Math C067 — Practice Test
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1. A coworker has designed a study with sample size 100. His margin of error is 5%. Which of the following sample sizes would give him a margin of error of roughly 1%?

   (a) 20 (b) 50 (c) 500 (d) 1000 (e) 2500 (f) 5000

   Solution:

   - Margin of error is roughly proportional to $\frac{1}{\sqrt{n}}$ where n is the sample size.
   - It is desired to multiply the margin of error by $\frac{1}{5}$,
   - so we need to multiply $\sqrt{n}$ by roughly a factor of 5,
   - which means multiplying n by roughly a factor of $5^2 = 25$,
   - so we need a sample of size roughly $25 \cdot 100 = 2500$.

   Note: Because the numbers in row ∞ of Table A-2 are slightly smaller than in row 99, a sample size of 2500 will result in a margin of error slightly less than 1%.

2. The College Board has designed a new test. Six randomly chosen subjects obtained scores of 7, 9, 10, 10, 11, 13.

   (a) Find a 99% confidence interval for the mean test score in the entire population of test takers.

   (b) For a margin of error of 0.4 what would be your confidence level?

   (c) The College Board sampled 10,000 subjects. Assuming they got the same mean and standard deviation as in the 6-person sample, give a 99% confidence interval for the mean test score in the entire population.

   (d) How large a sample would you need in order to get a margin of error of 0.03 with 99% confidence?

Solution:

- The sample size $n = 6$.
- Let $\overline{X}$ be a random variable denoting the average of 6 samples.
- The sample mean $\bar{x} = (7 + 9 + 10 + 10 + 11 + 13)/6 = 10$
- The sample variance $s^2 = (3^2 + 1^2 + 0^2 + 0^2 + 1^2 + 3^2)/5 = 20/5 = 4$.
- The sample standard deviation $s = \sqrt{4} = 2$.
- Therefore we estimate the population mean μ as 6 and σ as 2.
- Thefore we estimate $\sigma_{\overline{X}} = 2/\sqrt{6} \approx 0.8165$

(a)
- In Table A-2 we look in row $6 - 1 = 5$ and column $0.99/2 = 0.495$ to find that the margin of error is 4.03 standard deviations.
- The margin of error $E = 4.03\sigma_{\overline{X}} \approx 4.03 \cdot 0.8165 \approx 3.29$.
- Thus $[10 - 3.29, 10 + 3.29] = [6.71, 13.29]$ is a 99% confidence interval for the population mean μ.

(b)
- We estimate that 0.4 consists of approximately $0.4/0.8165 \approx 0.49$ standard deviations.
- In Table A-1 we look in row 0.4 and column 9 to find that the probability of being within 0.49 standard deviations to the right of the mean is approximately 0.1879
- Therefore the probability of being with 0.49 standard deviation to the right or left of the mean is approximately $0.1879 + 0.1879 = 0.3758$
- Therefore our confidence level is approximately 37.58%.

(c)
- The sample size is now 10000, so we estimate $\sigma_{\overline{X}} = \sigma/\sqrt{n} \approx 2/\sqrt{10000} = 2/100 = 0.02$
- In Table A-2 we look in row ∞ and column $0.99/2 = 0.495$ to find that the margin of error is 2.58 standard deviations.
- The margin of error $E \approx 2.58\sigma_{\overline{X}} \approx 2.58 \cdot 0.02 \approx 0.0516$.
- Thus $[10 - 0.0516, 10 + 0.0516] = [9.9484, 10.0516]$ is a 99% confidence interval for the population mean μ.

(d)
- $\sigma_{\overline{X}} = \sigma/\sqrt{n}$ so we estimate that $\sigma_{\overline{X}} \approx 2/\sqrt{n}$
- Looking in row ∞ column 0.495 of Table A-2, we find that the margin of error is approximately 2.58 standard deviations,
- so $E \approx 2.58 \cdot 2/\sqrt{n} = 5.16/\sqrt{n}$.
- We need $E = 0.03$ so

$$\begin{aligned} 0.03 &\approx 5.16/\sqrt{n} \\ \sqrt{n} &\approx 5.16/0.03 \\ \sqrt{n} &\approx 172 \\ n &\approx 172^2 \\ n &\approx 29584 \end{aligned}$$

3. You have been assigned to predict the results of a 2-candidate election. What sample size would you need in order to ensure a 4% margin of error with 95% confidence?

Solution:

- Let $\hat{P}$ denote the sampling proportion for n samples, where each sample has unknown success probability p.
- The standard deviation of $\hat{P}$ is $\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}}$.
- We don't have any estimate for p. To be conservative, we plug in $\frac{1}{2}$, which gives the largest possible value for $p(1-p)$, i.e.,
- $\sigma_{\hat{P}} \leq \sqrt{\frac{\frac{1}{2}(1-\frac{1}{2})}{n}} = \frac{1}{2}\frac{1}{\sqrt{n}}$.
- Looking in row ∞, column $0.95/2 = 0.475$ of Table A-1 we find that the margin of error consists of approximately 1.96 standard deviations, i.e.,
- $E \approx 1.96\frac{1}{2}\frac{1}{\sqrt{n}} = \frac{0.98}{\sqrt{n}}$
- We need $E = 0.04$ so

$$\begin{aligned} 0.04 &\approx \frac{0.98}{\sqrt{n}} \\ \sqrt{n} &\approx \frac{0.98}{0.04} \\ \sqrt{n} &\approx 24.5 \\ n &\approx 600.25 \end{aligned}$$

Rounding *up*, we get $n = 601$.