorconnectivity ensures that $G^{\prime}$ is still $k$-actor-connected. Based on Theorem 3, we have $G^{\prime} \cup \bar{G}$ as a $k$-connected graph. The second part follows directly from Theorem V. Therefore, the above property holds.

## VI. Implementation Issues

## A. Selection of priority

We assume that node priorities within $h$-hops are distinct. In the actual implementation, this condition can be relaxed. That is, nodes within $h$-hops can have the same priority. This will not cause errors because a node can go to sleep only if any two neighbors are connected by other $k$ paths with "higher" priorities. However, it will affect the efficiency of the algorithm, e.g., two nodes can cover each other's neighbors, but neither can go to sleep due to their identical priority. A natural choice for node priority is node ID, although other metrics can be used, such as energy level, where sensors can rotate their roles (active/sleeping) to balance energy consumption.

## B. Controlling the path length

In some real-time applications, it is vital for a detecting event to reach the corresponding actor(s) within certain time frames. To avoid generating a large path ratio, defined as the ratio of the length of a routing path in $G^{\prime}$ to that in $G$, we can restrict the length of each replacement path for each sleeping sensor. For example, we can set each replacement path to be
bounded by $h$ hops, then globally, the shortest path "stretch" of each sensor to an actor can be controlled.

## C. Static vs. dynamic implementation

Local rules can be implemented in a static or dynamic way. In static implementation, each node determines its status based on its $h$-hop information. In dynamic implementation, each node acts on a message originated from an actor. In such a message, the actor ID or even path information from the original actor to the current node can be piggybacked to assist the status determination of each node. The actor ID indicates the connectivity of a neighbor to a particular actor, even though the actor might be outside the $h$-hop neighborhood. Likewise, path information to an actor can be used for the local rule for $k$-actor-connectivity. Efficient reduction of active nodes is possible by judiciously selecting an appropriate time-out after receiving the first message at each sensor to gather more path information from actors.

If we allow propagation of node status, an active node can be treated as an actor which is useful for at-least-one actorconnectivity. Note that dynamic implementation resembles distributed implementation, which has several simple implementations for at-least-one and persistent actor-connectivity.

## D. Actor-initiated dynamic implementation

We use actor-initiated dynamic implementation for two simple cases: at-least-one and persistent actor-connectivity. The direct distributed implementation for at-least-one and persistent $k$-actor-connectivity are much more involved, since path information needs to be propagated. All dynamic implementations are not strictly local solutions with information propagation. However, they are used as baseline cases for comparison.

At-least-one actor-connectivity: (1) Each actor sends out an invitation message. (2) Each sensor responds to the first invitation only and forwards the invitation to its neighbors. (3) Sensors receiving responses are active, and sensors not receiving responses are put to sleep.

Although at-least-one actor-connectivity is simple, it does need some form of information propagation (in this case, an invitation). The number of invitation messages is equivalent to the number of sensors. The distributed implementation for persistent actor-connectivity is much more involved in terms of message complexity: it is the total number of actor-sensor connectivity.

Persistent actor-connectivity: (1) Each actor sends out an invitation message with its ID. (2) Each sensor responds to the first invitation for each ID and forwards the invitation to its neighbors. (3) Sensors receiving responses are active and sensors not receiving responses are put to sleep.

Maintaining $k$-actor-connectivity is much more involved since each active sensor needs to ensure the existence of node-disjoint paths to $k$ distinct actors. The complete path information needs to be propagated in the network, generating excessive traffic. Hence, we will not discuss this further.


Fig. 5. Examples of AID-per, AID-one, LR-per, and LR-one.

## VII. Related Work

The traditional WSNs usually contain only a single sink and perform the sensing in a distributed way. However, the management is centralized at the sink. WSANs contain actors in addition to a sink and perform both distributed sensing and management. WSANs [1] can be used as an integral part of some novel, low-cost, high-performance systems, and can provide the infrastructure of various applications [1]. In [11], it is shown that in a WSAN, the network capacity is increased compared with the pure ad hoc or sensor networks.

Energy-efficient routing protocols are a major research issue for energy constrained WSNs. WSANs have two unique coordinations compared with WSNs [1]: actor-actor and actorsensor coordinations. Therefore, routing protocols designed for WSANs should be both energy-efficient and coordinationsensitive. Additionally, the actor-related distributed coordi-

TABLE I
Simulation parameters.

| Network Area | $100 \times 100$ |
| :--- | :--- |
| Transmission Range | 25 |
| Node Degree | 12 |
| Number of Sensors | $n, 50$ to 300 |
| Number of Actors | $m, 2$ to 8 |
| Number of Hops | $h, 2$ to 4 |
| Connectivity Requirement | $k, 1$ to 6 |
| Number of Trials | 100 |

nation raises a new research issue. Most of the existing works focus on the design of a self-organizing framework for connecting sensors and actors. The existing solutions under this framework, however, are distributed but not localized.
Some approaches [12], [13], [14] construct tree-structures rooted at each actor in a distributed way. They can be viewed as a many (sensor)-to-one (actor) connection. The treestructure rooted at each sensor [9], [15] is also developed, which forms the many-to-many connection. Some other issues in WSANs are also discussed. For example, [5], [8] discuss control engineering problems and existing technologies in WSANs. In [6], [13], actor-actor coordination is addressed. [2] solves the topology control problem in WSANs considering both energy-efficiency and reliability.

The work proposed in this paper aims at minimizing the entire routing space instead of finding the exact routes from sensors to actors. Our work differs from the other works by considering a qualified minimal forwarding set for all the sensors, which meets the efficiency and reliability requirements. Note that in our paper we measure the routing energy consumption in terms of hop counts as opposed to distance. In the proposed algorithms, neither location nor distance information is needed. Only neighborhood information by exchanging "Hello" messages is necessary.
$k$-connectivity has also been studied in the context of connected dominating set (CDS) which meets both the coverage and connectivity conditions in this paper. [17] studies the $k$ connected coverage set problem. The objective is to find a small subset $C$ of the node set $V$, such that (1) each node in $V$ is dominated (i.e., covered) by at least $k$ different nodes in $C$, and (2) the nodes in $C$ are connected. [4] extends the result of [17] by considering an extra connectivity condition: nodes in $C$ are still connected after removing any $k-1$ nodes in $C$. Three localized algorithms are proposed in [4] based on a deterministic, a probabilistic, and a hybrid of deterministic and probabilistic approaches. The model considered in this paper further extends the notion of coverage and presents two types of connectivity preservation (to actors).

## VIII. Simulations

We evaluate the two proposed algorithms, Local Rule for persistent $k$-actor-connectivity (LR-per) and Local Rule for at-least-one $k$-actor-connectivity (LR-one) with different system parameters. We also simulate the Actor-Initiated Dynamic implementation for persistent actor-connectivity (AID-per) and at-least-one actor-connectivity (AID-one) to compare with the proposed local algorithms.


Fig. 6. Comparison of AID-per, AID-one, LR-per, and LR-one $(k=1)$.

## A. Simulation Environment

All simulations are conducted in randomly generated, static networks. To generate a network, $n$ sensors and $m$ actors are randomly placed in a $100 \times 100$ area. Networks that cannot form a $k$-actor-connected graph are discarded. The tunable parameters are as follows. (1) The number of sensors $n$. We vary $n$ from 50 to 300 to check the scalability of the algorithms. (2) The number of actors $m$. We vary $m$ from 2 to 8 . (3) The connectivity requirement $k$. We use 1 to 6 as the value of $k$. In each simulation, $k \leq m$. (4) The number of hops, $h$. The local algorithms use 2-hop neighborhood information in most of the simulations. We also increase $h$ to 3 and 4 to see the effect. When a node collects $h$ hops worth of information, it gets the network topology within its $h$-hops neighborhood except for the links between any two $h$-hop away nodes. Table I lists the simulation parameters. (5) The transmission range $r$. We use 25 as the value of $r$ in most of the simulations. (6) The average node degree $d$. We fix $d$ to 12 in the last simulation for the sensitivity analysis.

The following performance metrics are evaluated. (1) Active nodes. The number of active sensors, which represents the energy consumption. (2) Path ratio. The ratio of the average length of the routing paths in $G^{\prime}$ to that in $G$, which represents the routing latency of the system.

Figure 5 shows the selected active node set in a sample network. There are $n=30$ sensors (shown as circles) and $m=4$ actors (triangles), the active sensors are shown by bold circles, and the numbers in the sensors are the IDs. (a) and (b) are the results of AID-per and AID-one, with $k=1$. There are 28 and 9 active nodes, respectively. (c) and (d) are of LR-per and LR-one when $k=1$. There are 14 and 9 active nodes, respectively. (e) and (f) are of LR-per and LR-one when $k=2$. There are 26 and 21 active nodes, respectively.

(a) Active nodes in LR-per

(c) Path ratio in LR-per
(b) Active nodes in LR-one

(d) Path ratio in LR-one

Fig. 7. Performance of LR-per/LR-one with different $k(m=6, h=2)$.

## B. Simulation Results

Figure 6 shows the comparison of AID and LR $(k=1)$. (a) and (c) are the results of the number of active nodes and ratio of length of the routing paths with $m=2$, respectively. (b) and (d) show the results for $m=6$. In (a) we can see that methods for persistent connectivity have larger numbers of active nodes than those for at-least-one connectivity. AID-per selects more active nodes than LR-per. AID-one has less active nodes than LR-one only when the number of sensors is very small (smaller than 75). In (b), when $m=6$, the comparison results of the four algorithms remain the same with those in (a). However, AID-per has more active nodes with larger $m$, while the other three tend to have fewer active nodes. This is because in AID-per, each node needs to keep a shortest path to every actor with all the nodes on the path being active. In AID-one, a shortest path to an actor is needed for each node, and more actors help to reduce the length of this shortest path. Thus, fewer active nodes are necessary. For the LR-per and LR-one, since actors are viewed as nodes with the highest priorities, more actors lead to higher probability of (extended) replacement paths and hence of non-active nodes. Therefore, fewer active nodes are selected. In (a) and (b), more sensor nodes lead to an increased number of active nodes. However, the increasing tends to stop when the node density reaches a certain degree in AID-one, LR-per, and LR-one.
(c) and (d) are the ratio of the length of the routing path in original graph to that in the resultant graph (via only active nodes). The routing path is the shortest path from a sensor to a nearest actor, and the length is in terms of hop count. Since AID-per and AID-one always keep the nodes on the shortest path from a sensor to a nearest actor, the ratio is always 1. LR-per has smaller ratio than LR-one due to its larger active node set. Comparing (c) with (d) we can see that a larger $m$ results in a smaller ratio. Both in (c) and (d), more deployed


Fig. 8. Percentages of active nodes and the increasing of path ratio with different $k(n=300, m=6, h=2)$.
sensor nodes lead to an increased ratio. The increasing tends to stop when the node density reaches a certain degree.
Figure 7 shows the performance analysis of LR-per and LRone in terms of parameter $k(m=6)$. (a) and (b) are the sizes of the resultant active node sets of LR-per and LR-one. (c) and (d) are their corresponding path ratios. $k$ is increased from 1 to 6 in these figures. We can see that when $k$ is larger, more active nodes are necessary to achieve higher connectivity. However, compared with that of the deployed sensors, the increasing of the number of active nodes is slight. Also, LR-one needs fewer active nodes than LR-per, which is consistent with the previous simulation results. (c) and (d) show that a larger $k$ helps with a smaller path ratio due to more active nodes. However, this decrease of path ratio tends to stop when $k$ is large enough. We can see that when $k$ is 6 , the ratio is slightly larger than 1 (less than 1.05). There is little room to further reduce the ratio by increasing $k$. When $n$ is 50 , the original graph is hardly 6 -actor-connected, thus there are no simulation results in the figures when $n$ is 50 and $k$ is 6 .
Figure 8 is generated from Figure 7 to show the percentages of active nodes and the increasing of path ratio in LR-per and LR-one with the increasing of $k$. We can see that when the value of $k$ decreases, fewer active nodes are needed in both LR-per and LR-one, and the path ratios are increased as well. However, the increasing of the path ratio is insignificant compared to the decreasing of the number of active nodes until $k$ is small enough, that is, $k$ is decreased to 1 . Therefore, most of the time, the decrease of the number of the active nodes will not lead to the significant increase of the path ratio.
Figure 9 shows the results of LR-per and LR-one with different numbers of actors where $k$ is fixed as 2. (a) and (c) are the number of active nodes and path ratio in LR-per and (b) and (d) are those of LR-one. From (a) and (b) we can see that a larger $m$ results in a smaller number of active nodes. A decreasing of the number of active nodes caused by the increasing of the value of $m$ is more significant in LR-one than in LR-per. This is because in LR-per, although more actors do provide higher probability for (extended) replacement path and hence non-active node status, it also leads to the requirement of increased connectivity. (c) and (d) show that more actors result in a smaller path ratio. The path ratio of LR-per is smaller than that of LR-one due to its larger size of active nodes.
Figure 10 shows the performance of LR-per and LR-one

(a) Active nodes in LR-per

(c) Path ratio in LR-per

(d) Path ratio in LR-one

Fig. 9. Performance of LR-per/LR-one with different $m(k=2, h=2)$.
with different $h$. (a) and (b) are the numbers of active nodes in these two algorithms, and (c) and (d) are the path ratios of them. We can see that with more neighborhood information, fewer active nodes are necessary in both algorithms. However, this increase is relatively slight. When $h$ is 4 , a node can achieve almost the entire network topology, the performance can not be increased significantly. Therefore, in application a small value of $h$ is sufficient. The path ratios are decreased with the growth of the value of $h$ as in (c) and (d). However, the increase of the performance is not significant.
Figure 11 is for the sensitivity analysis. When the average node degree of the network is fixed, more deployed nodes lead to a smaller adjusted transmission ranges. Therefore, when the number of nodes increases, we can see the scalability performance of the algorithms. (a) and (b) are the selected active nodes in LR-per and LR-one, and (c) and (d) are the path ratios of them. We can see that the number of active nodes increases as the network scale grows in both LR-per and LR-one. However, the path ratios of them are relatively stable.
Simulation results can be summarized as follows:

1) LR-per has fewer active nodes than AID-per; AID-one has smaller number of active nodes than LR-one. But AID-per and AID-one are not localized.
2) Although the path ratios of LR-per and LR-one is not 1 (as in AID-per and AID-one), they are not significantly higher than 1 , and can be controlled by the value of $m$.
3) When $m$ is fixed, larger $k$ leads to larger active node set and smaller path ratio in LR-per and LR-one.
4) When $k$ is fixed, larger $m$ helps to reduce the number of active nodes and also path ratio. However, in LR-per the decreasing of number of active nodes by the increasing of the value of $m$ is insignificant.
5) The increase of the length of routing path in both LR-per and LR-one is insignificantly. Selecting only a subset of


Fig. 10. Performance of LR-per/LR-one with different $h(k=2, m=2)$.
nodes to be active introduces little data routing latency.
6) More local neighborhood information results in better performance in terms of both the number of active nodes and path ratio for both algorithms. However, a relatively small value, say 3 , of $h$ is enough to avoid overhead.
7) Under all circumstances, with the growth of the number of deployed nodes, the number of active nodes increases, but the path ratio tends to be stable. Therefore, all the proposed algorithms scale well.

## IX. Conclusion

We defined several sensor-actor connection requirements in wireless sensor and actor networks, proposed several local solutions to ensure different connection requirements, where each node makes its decision (regarding its active and sleeping mode) purely based on local information, and there is no information propagation during the decision process. We looked at several fault-tolerance extensions, the persistent and at-least-one $k$-actor connectivity, in which the network connectivity is still ensured in the presence of sensor or actor failures. We also proved the relationship between the regular $k$ vertex connectivity in graph theory and the proposed $k$-actor connectivity in the WSANs. Simulation results showed that LR-per and LR-one can both generate an efficient active node set, saving energy consumption by putting other nodes into sleep state without introducing much routing delay.

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