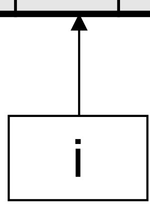
Building Java Programs
Chapter 13

Searching and Sorting

Sequential search

- **sequential search**: Locates a target value in an array/list by examining each element from start to finish.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

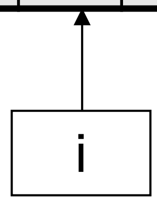
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



  - Notice that the array is sorted. Could we take advantage of this?

Sequential search

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

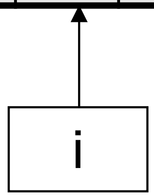



```
search(A[], thingToFind)
    start at the beginning
    for each item in A:
        if the item is what we're looking for:
            return its location

    if we got this far without returning already,
    what we're looking for isn't found
    return failure
```

Sequential search

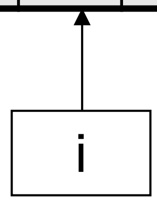
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103




```
public static int search(int A[], int thingToFind) {
     for (int i=0; i<A.length; i++) {
          if (A[i]==thingToFind) {
               return i;
          }
     }
     /* we didn't find it. return failure */
     return -1;
}
```

When Array Is Sorted

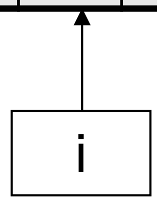
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



- This array is already sorted
- Do we really need to go through entire thing before quitting?
  - Suppose we’re searching for 38:
    - we know once we’ve reached A[10] that we didn’t find it

When Array Is Sorted

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

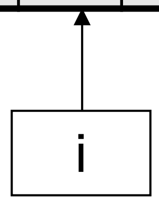



```
/* version 2. ONLY WORKS if A[] IS SORTED */
search(A[], thingToFind)
    start at the beginning
    for each item in A <= thingToFind:
        if the item is what we're looking for:
            return its location

    if we got this far without returning already,
    what we're looking for isn't found.
    return failure
```

When Array Is Sorted

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103




```
/* version 2. ONLY WORKS if A[] IS SORTED */
public static int search(int A[], int thingToFind) {
    for (int i=0; i<A.length && A[i]<=thingToFind; i++) {
      if (A[i]==thingToFind) {
          return i;
      }
    }
    return -1;
}
```

Searching a Sorted List

- What if we're ordering a pizza (and it's 1998)?
- Looking for phone number of Sammy's Pizza
- Start with the A's, then the B's, *etc*.?

Binary search (13.1)

- **binary search**: Locates a target value in a *sorted* array/list by successively eliminating half of the array from consideration.
  - How many elements will it need to examine?
  - Example: Searching the array below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

min (index 0) mid (index 8) max (index 16)

The `Arrays` class

- Class `Arrays` in `java.util` has many useful array methods:

Method name	Description
`binarySearch(`**array**`, `**value**`)`	returns the index of the given value in a *sorted* array (or < 0 if not found)
`binarySearch(`**array**`,` **minIndex**`, `**maxIndex**`, `**value**`)`	returns index of given value in a *sorted* array between indexes *min* / *max* - 1 (< 0 if not found)
`copyOf(`**array**`, `**length**`)`	returns a new resized copy of an array
`equals(`**array1**`, `**array2**`)`	returns `true` if the two arrays contain same elements in the same order
`fill(`**array**`, `**value**`)`	sets every element to the given value
`sort(`**array**`)`	arranges the elements into sorted order
`toString(`**array**`)`	returns a string representing the array, such as `"[10, 30, -25, 17]"`

- Syntax: `Arrays.`**methodName**`(`**parameters**`)`

Arrays.binarySearch

```
// searches an entire sorted array for a given value
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, value)

// searches given portion of a sorted array for a given value
// examines minIndex (inclusive) through maxIndex (exclusive)
// returns its index if found;  a negative number if not found
// Precondition: array is sorted
Arrays.binarySearch(array, minIndex, maxIndex, value)
```

- The `binarySearch` method in the `Arrays` class searches an array very efficiently if the array is sorted.
  - You can search the entire array, or just a range of indexes (useful for "unfilled" arrays such as the one in `ArrayIntList`)
  - If the array is not sorted, you may need to sort it first

Using `binarySearch`

```
// index     0   1   2   3    4    5    6    7    8    9   10   11   12   13   14   15
int[] a = {-4, 2, 7, 9, 15, 19, 25, 28, 30, 36, 42, 50, 56, 68, 85, 92};

int index  = Arrays.binarySearch(a, 0, 16, 42);    // index1 is 10
int index2 = Arrays.binarySearch(a, 0, 16, 21);    // index2 is -7
```

- `binarySearch` returns the index where the value is found
- if the value is *not* found, `binarySearch` returns:

  `-(insertionPoint + 1)`

  - where `insertionPoint` is the index where the element *would* have been, if it had been in the array in sorted order.
  - To insert the value into the array, negate `insertionPoint` + 1

```
int indexToInsert21 = -(index2 + 1);  // 6
```

Binary search code

```
// Returns the index of an occurrence of target in a,
// or a negative number if the target is not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    int min = 0;
    int max = a.length - 1;

    while (min <= max) {
        int mid = (min + max) / 2;
        if (a[mid] < target) {
            min = mid + 1;
        } else if (a[mid] > target) {
            max = mid - 1;
        } else {
            return mid;    // target found
        }
    }

    return -(min + 1);     // target not found
}
```

Recursive binary search (13.3)

- Write a recursive `binarySearch` method.
  - If the target value is not found, return its negative insertion point.

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

```
int index  = binarySearch(data, 42);   // 10
int index2 = binarySearch(data, 66);   // -14
```

Exercise solution

```
// Returns the index of an occurrence of the given value in
// the given array, or a negative number if not found.
// Precondition: elements of a are in sorted order
public static int binarySearch(int[] a, int target) {
    return binarySearch(a, target, 0, a.length - 1);
}

// Recursive helper to implement search behavior.
private static int binarySearch(int[] a, int target,
                                int min, int max) {
    if (min > max) {
        return -1;        // target not found
    } else {
        int mid = (min + max) / 2;
        if (a[mid] < target) {          // too small; go right
            return binarySearch(a, target, mid + 1, max);
        } else if (a[mid] > target) {  // too large; go left
            return binarySearch(a, target, min, mid - 1);
        } else {
            return mid;   // target found; a[mid] == target
        }
    }
}
```

Binary search and objects

- Can we `binarySearch` an array of Strings?
  - Operators like < and > do not work with `String` objects.
  - But we do think of strings as having an alphabetical ordering.

- **natural ordering**: Rules governing the relative placement of all values of a given type.

- **comparison function**: Code that, when given two values *A* and *B* of a given type, decides their relative ordering:
  - A < B,   A == B,   A > B

The `compareTo` method (10.2)

- The standard way for a Java class to define a comparison function for its objects is to define a `compareTo` method.
  - Example: in the `String` class, there is a method:

```
public int compareTo(String other)
```

- A call of **A**`.compareTo(`**B**`)` will return:

a value <	0	if **A** comes "before" **B** in the ordering,
a value >	0	if **A** comes "after" **B** in the ordering,
or	0	if **A** and **B** are considered "equal" in the ordering.

Runtime Efficiency (13.2)

- **efficiency**: A measure of the use of computing resources by code.
  - can be relative to speed (time), memory (space), etc.
  - most commonly refers to run time

- Assume the following:
  - Any single Java statement takes the same amount of time to run.
  - A method call's runtime is measured by the total of the statements inside the method's body.
  - A loop's runtime, if the loop repeats N times, is N times the runtime of the statements in its body.

Efficiency examples

```
statement1;
statement2;
statement3;

for (int i = 1; i <= N; i++) {
    statement4;
}

for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
}
```

- statement1–statement3: 3
- first loop: N
- second loop: 3N
- total: 4N + 3

Efficiency examples 2

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        statement1;
    }
}

for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;
    statement5;
}
```

N^2

$4N$

$N^2 + 4N$

- How many statements will execute if N = 10? If N = 1000?

Algorithm growth rates (13.2)

- We measure runtime in proportion to the input data size, N.
  - **growth rate**: Change in runtime as N changes.

- Say an algorithm runs $\mathbf{0.4N^3 + 25N^2 + 8N + 17}$ statements.
  - Consider the runtime when N is *extremely large* .
  - We ignore constants like 25 because they are tiny next to N.
  - The highest-order term (N^3) dominates the overall runtime.
  - We say that this algorithm runs "on the order of" N^3.
  - or $\mathbf{O(N^3)}$ for short ("**Big-Oh** of N cubed")

Complexity classes

- **complexity class**: A category of algorithm efficiency based on the algorithm's relationship to the input size N.

Class	Big-Oh	If you double N, ...	Example
constant	$O(1)$	unchanged	10ms
logarithmic	$O(\log_2 N)$	increases slightly	175ms
linear	$O(N)$	doubles	3.2 sec
log-linear	$O(N \log_2 N)$	slightly more than doubles	6 sec
quadratic	$O(N^2)$	quadruples	1 min 42 sec
cubic	$O(N^3)$	multiplies by 8	55 min
...	...	...	...
exponential	$O(2^N)$	multiplies drastically	$5 * 10^{61}$ years

Binary search (13.1, 13.3)

- **binary search** successively eliminates half of the elements.
  - *Algorithm:* Examine the middle element of the array.
    - If it is too big, eliminate the right half of the array and repeat.
    - If it is too small, eliminate the left half of the array and repeat.
    - Else it is the value we're searching for, so stop.
  - Which indexes does the algorithm examine to find value **22**?
  - What is the runtime complexity class of binary search?

index	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
value	-4	-1	0	2	3	5	6	8	11	14	22	29	31	37	56

Binary search runtime

- For an array of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  – How many divisions does it take?

- Think of it from the other direction:
  – How many times do I have to multiply by 2 to reach N?

  1, 2, 4, 8, ..., N/4, N/2, N

  – Call this number of multiplications "x".

  $2^x = N$

  $\mathbf{x = \log_2 N}$

- Binary search is in the **logarithmic** complexity class.

Sorting

- **sorting**: Rearranging the values in an array or collection into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
  - can be solved in many ways:
    - there are many sorting algorithms
    - some are faster/slower than others
    - some use more/less memory than others
    - some work better with specific kinds of data
    - some can utilize multiple computers / processors, ...
  - *comparison-based sorting* : determining order by comparing pairs of elements:
    - `<`, `>`, `compareTo`, ...

Sorting methods in Java

- The `Arrays` class in `java.util` has a static method `sort` that sorts the elements of an array

```
String[] words = {"foo", "bar", "baz", "ball"};
Arrays.sort(words);
System.out.println(Arrays.toString(words));
// [ball, bar, baz, foo]
```

Selection sort

- **selection sort**: Orders a list of values by repeatedly putting the smallest or largest unplaced value into its final position.

  The algorithm:
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.

  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    ...

  - Repeat until all values are in their proper places.

Selection sort example

- Initial array:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- After 1st, 2nd, and 3rd passes:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	**-4**	18	12	**22**	27	30	36	50	7	68	91	56	2	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	**2**	12	22	27	30	36	50	7	68	91	56	**18**	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	**7**	22	27	30	36	50	**12**	68	91	56	18	85	42	98	25

Selection sort code

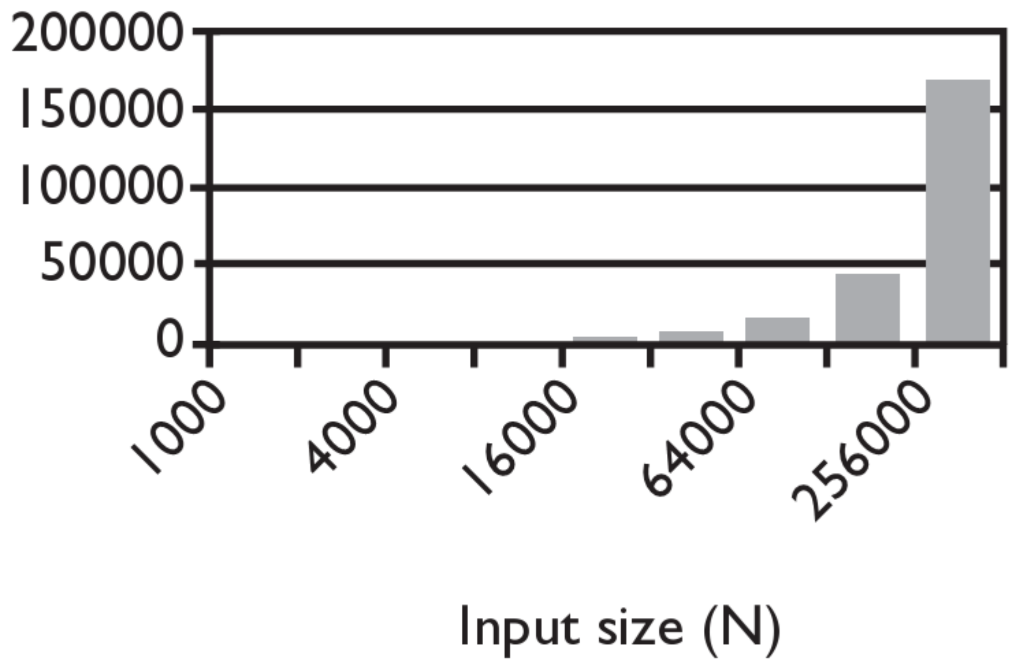
```
// Rearranges the elements of a into sorted order using
// the selection sort algorithm.
public static void selectionSort(int[] a) {
    for (int i = 0; i < a.length - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < a.length; j++) {
            if (a[j] < a[min]) {
                min = j;
            }
        }

        // swap smallest value its proper place, a[i]
        swap(a, i, min);
    }
}
```

Selection sort runtime (Fig. 13.6)

- What is the complexity class (Big-Oh) of selection sort?

N	Runtime (ms)
1000	0
2000	16
4000	47
8000	234
16000	657
32000	2562
64000	10265
128000	41141
256000	164985



Merge sort

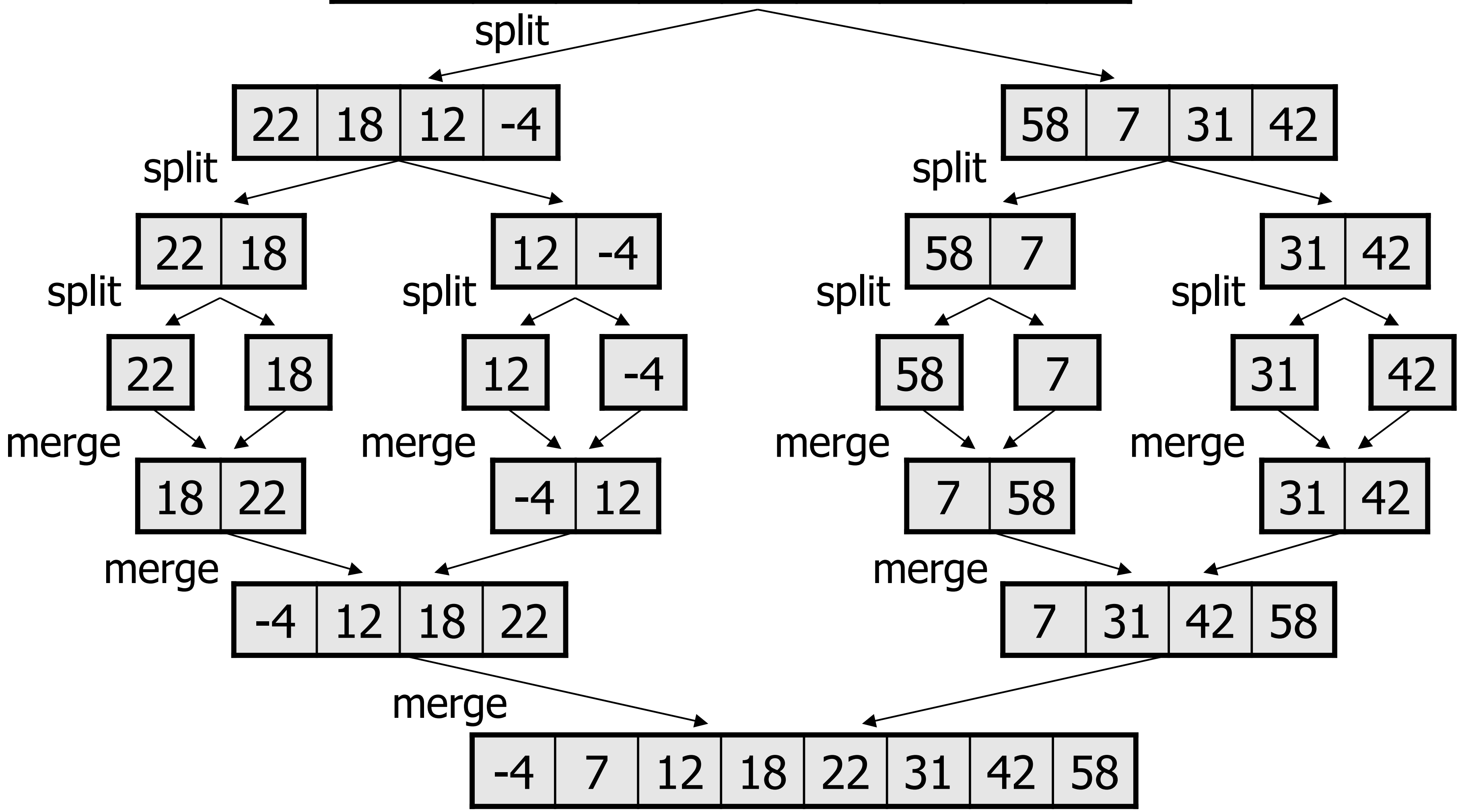
- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

Merge sort example

index	0	1	2	3	4	5	6	7
value	22	18	12	-4	58	7	31	42



Merging sorted halves

Subarrays								Next include	Merged array							
0	1	2	3	0	1	2	3		0	1	2	3	4	5	6	7
14 (i1)	32	67	76	**23** (i2)	41	58	85	14 from left	14 (i)							
14	**32** (i1)	67	76	**23** (i2)	41	58	85	23 from right	14	23 (i)						
14	**32** (i1)	67	76	23	**41** (i2)	58	85	32 from left	14	23	32 (i)					
14	32	**67** (i1)	76	23	**41** (i2)	58	85	41 from right	14	23	32	41 (i)				
14	32	**67** (i1)	76	23	41	**58** (i2)	85	58 from right	14	23	32	41	58 (i)			
14	32	**67** (i1)	76	23	41	58	**85** (i2)	67 from left	14	23	32	41	58	67 (i)		
14	32	67	**76** (i1)	23	41	58	**85** (i2)	76 from left	14	23	32	41	58	67	76 (i)	
14	32	67	76	23	41	58	**85** (i2)	85 from right	14	23	32	41	58	67	76	85 (i)

Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
public static void merge(int[] result, int[] left,
                                       int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

Merge sort code

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm.
public static void mergeSort(int[] a) {
    // split array into two halves
    int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
    int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

    // sort the two halves
    ...

    // merge the sorted halves into a sorted whole
    merge(a, left, right);
}
```

Merge sort code 2

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm (recursive).
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

        // sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

Merge sort runtime

- What is the complexity class (Big-Oh) of merge sort?

N	Runtime (ms)
1000	0
2000	0
4000	0
8000	0
16000	0
32000	15
64000	16
128000	47
256000	125
512000	250
1e6	532
2e6	1078
4e6	2265
8e6	4781
1.6e7	9828
3.3e7	20422
6.5e7	42406
1.3e8	88344