Math C067 Chapter 5–7 Review: Formulas and Methods

print and bring to final exam

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1. Descriptive Statistics for a Single Trial X

Expected Value / Mean (μ)

$$E[X] = x_1 \cdot p_1 + \cdots x_k \cdot p_k$$

Where $x_1, \ldots, x_k$ are all possible values for X and $p_i = \Pr[X = x_i]$.

Special case (Bernoulli trial): $E[X] = p$.

Variance (σ^2)

$$\text{Var}[X] = (x_1 - \mu)^2 \cdot p_1 + \cdots (x_k - \mu)^2 \cdot p_k$$

Where $x_1, \ldots, x_k$ are all possible values for X, $\mu = E[X]$, and $p_i = \Pr[X = x_i]$.

Special case (Bernoulli trial): $\text{Var}[X] = pq$.

Standard Deviation (σ)

$$\sigma = \sqrt{\text{Var}[X]}$$

Special case (Bernoulli trial): $\sigma_X = \sqrt{pq}$.

2. Descriptive Statistics for Repeated Trials

Expected Value / Mean (μ)

- If X is the *sum* of n samples from a distribution X_1, then $E[X] = n \cdot E[X_1]$. The formula for $E[X_1]$ is given in the previous section.
- Special case (*binomial distribution*): If X is the sum of n Bernoulli trials, where each Bernoulli trial has success probability p, then $E[X] = n \cdot p$.
- *Sample Mean* $\overline{X}$: If X is the *average* of n samples from a distribution X_1, then $E[X] = E[X_1]$. The formula for $E[X_1]$ is given in the previous section.
- Special case (*Sample Proportion* $\hat{P}$): If X is the *average* of n Bernoulli trials, where each Bernoulli trial has success probability p, then $E[X] = p$

Variance (σ^2)

- If X is the *sum* of n independent samples from a distribution X_1, then $\text{Var}[X] = n \cdot \text{Var}[X_1]$. The formula for $\text{Var}[X_1]$ is given in the previous section.
- Special case (*binomial distribution*): If X is the sum of n Bernoulli trials, where each Bernoulli trial has success probability p, then $\text{Var}[X] = n \cdot pq$
- *Sample Mean* $\overline{X}$: If X is the *average* of n independent samples from a distribution X_1, then $\text{Var}[X] = \text{Var}[X_1]/n$. The formula for $\text{Var}[X_1]$ is given in the previous section.
- Special case (*Sample Proportion* $\hat{P}$): If X is the *average* of n Bernoulli trials, where each Bernoulli trial has success probability p, then $\text{Var}[X] = pq/n$

Standard Deviation (σ)

- If X is the *sum* of n independent samples from a distribution X_1, then
$sigma_X = \sqrt{n} \cdot \sigma_{X_1}$
- Special case (*binomial distribution*): If X is the sum of n Bernoulli trials, where each Bernoulli trial has success probability p, then $\sigma_X = \sqrt{n} \cdot \sqrt{pq} = \sqrt{n \cdot pq}$
- *Sample Mean* $\overline{X}$: If X is the *average* of n independent samples from a distribution X_1, then $\sigma_X = \sigma_{X_1}/\sqrt{n}$
- Special case (*Sample Proportion* $\hat{P}$): If X is the *average* of n Bernoulli trials, where each Bernoulli trial has success probability p, then $\sigma_X = \sqrt{pq}/\sqrt{n} = \sqrt{pq/n}$

3. How to Estimate a Probability using the Standard Normal Distribution Table (A-1)

1. Make sure that the sample size n is large enough. With Binomial Distributions and Sample Proportions you need $np \geq 5$ and $nq \geq 5$. With other distributions you need $n \geq 30$.
2. Calculate μ and σ using one of the formulas in the previous section. Be sure to use the right formula.
3. Find the endpoints of your interval
    - If your distribution is a sum of trials (or, in particular, a Binomial Distribution) then each endpoint will need to have 0.5 added to it or subtracted from it. If you don't know which, draw a picture.
    - If your interval has only one endpoint then the other endpoint is ∞ or $-\infty$.
4. For each endpoint x, calculate the number of standard deviations between x and the mean, i.e., $(x-\mu)/\sigma$. (Make sure you subtract *before* you divide.)
5. Look up each of those two numbers in Table A-1 (ignore the sign when looking up). When looking up a number greater than 4 (or, in particular, ∞), you always get the value 0.5.
6. If μ is inside the interval then add the two numbers you got from Table A-1. If μ is outside the interval then subtract instead.