Non-Axiomatic Term Logic: A Computational Theory of Cognitive Symbolic Reasoning

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Abstract

- This paper presents **Non-Axiomatic Term Logic (NATL)**, as a theoretical computational framework of humanlike symbolic reasoning in artificial intelligence, based on Non-Axiomatic Logic (NAL) of NARS.
- This paper proposes "Term Representation Language" (TRL) that has higher expressive power than conventional term logic.

Non-Axiomatic Logic (NAL)

- In NAL, inference rules are given, but axioms (absolute propositions) do not exist.
- Even if reasoning is based on the same finite knowledge source, the results obtained can vary and can be updated due to the limited resource of time.
- In addition, “weak inference rules” other than deduction are provided, and the possibility of drawing invalid conclusions is allowed from the beginning.
- The concept of “truth” in NAL is what is reasonable in view of experience.

NAL - Syntax

- A proposition "S is P" is expressed by joining the terms S and P with the copula → in the form $S \rightarrow P$. This form of proposition is called a statement.

- When statement P implies statement Q, it is written as $P \Rightarrow Q$. $P \Rightarrow Q$ is also a (higher-order) statement, and the statement is also a term.

- When expressing the relationship of three or more terms, it is expressed like $U \times V \rightarrow R$. For example, "water resolves salt" is expressed as $water \times salt \rightarrow resolve$. $U \times V$ is treated as one term (product term) and R is called a relational term.

NAL - Semantics

- The semantics (true or false) of a statement is defined by a set of extensions (lower terms) and intensions (upper terms) of a term.
- Extension: $T^E = \{x|(x \in V_k) \wedge (x \rightarrow T)\}$
- Intension: $T^I = \{x|(x \in V_k) \wedge (T \rightarrow x)\}$
- The truth of $S \rightarrow P$ is said to be equivalent to whether $S^E \subseteq P^E and\ P^I \subseteq S^I$.

- However, the above definition of truth and falsehood can only work in an ideal situation (a situation in which there is no uncertainty) called inheritance logic. Since the premise of NAL is uncertainty, the above equivalence relation is generalized to incorporate uncertainty.

NAL - Inference rules

- The main reasoning in NAL is based on syllogistic rules that lead to one conclusion from two premises, but there are also rules that take only one statement, such as negation and conversion, and rules for structural transformations between product terms and relation terms.

- deduction : $\{S \rightarrow M, M \rightarrow P\} \vdash S \rightarrow P$
- induction : $\{S \rightarrow M, S \rightarrow P\} \vdash M \rightarrow P$
- abduction : $\{M \rightarrow P, S \rightarrow P\} \vdash S \rightarrow M$

Issues of Non-Axiomatic Logic

- The representation of terms
  - The reasoning process of NAL can be seen as the process of substituting or replacing one term with another. This substitution process in NAL is guided by the superficial match of terms or symbols and knowledge of similarity between symbols ($S \leftrightarrow T$).
  - There is problem in obtaining practical robustness, which is a common topic of symbol processing AI in the past.
  - It is also not easy to properly design a function that evaluates the similarity between symbols by fully considering the characteristics of the problem domain.
- Approach may solve
  - Embedding. Maps a set of symbols to points in a multidimensional continuous space (latent space) under specific constraints.

Issues of Non-Axiomatic Logic

- Useless inference
  - The algorithm of the reasoning system NARS based on NAL is very naive, and it is difficult to eliminate useless inference and obtain significant inference results within a valid time.
- Approach may solve
  - Train the reasoner by reinforcement learning to select statements and rules effectively.

Issues of Non-Axiomatic Logic

- The continuity and prototype hypothesis of copula
  - For example, $S \circ\!\rightarrow P$ means S is an individual case (that has a proper noun) of P, that is, S has no subterms (for example, $Tweety \circ\!\rightarrow bird$). However, in the case of nonliving things, it is often natural to think that an individual with a name has individual subspecies under it.
  - Whether the conditional inclusive relations (such as "If one likes pasta, the one likes spaghetti") and the causal relationship (like "If the wind blows, the tree shakes") can be treated collectively with the same implicative copula.
  - In human daily reasoning, there might be a continuous and diverse quantification recognition between "all" and "some", such as "most", "a lot", "a little", and so on.
- It seems better to think of copula as a family/prototype category in a continuous space.
  - The copulas given in a prior manner are kept minimally and other nuances (for example, $\circ\!\rightarrow$) are learned empirically.
  - A major challenge is how to design the truth functions with regard to copulas.

Issues of Non-Axiomatic Logic

- Separation and Coordination of Reasoning and Language Processing
  - It is not always necessary to process reasoning base on natural language sentences analyzed by morpheme units in a logical language.
  - What is necessary is recognition of similarities and relationships between terms that express meanings and concepts with appropriate granularity according to the situation.
  - Humans do not become able to speak after being able to think logically, nor are they able to think logically just because they can speak fluently.
  - It is possible to express grammatical structures finely with NAL product terms and relational terms, but it would require the introduction of complex mechanisms.
- A reasonable approach would be to regard the human's "reasoning system" and "language system" as separate and mutually dependent.
  - If the processing of grammatical elements can be left to the "language system", the construction of the "reasoning system" can be kept simple.

Term Representation Language

- In response to the issues in NAL presented in the previous section, this paper proposes Non-Axiomatic Term Logic (NATL).
- Term Representation Language (TRL) is a description model of human cognitive internal representations that use symbols to represent recognition and knowledge.
- NATL expresses the process of reasoning using knowledge representations explicitly structured by TRL.

Term Representation Language

- Term ($T_1, T_2, T_3, \ldots$)
  - A term represents an immediate perception of an experience, , an episodic memory of that experience, a semantic memory extracted and abstracted from an accumulation of episodic memories.
- Basic term ($B_1, B_2, \ldots$) and composed term
  - A term treated as the minimum unit of recognition is called a basic term.
  - The compound terms, statement terms, and linkage terms to be described below are collectively called composed terms.
- Compound term (C)
  - A composed term that expresses a relationship between multiple terms or a secondary recognition of a single term is called a compound term.
  - Let R denote the term representing these concepts of recognition and relationship, and let C denote the compound term $C : (R, T_1, T_2, \ldots, T_n)$.

Term Representation Language

- Statement term
  - A statement term S is a composed term of two arbitrary terms connected by a copula.
  - A statement term corresponds roughly to a statement in NAL and represents the categorization of a term.
  - The most basic copula expresses an is-a relationship. There are several variations of the is-a relation. TRL does not specify how many types of copulas exist.
- Linkage term
  - A linkage term L corresponds to what is called a higher-order statement or implication statement in NAL.
  - The copulas used in the linkage terms represent rules, causal relationships, etc., and can have many different kinds, but TRL does not specify how many kinds there are.
- Variable term
  - Variable terms are represented by lowercase letters x, y, . . .

Non-Axiomatic Term Logic (NATL)

Term Representation Language			Non-Axiomatic Term Logic	
Term	Basic term		$\mathcal{C}$	Thing term
	Composed term	Compound term		
		Statement term	$\mathcal{S}$	Logic term
		Linkage term	$\mathcal{L}$	
	Variable term			

Table 1: Term Types and Correspondences between TRL and NATL

- Reasoning based on NATL draws conclusions from prior knowledge by performing substitution operations between one logical term and another term (thing term or logical term). Substitution operations are allowed when two terms can be unified, including when the two terms are identical.
- Here, we assume soft unification based on embedding vectors, rather than hard unification by exact string matching.

Reasoning and logic in NATL

- Reasoning in NATL as a combination of classes of two terms is divided into the following five types. That is, $\mathcal{S} \cdot \mathcal{S}, \ \mathcal{S} \cdot \mathcal{C}, \ \mathcal{S} \cdot \mathcal{L}, \ \mathcal{L} \cdot \mathcal{C}, \ \mathcal{L} \cdot \mathcal{L}$.

- NATL is a theory that argues that inferences that humans make in daily life can be completely described by five types of symbolic operations on three classes of knowledge representations.

Reasoning types

- $\mathcal{S} \cdot \mathcal{S} \rightarrow \mathcal{S}$

$$\begin{array}{ll} S_1: & human \rightarrow animal \\ S_2: & animal \rightarrow mammal \\ \hline S_3: & human \rightarrow mammal \end{array}$$

$$\begin{array}{ll} S_4: & Lily \rightarrow swan \\ S_5: & Lily \rightarrow white \\ \hline S_6: & swan \rightarrow white \end{array}$$

$$\begin{array}{ll} S_7: & these\text{-}beans \rightarrow white \\ S_8: & beans\text{-}from\text{-}the\text{-}bag \rightarrow white \\ \hline S_9: & these\text{-}beans \rightarrow beans\text{-}from\text{-}the\text{-}bag \end{array}$$

Reasoning types

- $\mathcal{S} \cdot \mathcal{C} \rightarrow \mathcal{C}$

$$\begin{array}{ll} S_1: & \textit{polar-bear} \rightarrow \textit{white} \\ C_1: & (\textit{likes}, \textit{John}, \textit{white}) \\ \hline C_2: & (\textit{likes}, \textit{John}, \textit{polar-bear}) \end{array}$$

  - Although reverse inferences such as the following are also allowed, generally the certainty of inference results is expected to be low.

$$\begin{array}{ll} S_1: & \textit{polar-bear} \rightarrow \textit{white} \\ C_2: & (\textit{likes}, \textit{John}, \textit{polar-bear}) \\ \hline C_1: & (\textit{likes}, \textit{John}, \textit{white}) \end{array}$$

Reasoning types

- $\mathcal{S} \cdot \mathcal{L} \rightarrow \mathcal{C}/\mathcal{S}/\mathcal{L}$
  - / in $\mathcal{C}/\mathcal{S}/\mathcal{L}$ means exclusive disjunction

$$\begin{array}{ll} S_1: & \textit{polar-bear} \rightarrow \textit{white} \\ L_1: & (x \rightarrow \textit{white}) \Rightarrow (\textit{likes}, \textit{John}, x) \\ \hline C_2: & (\textit{likes}, \textit{John}, \textit{polar-bear}) \end{array}$$

$$\begin{array}{ll} S_2: & \textit{weather-of-today} \rightarrow \textit{bad} \\ L_2: & (\textit{weather-of-the-day} \rightarrow \textit{bad}) \Rightarrow \textit{no-school} \\ \hline B_1: & \textit{no-school} \end{array}$$

- Although *weather-of-today* and *weather-of-the-day* are not exactly the same concept, the premise here is to allow the unification of them in an appropriate inferential context. When it is allowed is determined empirically by the reasoner

Reasoning types

- $\mathcal{C} \cdot \mathcal{L} \rightarrow \mathcal{C}/\mathcal{S}/\mathcal{L}$
  - When one of the two element terms of a linkage term and the thing term are unifiable, the remaining element terms of the linkage items are taken as a conclusion. In this case, the class of the consequent terms depend on the given linkage terms.

$$
\begin{array}{ll}
C_1: & (likes, John, polar\text{-}bear) \\
L_1: & (likes, x, polar\text{-}bear) \Rightarrow (likes, x, penguin) \\
\hline
C_2: & (likes, John, penguin)
\end{array}
$$

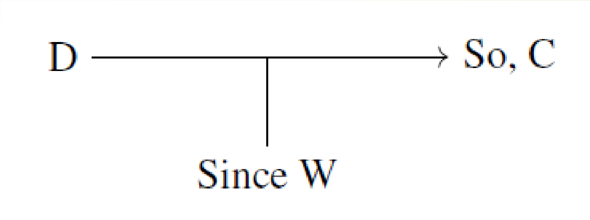
Reasoning types

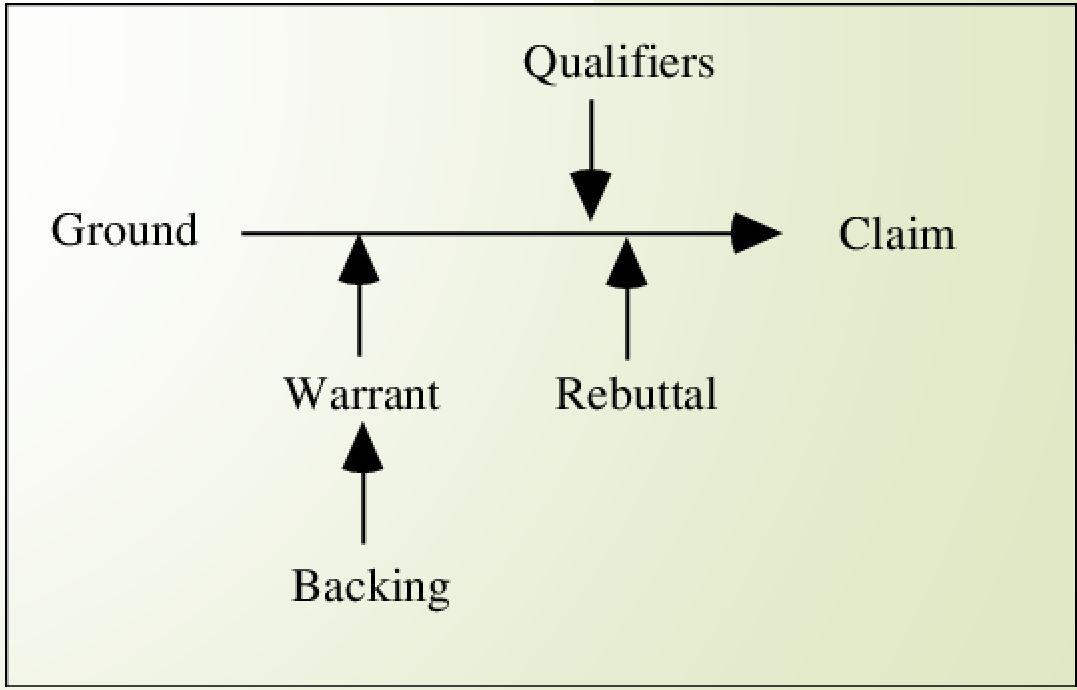
- $\mathcal{L} \cdot \mathcal{L} \rightarrow \mathcal{L}$
  - Like $\mathcal{S} \cdot \mathcal{S} \rightarrow \mathcal{S}$, two remaining element terms construct a new linkage term via the two element terms that are unifiable. Here only give a deductive example.

$$\begin{array}{ll} L_1: & (likes, x, polar\text{-}bear) \Rightarrow (likes, x, penguin) \\ L_2: & (likes, y, penguin) \Rightarrow (likes, y, dolphin) \\ \hline L_3: & (likes, x, polar\text{-}bear) \Rightarrow (likes, x, dolphin) \end{array}$$

Validity and Effectiveness of NATL

- Toulmin's diagram
  - D, datum, (or G, Ground) is the basis of an argument, C is the claim of the argument, W is the warrant that enables the claim to be drawn from the datum.
  - The extended version has more elements.
- Toulmin's argumentation scheme is also based on syllogism, but W is not an absolute rule.





Validity and Effectiveness of NATL

- Example:
  - D: It is raining. C: You should take an umbrella. W: It is bad to get wet. In this example, if W sets to "It is good to get wet.", the conclusion will be "You should not take an umbrella."

$$\begin{array}{ll} S_D: & \textit{weather-of-the-day} \rightarrow \textit{raining} \\ S_W: & \textit{getting-wet} \rightarrow \textit{bad} \\ C_C: & (\textit{take}, \textit{You}, \textit{umbrella}) \end{array}$$

  - C_C cannot be derived directly from S_D and S_W. NATL can explain the reason why $S_D \Rightarrow C_C$ can be claimed with the following three pieces of knowledge L_1, L_2, L_3.

$$\begin{array}{ll} L_1: & (\textit{causal-and}, x, \textit{bad}) \Rightarrow (\textit{avoid}, \textit{people}, x) \\ L_2: & (\textit{weather-of-the-day} \rightarrow \textit{raining}) \Rightarrow \textit{getting-wet} \\ L_3: & (\textit{have}, x, \textit{umbrella}) \Rightarrow (\textit{avoid}, x, \textit{getting-wet}) \end{array}$$

$$\begin{array}{ll} \{S_D, L_2\} \vdash & B_1 : \textit{getting-wet} \\ \{S_W, B_1\} \vdash & B_2 : \textit{bad} \\ \{B_1, B_2\} \vdash & C_1 : (\textit{causal-and}, B_1, B_2) \\ \{C_1, L_1\} \vdash & C_2 : (\textit{avoid}, \textit{people}, \textit{getting-wet}) \\ \{C_2, L_3\} \vdash & C_3 : (\textit{have}, \textit{people}, \textit{umbrella}) \end{array}$$

Applications and Challenges

- Other Applications
  - Solving math problems heuristically
  - Internal Representation Language of Multimodal AI Systems
  - Understanding and generation of analogy and metaphor
- Computer Implementation and Future Challenges
  - Semantic vector representations
  - Unification between similar basic terms
  - Reasoner
  - Translation from natural language to term representation language

Thank you