The Role of Copulas in Reasoning

Pei Wang
Department of Computer and Information Sciences
Temple University, USA
https://cis.temple.edu/~pwang/

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1 Introduction

The usage of copulas is one of the defining features of term logic, where a typical sentence has a "subject-copula-predicate" format, and a typical inference rule corresponds to a certain copula combination in the premises, as in Aristotle's Syllogistic [Aristotle, 1882]. On the contrary, in predicate logic and propositional logic, copulas do not play such central roles [Copi et al., 2013].

While it is widely believed that predicate logic is more capable than term logic both in representation and inference, in the following I will challenge this conclusion from the perspective of artificial intelligence. I will argue that for certain types of working environments and inference tasks, term logic provides a better framework than predicate logic, and a key contribution is made by copulas.

This paper starts with a description of copulas in various term logic models, as well as how their functions and roles are replaced in predicate logic models. This description also relates to other models of reasoning which are not often considered as "logic". After that, the study of reasoning in artificial intelligence is surveyed and analyzed, which leads to a special requirement for logic. A model designed to meet this requirement is described, in which copulas play a fundamental role. This new logic, "Non-Axiomatic Logic (NAL)", is compared with the traditional models, with its features and capabilities discussed.

Though many of the conclusions of this paper have appeared in my previous publications [Wang, 2000, Wang, 2004, Wang, 2006, Wang, 2013, Wang, 2019b], this writing is my first attempt to give the role of copulas in reasoning a more comprehensive and detailed treatment.

2 Copulas in Logic

In linguistics, a "copula" is a verb that binds the subject of a sentence to the predicate (though it is sometimes considered part of the predicate), such as the various forms of "being" (am, are, is, was, were) in English.

In term logic, a *copula* indicates a logical relation between the subject term and the predicate term in a proposition (or statement), so a copula c can form a proposition ScP with two terms S and P. The order of these two terms is conventional. Though usually the subject term is put first, with the predicate term indicating its type or property, Aristotle also turned the order around by considering "When one thing is predicated of another" [Aristotle, 1882].

A logical copula is not ambiguous like many linguistic copulas. For instance, the copula "is" in English can mean identity, subsumption, or something else, but they will each be represented as a different logical copula. When there is only one copula involved, it can be omitted in the representation, though the logical relation remains, which is represented implicitly. On the other hand, additional factors can be included in copulas to keep the ScP format for propositions. For example, such a view can be taken for Aristotle's Syllogistic, by considering the forms a, e, i, and o as copulas in a broad sense.

Based on such a representation format, the most obvious inference rule is a deduction rule that corresponds to the transitivity of a copula. In Aristotle's logic, such a syllogism is known as "Barbara" and can be written as

$$\{MaP, SaM\} \vdash SaP$$

In this inference rule, any terms can serve as S, P, or M, so what distinguishes this rule from the other rules is only the copulas and locations of the shared term M in the premises. It is well-known that when terms are interpreted as sets and the copulas as set relations (a, e, i, and o as *subset*, *disjoint*, *intersect*, and *difference*, respectively), all valid syllogisms in Aristotle's logic can be proved as theorems in set theory [Lukasiewicz, 1951, Beziau, 2017]. At the same time, the meaning of a copula involved is fully revealed by the inference rules in which it appears.

Since mathematical logic [Frege, 1999, Whitehead and Russell, 1910] came, the dominance of term logic was taken over by predicate logic, which reserves no special role for copulas. As far as mathematical knowledge is concerned, the "predicate with argument" format, $P(a_1, \ldots, a_n)$, is more suitable than the ScP format. Though both formats have a "predicate" P, it is defined and handled very differently in the two. The "subject vs. predicate" distinction in term logic is local and relative, so the same term can be a subject in one proposition and a predicate in another. On the contrary, the "argument vs. predicate" distinction in predicate logic is global and absolute, as these two types of terms belong to disjoint domains.

In predicate logic, it is possible to express a traditional copula as a predicate, which is a reason for many people to consider predicate logic as having a higher expressive capability, as in Aristotle's logic there is no obvious way to express a non-copula relation among terms. However, even if a predicate has the same referential semantics as a logical copula, it is still not directly recognized and processed by the inference rules. Predicate logic mainly depends on the inference rules of propositional logic, which are defined and justified according to the truth-values of the propositions involved, without restrictions and requirements on the syntactic structures of these propositions. For example, the truth-value of an implication proposition $p \rightarrow q$ is fully determined by the truth-values of p and q, without considering their contents. Overall, the validity of inference rules is based on the requirement that they can only derive true conclusions from true premises [Stebbing, 1950, Copi et al., 2013]. Since truth functions are taken as fundamental, many conceptual relations in term logic are rewritten as propositional relations in predicate logic. For example, SaP in the former usually becomes $(\forall x)(S(x) \rightarrow P(x))$ in the latter [Stebbing, 1950], and other copulas are transformed into truth-value relations in similar ways.

For the intended usage of mathematical logic, the advantages of predicate logic over term logic are obvious, and consequently, term logic is often considered as only of historical values. The most noticeable exception of this consensus is "Term Functor Logic" (TFL) [Sommers, 1982, Sommers and Englebretsen, 2000, Englebretsen, 1981, Englebretsen, 1996], which is functionally equivalent to first-order predicate logic while keeping the naturalness of term logic, partly because of the use of copulas. In TFL, "the logical forms of statements that are involved in inferences as premises or conclusions can be construed as the result of connecting pairs of terms by means of a logical copula (functor)" [Sommers and Englebretsen, 2000].

3 Logic and Artificial Intelligence

The logic considered in this paper is evaluated mainly according to the needs of Artificial Intelligence (AI). Generally speaking, the objective of AI research is to construct computer systems that work like the human mind, though there are very different opinions on which aspects the similarity should be [Wang, 2019a]. For the researchers who want computers to follow the same "laws of thought" as humans, it is natural to look for inspirations and tools from logic, which "is concerned with the principles of valid inference" from the beginning [Kneale and Kneale, 1962].

It is obvious that except for tasks like theorem proving, *classical logic* (first-order predicate logic) does not fully meet the needs of AI, which often deals with problems and tasks outside mathematics. Nevertheless, many researchers still take classic logic as a proper starting point, with the hope that some extensions and revisions will solve the problems in AI while keeping the virtues of classic logic, such as its certainty and exactness [McCarthy, 1989, Nilsson, 1991]. Among the various *non-classical logics* explored in AI, there are *nonmonotonic logic* for defeasible reasoning [Reiter, 1987] and *probabilistic logic* for uncertain reasoning [Nilsson, 1986].

As for the current discussion, the most relevant work is that of *description logic* [Brachman and Schmolze, 1985], where the primary focus is to represent human knowledge in a logical language suitable for computerized reasoning. Among the modifications it makes in classical logic, a major addition is to explicitly support the taxonomic structure among concepts using an "IS-A" relation [Brachman, 1983], which is basically a logical copula, as it is directly recognized and processed by the inference rules. Though this relation can be expressed in other ways, many knowledge representation frameworks (such as "ontology", "knowledge graph", etc.) choose to directly express and handle it to achieve higher naturalness and efficiency. The result is a hybrid of term logic and predicate logic, where the former supports a taxonomic hierarchy, and the latter handles additional knowledge besides the hierarchical relations.

Though I share opinions with the above works, my own logic model has been motivated by different considerations. My understanding of "intelligence" is "the capability of working with insufficient knowledge and resources and adapting to the environment" [Wang, 2019a]. Based on this understanding, I construct a normative theory and a reasoning model to specify how such a system should work, no matter whether it is naturally evolved (like the human mind) or artificially designed (like a computer system) [Wang, 2006].

In this context, "insufficient knowledge" means that the system's knowledge cannot always contain or derive correct or even satisfactory answers for the questions it faces and that what the system "knows" can be wrong, due to incorrect information, biased data, or changes in the environment. It follows that the system's reasoning process is no longer "from truth to truth", as in classical logic or Aristotelian logic.

On the other hand, "insufficient resources" is mainly about the restrictions on computational time and space. As the system may get a new task at any moment and each task has response-time demands (such as a deadline or "as soon as possible"), it often cannot consult all relevant knowledge when processing a task, nor to be specified as a predetermined procedure with a fixed resource demand. It means the system's reasoning processes in task processing cannot depend on task-level algorithms.

However, it does not mean that in such situations all reasoning activities are equally valid (or invalid). It is arguable that such a working environment is the normal case for the human mind, and our existence shows that the regularities observed in the human reasoning process are not completely irrational, even though not "truth-preserving" in the traditional sense. What I have tried is to re-establish the validity of reasoning and rationality of thinking on the foundation of *adaptation*. For a system to be adaptive in its lifetime, it must predict the future according to its past experience and behave accordingly, even if the future and the past cannot be taken as the same. Furthermore, the system needs to spend its available resources in an efficient manner and also manage them according to its own experience.

This type of adaptation requires a "Concept-Centered Knowledge Representation (CCKR)", where a *concept* is an abstraction of an experience segment. The use of concepts allows the system to recognize the partial similarity between the current situation and various past situations, as well as to selectively organize experience in an efficient manner. Since concepts have different levels of abstractness, the same experience can be represented with various granularity, scope, focus, etc. to meet different needs of the system. Unlike the concepts in the "symbolic AI" tradition, the meaning of a concept in CCKR is not determined by the object or event it refers to in the world, but by its location in experience, as revealed by its relations with other concepts.

According to this "Experience-Grounded Semantics (EGS)", the system's knowledge does not provide a description of the world "as it is" but "as the system knows", and the extent to which a statement is "true" cannot be judged by comparing it with facts or future observations (which are not available to the system) but with evidence collected from (past) experience. Reasoning serves the purpose of transforming knowledge to related situations, and valid inference rules are those whose conclusions correctly summarize the evidence provided by the premises. This type of logic is fundamentally different from traditional logic (no matter whether they are built as predicate logic or term logic), while still keeping the normative and formal nature of logic [Wang, 2004, Wang, 2019b].

4 Copulas in NAL

Based on the above considerations, in my logic NAL [Wang, 2013] a *concept* is an internal entity (a data structure in the reasoning system) that corresponds to a recognizable component in the experience of the system. It can be a perceived pattern of stimuli, an executable operation, a word or phrase in a (communication) language, or a combination of them. Each concept is uniquely identified within the system by a *term*, which in the simplest form is a string of characters.

A new term can enter the system from the environment, or be composed by the system itself from some existing terms using a term connector. Each compound term corresponds to a concept, which is semantically related to the concepts identified by the terms that are the components of the compound. Among the infinite number of conceptual relations, a small number is directly recognized and processed by the inference rules. Copulas are among them and indicate various forms of substitutability of terms, as well as the concepts identified by them.

The most fundamental copula in NAL is *inheritance*, which is a reflexive and transitive relation between two terms, and is written as "$\rightarrow$". The statement "$S \rightarrow P$" is intuitively close to the "SaP" in Aristotle's syllogism, indicating that S is a specialization of P and that P is a generalization of S. By defining the *extension* and *intension* of a term as sets containing its specializations and generalizations, respectively, "$S \rightarrow P$" also indicates "S inherits the intension of P" and "P inherits the extension of S", which is why this copula is named "inheritance".

The above idealized "complete inheritance" can be extended to cover "incomplete inheritance" between terms (concepts), which is the normal case for systems working with insufficient knowledge and resources. In NAL, the idealized (binary-valued) inheritance copula is used to define evidence for a realistic (multi-valued) *inheritance* copula, also written as "$\rightarrow$". The positive evidence for statement "$S \rightarrow P$" are the terms in the *extensional intersection* and the *intensional intersection* of S and P, while the terms in the *extensional difference* of S and P and the *intensional difference* of P and S are negative evidence of the statement. The truth-value of a statement is defined as a pair of values in

[0, 1], including a *frequency* that is the proportion of positive evidence among all available evidence, and a *confidence* that is the proportion of currently available evidence among all evidence at a fixed "evidential horizon" after a constant amount of future evidence is collected.

Basic inference rules of NAL are syllogistic, each with a truth-value function calculating the evidential support the premises provide for the conclusion. To establish these functions, all involved values in [0, 1] are treated as extended Boolean values in $\{0, 1\}$, as explained in detail in [Wang, 2013]. Therefore, corresponding to the Aristotelian syllogism "Barbara", there is the NAL rule

$$deduction : \{M \to P, S \to M\} \vdash S \to P$$

which specifies the transitivity of the (multi-valued) *inheritance* copula.

Following the insight of Peirce, induction and abduction in NAL are obtained from deduction by exchanging the conclusion in the above rule with each of the two premises, respectively:

$$induction : \{S \to P, S \to M\} \vdash M \to P$$

$$abduction : \{M \to P, S \to P\} \vdash S \to M$$

Though Peirce only suggested the cognitive functions of these non-deductive inference types (induction for generalization, and abduction for explanation), using experience-grounded semantics these rules can be justified in the same manner as deduction since they are exactly how evidence is collected by comparing the extension and intension, respectively, of the two terms in the conclusion, as specified earlier.

The following revision rule pools evidence from distinct sources:

$$revision : \{S \to P, S \to P\} \vdash S \to P$$

These four rules form a minimum NAL that can summarize its experience for adaptation while working with insufficient knowledge and resources.

The other inference rules are mostly designed to handle various types of compound terms. For example, if two terms A and B have a relation R that is not *inheritance* (or any other copulas), then the relation is expressed as a set of ordered pairs including that of A and B, that is, $(A \times B) \to R$, like in set theory. This statement still keeps the "subject-copula-predicate" format, though the subject term is a compound $(A \times B)$, whose meaning is partially determined by its (intrinsic) syntactic relations with its components A and B, and partially determined by its (acquired) semantic relation with R. Here the meaning of R is completely obtained from the system's experience, which is fundamentally different from how a copula (such as "$\to$") gets its meaning, though both represent "conceptual relations" and are represented as "predicates" in predicate logic.

Overall, there are four basic copulas in NAL. The *similarity* copula ($\leftrightarrow$) is a symmetric version of the *inheritance* copula ($\to$), and their isomorphic form between two statements (terms with a truth-value) are the *implication* copula (⇒) and the *equivalence* copula (⇔), which intuitively represents "if-then" and "if-and-only-if", respectively. While the first pair of copulas specifies the substitutability of the *meanings* (i.e., extensions and intensions) of the two terms linked, the second pair specifies the substitutability (derivability) of their *truth-values*. The inference rules of NAL correspond to different combinations of the copulas in the premises, just like in the case of Aristotle's syllogisms.

5 Comparisons and Discussions

Though the previous description of NAL is brief and highly simplified, it still provides enough materials for the role of copulas to be discussed.

First, the four basic copulas are represented and processed very differently from other conceptual relations, as the copulas are innate to the system and directly processed by the inference rules, with experience-independent (operational) meaning. As constants in NAL, they are at the meta-level, while the other relations are object-level concepts, with experience-grounded (acquired) meaning.

The dependency of copulas in NAL corresponds to the view of seeing "reasoning" as concept-substituting, which allows an adaptive system to see the novel (current) situation as familiar (past) situations. This ability of "seeing something as something else" has been suggested as at the center of cognition and intelligence by Ulam [Rota, 1989] and Hofstadter [Hofstadter, 1995, Hofstadter, 2001], and Peirce also suggested that "Logic may be considered as the science of identity" [Peirce, 1986]. This point of view is subtly different from the traditional view of seeing "reasoning" as demonstrations or derivations of new truth from known truth [Aristotle, 1882, Whitehead and Russell, 1910, Kneale and Kneale, 1962].

The most representative case of the traditional view is the reasoning in axiomatic systems, where the truth of axioms is either self-evident or widely accepted, from which the theorems are derived by truth-preserving (deductive) inference rules with guaranteed truth. This feature is shared by many other reasoning systems, where the initial promises are not called "axioms" but "presumptions", "facts", "ground truth", and so on, from which the conclusions (usually not called "theorems") are derived. Though the truth value of these initial promises can be challenged outside the system, within the system they are nevertheless treated just like axioms, and all of these systems do not consider the insufficiency of knowledge and resources as described previously.

This is exactly where NAL is different. Given its assumption on the insufficiency of knowledge and resources, its relation with traditional logic is analogous to the relation between Non-Euclidean geometry and Euclidean geometry, which is why it was named "Non-Axiomatic Logic" (NAL) [Wang, 1995, Wang, 2013]. In this "axiomatic vs. non-axiomatic" distinction, the key is not on whether a system has "axioms" (according to the common definition), but whether there is "axiom-like" knowledge serving as the standard of truth within the system.

Under the assumption of insufficient knowledge and resources, a system must abstract experience to various levels to become concepts, and the evaluation of the identity, or substitutability, between concepts becomes a central task. Since such relations are domain-independent, they can be captured at the meta-level with justifiable patterns and rules, in spite of the inevitably uncertain and ever-changing nature of the concepts involved. Consequently, it is necessary to separate copulas from ordinary conceptual relations and to build logic systems around the copulas.

The above conclusion does not mean that the copula-free predicate logic is "wrong", but takes it as a different type of reasoning. In axiomatic systems, the focus is on the truth-value relations among the propositions, evaluated according to a given set of axioms or axiom-like propositions that cannot be challenged during the system's lifetime. When the propositions are symbolic and formal, an *interpretation* is needed to map the symbols within the system to concrete entities outside, usually approximately, for the symbols to become meaningful. This treatment gives the system's conclusions the desired certainty, as well as the possibility of applying them to multiple situations under different interpretations. These features are indeed hoped for in mathematics and other formal theories, like logic, but run into various problems when used outside these domains.

Besides the inability to cover non-deductive inference, another well-known issue of classic logic is the lack of semantic relevance between premises and conclusions. As revealed by the paradoxes of implication, many "logically correct" conclusions are intuitively awkward. The proposed solutions, in the form of various models of "relevance logic" [Mares, 2020] and "paraconsistent logic" [Priest et al., 1989], still attempt to resolve the issue in predicate logic, though it can be argued that the issue does not even exist in term logic [Steinkrüger, 2015]. This is the case exactly because of the use of copulas. In a logic like NAL, within a statement a copula relates two terms in meaning, and in a syllogistic inference rule, the two premises must have a shared term for the inference to happen, and the shared term ("middle term") also relates the other two terms (in the conclusion) together in meaning. The inference rule still determines the truth-value of the conclusion according to those of the premises, but that is under the condition that the terms involved are all semantically related. Unlike in propositional/predicate logic, in term logic it is invalid to replace one premise in an inference rule with another statement that happens to have the same truth-value.

A related feature is that inference in a term logic like NAL is not purely "formal" or "syntactic", but also "semantic" in the sense that all the involved terms have their meaning partially used (in the premises) and generated (in the conclusions). Consequently, NAL provides a model of categorization (conceptualization) that explains the forming and evolving of concepts, which happen together with reasoning as different aspects of the same process, rather than as separate processes carried out by separate mechanisms. This closeness to categorization also explains why a term logic uses a "categorical language" that is closer to natural languages than a predicate logic is [Sommers, 1982].

However, the conclusion that "term logic provides a more suitable framework for empirical reasoning" is not a suggestion to return to Aristotle's Syllogisms, which is only a (binary) deductive system. Peirce added induction and abduction in their term logic form in an elegant manner and pointed out their cognitive functions, but still did not justify their validity as inference rules. Later, these two names are usually only used with cognitive functions, while their formalization is moved into the framework of predicate/propositional logic, rather than in term logic [Flach and Kakas, 2000].

NAL, as a normative model designed for adaptive systems, differs from traditional logic (both in the term logic tradition and the predicate logic tradition) by assuming insufficient knowledge and resources, which leads to its concept-centered representation and experience-grounded semantics. In this context, a copula no longer corresponds to an objective relation between objects/events (or their classes) outside the system, but to substitutability between elements (or their patterns) within the system's experience, and this substitutability can be partial and uncertain. The Aristotelian forms a, e, i, and o all correspond to the *inheritance* copula in NAL, with their differences indicated by the truth-value that indicates evidential support. Since traditional logic assumes that all relevant evidence is already available, they only need two quantifiers to distinguish the "for all" and "there exist" situations, while in NAL, the *amount* of evidence needs to be quantitatively measured, and the system can never be sure that no new evidence will be found for a specific statement.

Given this understanding of copulas, in NAL all types of inference can be uniformly justified as using the evidence provided by the premises to support the conclusion, and the difference between deductive and non-deductive inference becomes *quantitative* (as indicated by the confidence value of the conclusion), rather than *qualitative* (as whether truth-preserving). Without this "concept substitution" view of reasoning, it is difficult (if not impossible) to use predicate logic with experience-grounded semantics, which is necessary for adaptive systems.

This is also where the key difference between NAL and TFL exists. Though both models extend and revise Aristotle's logic and increase the expressing capability of term logic (by introducing singular terms, relational terms, unanalyzed statements, compound statements, and so on), they are designed with different objectives in mind. Consequently, they make different assumptions on the sufficiency of knowledge and resources, which lead to other differences in the design, including their usages of copulas.

6 Conclusions

Non-Axiomatic Logic (NAL) [Wang, 1995, Wang, 2013] attempts to provide a normative model for reasoning in an adaptive system that must deal with the insufficiency of knowledge and resources [Wang, 2004, Wang, 2019b]. Unlike traditional logic, NAL cannot achieve absolute certainty by depending on a set of axiom-like propositions and only carries out inference that is truth-preserving as defined in model-theoretic semantics [Barwise and Etchemendy, 1989]. Instead, it uses concept-centered representation and experience-grounded semantics [Wang, 2005]. According to this point of view, knowledge is mainly about the substitutability between concepts, and reasoning is mainly about the spreading of such substitution relations.

As copulas correspond to types of conceptual substitutability without restricting the content of the concepts involved, they provide the basis for a formal language and inference rules defined on the language for the above purpose. The language allows uncertainty caused by conflicting evidence and changing situations, and the rules cover inconclusive inference based on partial evidence. Furthermore, copula-based representation and inference guarantee the semantic relevance of the premises and the conclusion in each inference step.

Axiomatic reasoning in mathematics (and other formal theories) and non-axiomatic reasoning in everyday life (and empirical theories) require different logic models. The practice of NAL shows that copulas can play a central role in the latter type of logic, as well as provides new materials in the discussions about the relationship between logic and thinking [Kneale and Kneale, 1962, Gabbay and Woods, 2001].

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