Multichannel Broadcast Via Channel Hopping in Cognitive Radio Networks

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***Abstract*—Broadcast is a mandatory service for wireless communications, through which the signals or contents are usually propagated as broadcasts to groups of users that subscribe to the service. However, in cognitive radio networks where secondary users (SUs) opportunistically access the licensed spectrum bands, the broadcast delivery via a single broadcast channel cannot be reliably guaranteed when the broadcast channel is reclaimed by a primary user (PU, or licensed user) that has a higher priority of accessing the spectrum or when an SU moves into a region where PU transmissions are active. As a result, broadcast failures occur between the base station (BS) and the SU. To address this problem, the BS has to broadcast on more than one channel to avoid colliding with PUs and ensure that all users can correctly receive the broadcasted content. In this paper, we propose a multichannel broadcast protocol, i.e., *Mc-Broadcast*, that enables a network BS to broadcast over multiple channels via a channel hopping (CH) process such that the broadcasts can be successfully delivered to SUs. The CH sequence for Mc-Broadcast is generated using a mathematical construct called *Langford pairing*, which allows the BS to significantly reduce the broadcast latency and given a customized number of broadcast channels. Our analytical and simulation results show that the proposed method reduces the broadcast latency, and it is robust to the broadcast failure caused by PU transmissions under various network conditions.**

***Index Terms*—Broadcast, channel hopping, cognitive radio network, Langford pairing.**

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I. Introduction

BROADCAST in wireless networks is typically offered as a push-type service for distributing important control information from one source to other users in the same network, e.g., from the base station (BS) to a group of users that share radio resources. Moreover, broadcast enables the low-cost delivery of large volumes of popular content (e.g., multimedia content) to users in a wireless network. There are many existing solutions to broadcast in the market, including mobile TV broadcasting (DVB-H) [12], audio casting, massive software updates, content delivery over WiMax [10], and broadcast (or multicast) service offered by the Third-Generation Partnership Project (3GPP) in long-term evolution cellular networks [7].

A single channel, which is referred to as a *broadcast channel*, is usually used by a BS to distribute the same content to a group of users that subscribe to the same service [5]. Meanwhile, a BS may employ multiple broadcast channels for delivering different contents to groups of users that subscribe to different services. A user or a content subscriber is able to successfully receive the broadcasted content when 1) the broadcast channel is available and 2) when the user is located within the transmission range of the BS.

In cognitive radio (CR) networks with secondary users (SUs), the *broadcast failure problem* can occur due to the temporal and spatial variations in channel availability. Specifically, the primary users (PUs, or licensed users) may reclaim the spectrum band where broadcast channels reside, and the unlicensed users have to vacate this channel according to the requirement for protection of licensed users in CR networks. On the other hand, an SU is likely to move from one region where no licensed users exist to a region where licensed users are present. In either case, the broadcast channel becomes unavailable, thereby leading to unsuccessful deliveries of broadcasts.

A vast majority of existing work has focused on tackling this problem in *multihop or ad hoc* CR networks. Instead of relying on a single broadcast channel, the control information is transmitted over a preselected set of broadcast channels, which can be derived based on the neighbor graphs [11]. To determine the minimum broadcast schedule length for a CR network, two heuristics are presented, and they can produce schedules that have either optimal or near-optimal lengths [2]. In [3], a mixed broadcast scheduling algorithm is proposed under the unit disk graph model, which combines the unicast and broadcast collaboratively to obtain a small broadcast latency. To broadcast over multiple channels, the channel hopping (CH) technique is used by CRs without requiring knowledge of the global

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network topology or the requirement of time synchronization information [17]–[19].

We focus on the broadcast failure problem in an *infrastructure-based (or cellular)* CR network. To guarantee the successful broadcast, a BS has to employ a *multichannel broadcast protocol*, i.e., it delivers contents over multiple broadcast channels using broadcast radios, to reduce the chance of colliding with PUs' transmissions in the spatial or temporal domain.

In an infrastructure-based CR network, the BS can be equipped with multiple radio interfaces. The rapidly diminishing prices of the radios have made it feasible to equip a wireless node with multiple radios. Providing the BS with one or more multichannel radios offers a promising avenue for significantly reducing the latency before successful broadcast delivery and enhancing the network capacity by simultaneously exploiting multiple orthogonal channels through different radio interfaces and mitigating interferences through proper channel assignment. However, there is a limit (an optimal value) for the number of radios. We will show later that the optimal number of radios is twice the channel number, via both analytical and simulation results.

There are two design challenges for devising such a multichannel broadcast protocol: 1) How many broadcast channels are needed to guarantee successful broadcast delivery? 2) What is the minimum broadcast latency given a number of broadcast channels?

In this paper, we present a multichannel broadcast protocol, which is called *Mc-Broadcast*, for delivering contents to SUs in an infrastructure-based CR network. Every broadcast radio at a BS selectively transmits over a number of channels via a CH process. The CH sequence is generated using a mathematical construct called *Langford pairing* (LP), which meets the two aforementioned design challenges—The BS is free to customize the number of broadcast channels; meanwhile, the induced broadcast latency can be significantly reduced given a number of broadcast channels. Our analytical and simulation results show that Mc-Broadcast incurs a small broadcast latency, and it guarantees a high successful delivery ratio under various network conditions.

The rest of this paper is organized as follows: We provide background knowledge in Section III. In Section IV, we present the problem formulation in the design of Mc-Broadcast. We present the two broadcast protocols in Section V. In Section VI, we evaluate the performance of Mc-Broadcast using simulation results. We conclude this paper in Section VII.

II. Related Work

The problem of devising multichannel broadcast protocols has not yet been largely studied in the context of an infrastructure-based CR network. To the best of our knowledge, only a limited number of papers have so far tackled the broadcast algorithm design problem in infrastructure-based CR networks. Nevertheless, the solutions are constructed on the basis of assumptions such as the accessibility to and availability of all possible common channels (some protocols only support ensured delivery on a subset of common channels) or the existence of a global (network-wide) or local common broadcast/control channel. In [21], Yang *et al.* proposed a deterministic rendezvous scheme that leverages the deterministic rendezvous sequence (DRSEQ). In [15], Lin *et al.* proposed a jump-stay-based CH protocol for guaranteed broadcast delivery. A network that uses DRSEQ or the jump-stay-based algorithm can ensure successful delivery despite any amount of clock drift between the BS and user nodes. However, neither of them can guarantee broadcast delivery on all possible channels. If PU activity occupies a subset of common channels that both the BS and the user node can sense, a CH algorithm that fails to guarantee broadcast delivery on all possible channels will not enable the BS and the user node to establish delivery links on the rest of common channels that have not yet been occupied due to PU traffic. Arachchige *et al.* proposed a broadcast protocol on the basis of integer linear programming [1]. However, the proposed solutions in [1] impractically assume a common signaling channel for the whole network. The use of a single common control channel simplifies the broadcast delivery establishment process but it creates a single point of failure—It may become unavailable due to the appearance of licensed user signals, and furthermore, a broadcast protocol based on a single common control channel is subject to the jamming problem. In [14], Lazos *et al.* proposed a distributed cluster agreement algorithm called spectrum-opportunity clustering (SOC) that aims to locally establish a common control channel in each cluster and, thus, may be regarded as an alternative. However, SOC requires *a priori* knowledge regarding spectrum sensing capabilities of all secondary nodes, which can be obtained only after some message exchanges happen. In [4], Cormio and Chowdhury proposed a scheme that allows altering the hopping sequence based on the PU activity in the channels so as to find a common control channel, while they become infeasible due to some inherent limitations in various and complicated network environments. In view of these drawbacks, the nodes fail to ensure successful broadcast delivery under certain conditions, which rules it out in broadcast scenarios. In [19], a distributed broadcast protocol without assuming a common control channel is proposed. However, it is devised for multihop CR ad hoc networks while this paper delves into the issues in infrastructure-based CR networks.

III. Technical Background

Here, we provide the background knowledge relevant to LP [13] and the extended LP (ELP).

A. LP

Definition of LP: Given an integer n, LP is a sequence of length $2n$ that consists of two 1's, two 2's, ..., and two n's and satisfies that there are exactly one number between the two 1's, exactly two numbers between the two 2's, ..., and exactly n numbers between the two n's.

Formally, an *LP*, $\{l_i\}_{0\leq i\leq 2n-1}$ of order n, also called a *Langford sequence*, is a permutation of the sequence of $2n$ integers $\{1, 1, 2, 2, 3, 3, \ldots, n, n\}$, and it satisfies the *Langford property*: If $l_i = l_j, 0 \leq i < j \leq 2n-1$, then $j - i = l_i + 1$.

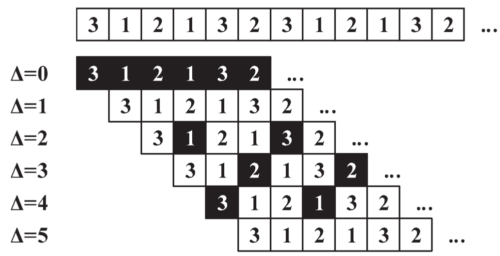

Fig. 1. We put two copies of the LP {3,1,2,1,3,2} together and shift one of them by Δ grids. This figure shows different situations with Δ ranging from 0 to 5.

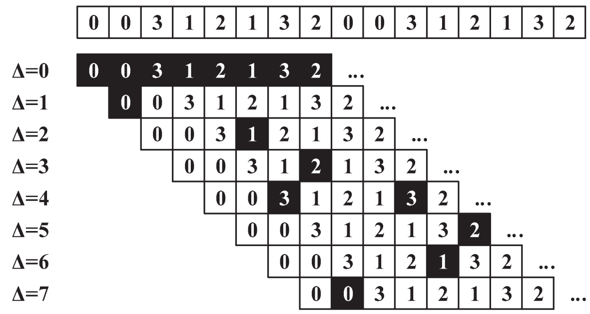

Fig. 2. We put two copies of the ELP {0,0,3,1,2,1,3,2} together and shift one of them by Δ grids. This figure shows different situations with Δ ranging from 0 to 7.

For example, the sequence $l = \{3, 1, 2, 1, 3, 2\}$ is an LP of order $n = 3$. There is only one number (that is 2) between the two 1's, two numbers (they are 1 and 3) between the two 2's, and three numbers (they are 1, 2, and 1) between the two 3's. Given $i = 0$ and $j = 4$, we have $l_0 = l_4 = 3$, and $j - i = 3 + 1$; given other combinations of i and j, the sequence l also satisfies the Langford property.

With respect to the existence of LP of given order n, the following lemma gives a sufficient and necessary condition.

Lemma 1: An LP of order n exists if and only if n is congruent to 0 or 3 modulo 4.

For the proof of Lemma 1, please refer to [16, Th. 1].

Drawback of LP and motivation of ELP: In Fig. 1, we put two copies of the LP {3, 1, 2, 1, 3, 2} together and shift one of them by Δ grids. Fig. 1 shows different situations with Δ ranging from 0 to 5. According to the construction of LP, there are 1 number between the two 1's, when $\Delta = 1 + 1 = 2$, the second copy's second grid with a "1" inside aligns with the first copy's fourth grid with a "1" inside. Since there are 2 numbers between the two 2's, when $\Delta = 2 + 1 = 3$, the second copy's third grid with a "2" inside aligns with the first copy's sixth grid with a "2" inside. We hope that for all possible Δ's, there always exist two aligned grids of the two copies with the same number inside the grids. In Fig. 1, this is false when $\Delta = 1$ or $\Delta = 5$. This is because there are no neighboring positions with the same number in an LP. Thus, we insert two 0's at the beginning of an LP and get an ELP {0, 0, 3, 1, 2, 1, 3, 2}. In Fig. 2, we can observe that for all possible Δ's, there always exist two aligned grids in the two copies with the same number inside the two grids. In Section III-B, we introduce the notion of the ELP.

B. ELP

Slightly different from LP, we require that an ELP contain two 0's (note that an LP does not contain any 0's) and that these two 0's be neighboring. Formally, we define an *ELP*, $\{l'_i\}_{0 \le i \le 2(n+1)-1}$ of order n, as a permutation of the sequence of $2(n+1)$ integers, i.e.,

$$\{0, 0, 1, 1, 2, 2, \ldots, n, n\}.$$

The sequence satisfies the Langford property, i.e., if $l_i = l_j, 0 \le i < j \le 2(n+1) - 1$, then $j - i = l_i + 1$.

For example, the sequence

$$l' = \{0, 0, 3, 1, 2, 1, 3, 2\}$$

is an ELP of order $n = 3$.

Given an LP $\{l_i\}_{0 \le i \le 2n-1}$ of order n, we can easily construct an ELP $\{l'_i\}_{0 \le i \le 2(n+1)-1}$ of the same order by inserting two integers $l'_0 = l'_1 = 0$ at the beginning of the original LP, i.e., by letting $l'_i = 0$ when $i = 0$, 1 and $l'_i = l_{i-2}$ when $1 < i \le 2n + 1$. It immediately follows from Lemma 1 that an ELP of order n exists if n is congruent to 0 or 3 modulo 4.

Given an ELP, l', of order n, any integer $k \in [0, n]$ appears exactly twice in the ELP. Suppose $l'_i = l'_j = k$ and $i < j$, we define l'_i and l'_j as the k-valued pair in the ELP, and $d(k) = j - i$ denotes the distance between the two integers in the k-valued pair. As a result, we have the following lemma.

Lemma 2: Given an ELP, l', of order n, there exists exactly one k-valued pair, for each $k \in [0, n]$.

Inspired by the conclusion in Lemma 2, we use ELP to generate CH sequences for the BS and users, which will be described in Section V.

IV. Problem Formulation

A. System Model

1) Multichannel Broadcast: In a CR network, the BS and the users in the BS's service area are SUs, and they are equipped with CRs operating over broadcast channels that are licensed to the PU. Due to PU's activities, a broadcast channel may become unavailable at any time. Therefore, the BS has to broadcast the content over multiple channels to ensure the successful delivery to users, and we call such a process a *multichannel broadcast* process.

Suppose there are N broadcast channels, labeled as $0, 1, 2, \ldots, N - 1$. The BS is equipped with multiple broadcast radios, labeled as $r_1, r_2, \ldots, r_R$, where R is the total number of the BS's broadcast radios. There are U users $s_1, s_2, \ldots, s_U$ in the service area. Every user is equipped with a single radio interface.

2) Broadcast Via CH: To implement the multichannel broadcast protocol, a BS's broadcast radio or a user's radio can hop across multiple broadcast channels to deliver or to receive the broadcast content. Thus, we use the CH sequence to define the order with which a BS's broadcast radio (or a user's radio) visits the set of broadcast channels.

We consider a time-slotted communication system, where a global system clock exists. The local clock of each node may be synchronized to the global clock or may differ with the global clock by a certain amount of clock drift. A radio is assumed to be capable of hopping between different channels according to a CH sequence and its local clock. A packet can be exchanged between two radios if they hop onto the same channel in the same time slot.

Then, we represent a CH sequence u of period T as a sequence of channel indexes

$$u = \{u_0, u_1, u_2, \ldots, u_i, \ldots, u_{T-1}\}$$

where $u_i \in [0, N-1]$ represents the channel index of the ith time slot of CH sequence u. If $u_i = u_j, \forall\, i, j \in [0, T-1]$, the radio using u as its CH sequence stays on the same channel and does not hop.

Given two CH sequences of period T, u, and v, if there exists $i \in [0, T-1]$ such that $u_i = v_i = h$, where $h \in [0, N-1]$, we say that a *broadcast delivery* occurs between u and v in the ith time slot on broadcast channel h. The ith time slot is called a *delivery slot*, and channel h is called a *delivery channel* between u and v.

Given N channels, let $\mathcal{C}(u, v)$ denote the *set of delivery channels* between two CH sequences u and v. The cardinality of $\mathcal{C}(u, v)$ is called the *number of broadcast delivery channels*, which is denoted by $|\mathcal{C}(u, v)|$, and $|\mathcal{C}(u, v)| \in [0, N]$. The number of broadcast delivery channels measures the number of channels in which successful broadcast delivery occurs, i.e., the diversity of broadcast delivery channels.

Let $\mathcal{T}(u, v)$ denote the *set of delivery slots* between two CH sequences u and v, and $|\mathcal{T}(u, v)| \in [0, T]$. The cardinality of $\mathcal{T}(u, v)$ reflects the number of time slots in which successful broadcast delivery occurs within a period.

3) Broadcast by Multiple Radios: To reduce the broadcast latency, the BS is allowed to use a set of broadcast radios, denoted by $\mathcal{B} = \{r_1, r_2, r_3, \ldots, r_R\}$. Since the BS has multiple radios, the broadcast delivery occurs between the BS and a user s if the broadcast delivery occurs between one of the BS's broadcast radios and the user s's radio, i.e., the broadcast delivery occurs between a radio set $\mathcal{B}$ and a user s if there exists a radio $r_i \in \mathcal{B}$ such that a broadcast delivery occurs between the CH sequences of radios r_i and s. To simplify the notation, we simply use r_i to denote the CH sequence of the BS's broadcast radio $r_i \in \mathcal{B}$ and use s to denote the CH sequence of user s's radio.

The *set of broadcast delivery channels* between the BS with its set of broadcast radios $\mathcal{B}$ and a user s is the union of the sets of broadcast delivery channels between each broadcast radio of the BS and the user s's radio, i.e., let $\mathcal{C}(\mathcal{B}, s) = \bigcup_{r \in \mathcal{B}} \mathcal{C}(r, s)$ denote the set of broadcast delivery channels between the BS with its set of broadcast radios $\mathcal{B}$ and the user s's radio, and the cardinality of $\mathcal{C}(\mathcal{B}, s)$ is called the *number of delivery channels*, denoted by $|\mathcal{C}(\mathcal{B}, s)|$, and $|\mathcal{C}(\mathcal{B}, s)| \in [0, N]$.

Similarly, the *set of delivery slots* between the BS with its set of broadcast radios $\mathcal{B}$ and a user s is the union of the sets of delivery slots between each broadcast radio of the BS and the user s's radio, i.e., let $\mathcal{T}(\mathcal{B}, s) \triangleq \bigcup_{r \in \mathcal{B}} \mathcal{T}(r, s)$ denote the *set of delivery slots* between the BS with its set of broadcast radios $\mathcal{B}$ and the user s's radio, and $|\mathcal{T}(\mathcal{B}, s)| \in [0, T]$.

4) Asynchronous Multichannel Broadcast System: Given a CH sequence u, we use rotate(u, k) to denote a *cyclic rotation* of CH sequence u by k time slots, i.e.,

$$\text{rotate}(u, k) = \{v_0, \ldots, v_j, \ldots, v_{T-1}\}$$

where $v_j = u_{(j+k) \bmod T}$, $j \in [0, T-1]$. For example, given $u = \{0, 1, 2\}$ and $T = 3$, $\text{rotate}(u, 2) = \text{rotate}(u, -1) = \{2, 0, 1\}$.

We define an *asynchronous multichannel broadcast (AMB)* system $\mathcal{M}$ with CH period T as an ordered pair $(\mathcal{B}, \mathcal{U})$:

- $\mathcal{B}$ is the set of CH sequences of period T used by broadcast radios of the BS. Suppose $\mathcal{B} = \{r_1, r_2, r_3, \ldots, r_R\}$, where R is the number of the BS's broadcast radios, and the BS's broadcast radio r_i uses the CH sequence r_i in $\mathcal{B}$.
- $\mathcal{U}$ is the set of CH sequences of period T used by the users. Suppose $\mathcal{U} = \{s_1, s_2, s_3, \ldots, s_U\}$, where U is the number of users, and the user s_j's radio uses the CH sequence s_j in $\mathcal{U}$.

Recall that we simply use r_i and s_j to denote the CH sequences of the BS's broadcast radio $r_i \in \mathcal{B}$ and user s_j's radio, respectively.

An AMB system must satisfy the *rotation closure property*: $\forall\, k, l \in [0, T-1]\ \forall\, s \in \mathcal{U}$, there exists $r \in \mathcal{B}$ such that $|\mathcal{C}(\text{rotate}(s, k), \text{rotate}(r, l))| \geq 1$. Thus, we map the design problem of CH sequences of the BS's broadcast radios and users' radios to the design problem of an AMB system with the rotation closure property. The rotation closure property implies that for all possible clock drifts between the BS and users, every user can have successful broadcast delivery with the BS, i.e., with one of the BS's broadcast radios.

In other words, in an AMB system, the BS, with each broadcast radio using CH sequences in $\mathcal{B}$, can deliver broadcast messages to all users using CH sequences in $\mathcal{U}$ via a CH process for all possible clock drifts.

B. Performance Metrics

Given an AMB system $\mathcal{M} = (\mathcal{B}, \mathcal{U})$, we introduce the following metrics for evaluating its performance.

- **Delivery channel diversity**: The delivery channel diversity for an AMB system measures the lower bound of the number of delivery channels between the BS and an arbitrarily given user for all possible clock drifts. The delivery channel diversity, which is denoted by DIV$(\mathcal{M})$, is the minimum number of delivery channels $|\mathcal{C}(\mathcal{B}, \text{rotate}(s, k))|$ for every $s \in \mathcal{U}$ and every $k \in \mathbb{Z}$, i.e.,

$$\begin{aligned}\text{DIV}(\mathcal{M}) &= \min_{s \in \mathcal{U}, k \in \mathbb{Z}} |\mathcal{C}\left(\mathcal{B}, \text{rotate}(s, k)\right)| \\ &= \min_{s \in \mathcal{U}, k \in \mathbb{Z}} \left| \bigcup_{r \in \mathcal{B}} \mathcal{C}\left(r, \text{rotate}(s, k)\right) \right|.\end{aligned}$$

- **Broadcast latency**: To quantify the broadcast latency, we define the maximum broadcast latency for a given AMB system as the upper bound of the latency before the first successful broadcast delivery between the BS and an arbitrary user on at least one channel for all possible clock drifts, which can be computed by

$$\max_{s\in\mathcal{U},k\in\mathbb{Z}} \left[\min \mathcal{T}\left(\mathcal{B}, \text{rotate}(s,k)\right)\right].$$

- **Delivery ratio**: To measure the proportion of delivery slots in a period, we define the delivery ratio for a CH sequence pair. The delivery ratio for a CH sequence pair r and s, which is denoted by
- $\rho(r,s)$, is

$$\min_{k,l\in\mathbb{Z}} \left(\frac{|\mathcal{T}\left(\text{rotate}(r,k), \text{rotate}(s,l)\right)|}{T}\right).$$

  Then, we introduce the delivery ratio for an AMB system $\mathcal{M} = (\mathcal{B},\mathcal{U})$, which measures the minimum proportion of delivery slots in all time slots.
- To be precise, the delivery ratio is

$$\rho(\mathcal{M}) \triangleq \min_{s\in\mathcal{U},k,l\in\mathbb{Z}} \frac{\sum_{r\in\mathcal{B}} |\mathcal{T}\left(\text{rotate}(r,k), \text{rotate}(s,l)\right)|}{|\mathcal{B}|T}.$$

C. Optimal AMB System

To reduce the chance of broadcast failure due to the presence of PU signals, a BS needs to maximize the number of distinct delivery channels (i.e., achieve full delivery channel diversity). We would ideally want an AMB system to guarantee N distinct delivery channels between a BS's radio and any users when N channels are available.

Our objective is to devise an *optimal* AMB system as a system that has the maximum delivery channel diversity, minimum broadcast latency, and the maximum delivery ratio during a period.

V. Extended Langford Pairing-Based Broadcast Protocols

Here, we construct an AMB system $\mathcal{M} = (\mathcal{B},\mathcal{U})$ based on the ELP. To illustrate our design of the ELP-based AMB system, we first investigate a simple scenario in which the BS has a single radio, i.e., $|\mathcal{B}| = 1$, and there is only a single user, i.e., $|\mathcal{U}| = 1$. Then, we address the general scenario where $|\mathcal{B}|$ and $|\mathcal{U}|$ are generally greater than 1.

A. Fundamentals of ELP-Based CH Sequences

Here, we mainly discuss the ELP-based CH protocol for AMB systems with a single radio pair, i.e., $\mathcal{M} = (\{r\},\{s\})$, where r is the only BS radio, and s is the only user.

1) CH Sequence Generation: We focus on the simple scenario where $|\mathcal{B}| = |\mathcal{U}| = 1$, i.e., $\mathcal{B} = \{r\}$ and $\mathcal{U} = \{s\}$. By investigating the simple scenario, we establish mathematical properties of ELP that are pivotal for the design of a general AMB system that is to be discussed later.

To begin with, we explain why ELP rather than the orginal LP is leveraged. We consider the original LP. If N is congruent to 0 or 3 modulo 4, by Lemma 1, there exists an LP $\{l_i\}_{0\le i\le 2N-1}$ of order N. Suppose both r and s use the CH sequence $\{l_i - 1\}_{0\le i\le 2N-1}$ of period $2N$. If s is one time slot ahead, the broadcast delivery cannot occur between r and s. For example, suppose the channel number $N = 3 \equiv 3 \pmod 4$ and $\{3, 1, 2, 1, 3, 2\}$ is an LP of order 3. Both r and s use the CH sequence $\{2, 0, 1, 0, 2, 1\}$ of period 6. If s is one time slot ahead, the broadcast delivery cannot occur between r and s, i.e., $|\mathcal{C}(r, \text{rotate}(s,1))| = 0$.

However, we can construct an AMB system by using ELP. If the channel number N is congruent to 0 or 1 modulo 4, then $N - 1$ is congruent to 0 or 3, and by Lemma 1, there exists an ELP $\{l'_i\}_{0\le i\le 2N-1}$ of order $N - 1$. For example, when $N = 4$, the ELP-based CH sequence is

$$u = \{0, 0, 3, 1, 2, 1, 3, 2\}.$$

When $N \not\equiv 0, 1 \pmod 4$, we can easily use the *downsizing scheme* or the *padding scheme* to transform it into an AMB system design problem with the channel number N' congruent to 0 or 1 modulo 4.

Downsizing scheme: Suppose the channel number $N \not\equiv 0, 1 \pmod 4$, let N' be $\max\{N' \le N : N' \equiv 0, 1 \pmod 4\}$. It is easy to see that $|N - N'| \le 2$. The donwsizing scheme will limit the set of broadcast channels to a N'-element subset of the original broadcast channel set, e.g., we use $\{0, 1, 2, \ldots, N' - 1\}$ as the new broadcast channel set.

Padding scheme: Suppose the channel number $N \not\equiv 0, 1 \pmod 4$, let N' be $\min\{N' \ge N : N' \equiv 0, 1 \pmod 4\}$. It is easy to see that $|N - N'| \le 2$. In contrast with the downsizing scheme, the padding scheme introduces $N' - N$ more channels but maps them to the original broadcast channels in $\{0, 1, 2, \ldots, N - 1\}$. For example, suppose $N = 7$ and we have $N' = 8$. Now, we introduce one more channel, i.e., channel 7, and adjoin channel 7 to the original broadcast channel set $\{0, 1, 2, \ldots, 6\}$, but channel 7 is an alias of channel 0, i.e., it is mapped to channel 0.

For instance, if $N \equiv 2 \pmod 4$, we can use the downsizing scheme and $|N - N'| = 1$; if $N \equiv 3 \pmod 4$, we can use the padding scheme and $|N - N'| = 1$. This will only lead to a very mild degradation in performance since $|N - N'| = 1$.

With the aid of the downsizing scheme and the padding scheme, we can focus on the AMB design problem with the channel number N' congruent to 0 or 1 modulo 4.

2) Properties of ELP CH Sequences: In an AMB system with a single radio pair (the BS has only one broadcast radio r, and there is only one user s), suppose both r and s use the ELP $\{l'_i\}_{0\le i\le 2N'-1}$ of period $2N'$ as their CH sequences, where $N' \equiv 0, 1 \pmod 4$. Theorem 3 shows that the broadcast delivery between r and s can always occur for all possible clock drifts and that the set of broadcast delivery channels is determined by the clock drift (in the statement of Theorem 3, $k - l$ is the clock drift). Theorem 3 is the basis of achieving delivery channel diversity that will be addressed later in that we can manipulate the set of broadcast delivery channels by deliberately manipulating the clock drift.

Theorem 3: $u = \{l'_i\}_{0\le i\le 2N'-1}$ is an ELP of order $N' - 1$.

1) If $k - l \equiv 0 \pmod{2N'}$, then $\text{rotate}(u,k) = \text{rotate}(u,l)$ and $\mathcal{C}(\text{rotate}(u,k), \text{rotate}(u,l)) = \{0, 1, 2, \ldots, N' - 1\}$.

2) If $k - l \equiv g \not\equiv 0 \pmod{2N'}$, where $|g| \leq N'$, then $\mathcal{C}(\text{rotate}(u,k), \text{rotate}(u,l)) = \{|g| - 1\}$.

The following shows an immediate corollary that for all possible clock drifts, the two radios using the same ELP will always have successful broadcast delivery.

Corollary 4: $u = \{l'_i\}_{0 \leq i \leq 2N'-1}$ is an ELP of order $N' - 1$. $\forall\, k, l \in \mathbb{Z}$, $|\mathcal{C}(\text{rotate}(u,k), \text{rotate}(u,l))| \geq 1$, i.e., $\text{rotate}(u,k)$ and $\text{rotate}(u,l)$ can always have at least one delivery slot.

Theorem 5 discusses the relation between the clock drift, i.e., $k - l$, and the set of delivery slots. It shows that the cardinality of the set of delivery slots can only be 1, 2, and $2N'$. Note that the cardinality of the set of delivery slots is always greater than 1, i.e., the broadcast delivery can always occur between r and s if they use the same ELP.

Theorem 5: $u = \{l'_i\}_{0 \leq i \leq 2N'-1}$ is an ELP of order $N' - 1$.

1) If $k - l \equiv 0 \pmod{2N'}$, then $\mathcal{T}(\text{rotate}(u,k), \text{rotate}(u, l)) = \{0, 1, 2, 3, \ldots, 2N' - 1\}$.
2) If $k - l \equiv g \not\equiv 0 \pmod{2N'}$, where $|g| \leq N'$, then
   a) if $|g| < N'$, then $|\mathcal{T}(\text{rotate}(u,k), \text{rotate}(u,l))| = 1$;
   b) if $|g| = N'$, then $|\mathcal{T}(\text{rotate}(u,k), \text{rotate}(u,l))| = 2$.

B. Simple Broadcast (S-Broadcast) Scheme for a Single CH Sequence Pair

Here, we propose a simple broadcast scheme, which is called *S-Broadcast*, for the simple scenario of the AMB system $(\mathcal{M} = (\{r\}, \{s\}))$ design problem.

Motivation: By Theorem 3, if the broadcast radio r and the user radio s both use the same ELP $u = \{l'_i\}_{0 \leq i \leq 2N'-1}$, the successful broadcast delivery between them is guaranteed; however, delivery channel diversity is not ensured. If the broadcast radio uses cyclic rotated copies of u and the user radio uses periodically extended u, the delivery channel diversity will be increased.

To be precise, the CH sequences for the broadcast and user radios can be generated as follows.

1) The broadcast radio generates its CH sequence

$$r = \prod_{f=1}^{F} \text{rotate}(u, o_f)$$

   where $\{o_f\}_{1 \leq f \leq F}$ is a sequence of integers that are used to deliberately manipulate the clock drift, and $\prod_{f=1}^{F} \theta_f$ denotes the concatenation of strings $\theta_1, \theta_2, \theta_3, \ldots, \theta_F$, i.e., $\prod_{f=1}^{F} \theta_f = \theta_1 \| \theta_2 \| \theta_3 \| \cdots \| \theta_F$.
2) The user radio generates its CH sequence as $s = \prod_{f=1}^{F} u$, which is the periodic extension of u.

We define the *delivery channel determination function* such as

$$\delta : \mathbb{Z} \to \mathbb{Z}.$$

For $k \in \mathbb{Z}$, $k \equiv g \pmod{2N'}$, where $|g| \leq N'$, then $\delta(k) = |g| - 1$. For example, when $N' = 4$, $\delta(1) = 0$, $\delta(2) = 1$, $\delta(3) = 2$, $\delta(4) = 3$, $\delta(5) = 2$, $\delta(6) = 1$, $\delta(7) = 0$, and in particular, $\delta(0) = -1$.

Theorem 6 shows the estimation of delivery channel diversity of the AMB system $\mathcal{M} = (\{r\}, \{s\})$.

Theorem 6: $u = \{l'_i\}_{0 \leq i \leq 2N'-1}$ is an ELP of order $N' - 1$. $r = \prod_{f=1}^{F} \text{rotate}(u, o_f)$, $s = \prod_{f=1}^{F} u$, where $\{o_f\}_{1 \leq f \leq F}$ is a sequence of integers.

1) If $\forall\, k \in \mathbb{Z}$, $\exists 1 \leq f \leq F$, s.t. $o_f - k \equiv 0 \pmod{2N'}$, then the delivery channel diversity for the AMB system $\mathcal{M} = (\{r\}, \{s\})$ is N'.
2) If $\exists k_0 \in \mathbb{Z}$, s.t. $\forall\, 1 \leq f \leq F, o_f - k_0 \not\equiv 0 \pmod{2N'}$, then the delivery channel diversity $\text{DIV}(\mathcal{M})$ for the AMB system $\mathcal{M} = (\{r\}, \{s\})$ satisfies $\min_{k \in \mathbb{Z}} |\{\delta(o_f - k)\}_{1 \leq f \leq F}| \leq \text{DIV}(\mathcal{M}) \leq |\{\delta(o_f - k_0)\}_{1 \leq f \leq F}| \leq F$.

S-Broadcast: Our proposed broadcast protocol for the simple scenario of the AMB system $(\mathcal{M} = (\{r\}, \{s\}))$ design problem, i.e., S-Broadcast, is an asynchronous CH-based channel broadcast protocol that achieves the broadcast latency at most $2N' - 1$, the delivery ratio $\rho = (1/N')$, and full diversity. According to the design of S-Broadcast

- $r = \prod_{f=1}^{2N'} \text{rotate}(u, f - 1)$;
- $s = \prod_{f=1}^{2N'} u$.

Two examples illustrating the CH sequences of S-Broadcast when $N' = 4$ are shown in Fig. 3.

Theorem 7 shows the performance metrics of S-Broadcast. We will prove that an AMB system that implements S-Broadcast has bounded broadcast latency and can achieve full delivery channel diversity. In addition, we calculate its delivery ratio.

Theorem 7: $\mathcal{M} = (\{r\}, \{s\})$ is an AMB system that implements S-Broadcast. Then, we have the following.

1) The broadcast latency is, at most, $2N' - 1$.
2) The delivery ratio $\rho = (4N'/4N'^2) = (1/N')$.
3) The delivery channel diversity is N'.

C. Multichannel Broadcast (Mc-Broadcast) Scheme for Multiple CH Sequence Pairs

In the general AMB system design problem, $|\mathcal{B}|$ and $|\mathcal{U}|$ are generally greater than 1, i.e., $\mathcal{B} = \{r_1, r_2, r_3, \ldots, r_R\}, \mathcal{U} = \{s_1, s_2, s_3, \ldots, s_U\}$, $R, U \geq 1$. Here, we first design two ELP-based CH protocols, i.e., *A-Broadcast* and *L-Broadcast*, for the case $R \geq 2N'$ and the case $R < 2N'$, respectively. In Theorems 9 and 10, we theoretically evaluate their performance.

Then, we propose a multichannel broadcast protocol, which is called *Mc-Broadcast*, that combines both of the two given ELP-based CH protocols, i.e., it adopts A-Broadcast if $R \geq 2N'$, and it adopts L-Broadcast if $R < 2N'$.

Suppose $u = \{l'_i\}_{0 \leq i \leq 2N'-1}$ is an ELP of order $N' - 1$. Theorem 3 shows that broadcast delivery can occur between any two cyclic rotation copies of u, e.g., between $\text{rotate}(u,k)$ and $\text{rotate}(u,l)$, i.e., if $\forall\, 1 \leq i \leq R$, $r_i = \text{rotate}(u, o_i)$, and $\forall\, 1 \leq j \leq U$, $s_j = u$, $\mathcal{M} = (\mathcal{B}, \mathcal{U})$ is an AMB system. However, full delivery channel diversity $(\text{DIV}(\mathcal{M}) = N')$ is not necessarily guaranteed, and we expect to reduce the broadcast latency. To take the advantage of multiple broadcast radios of the BS, it would be beneficial to select o_i's properly to achieve full delivery channel diversity, reduce the broadcast latency

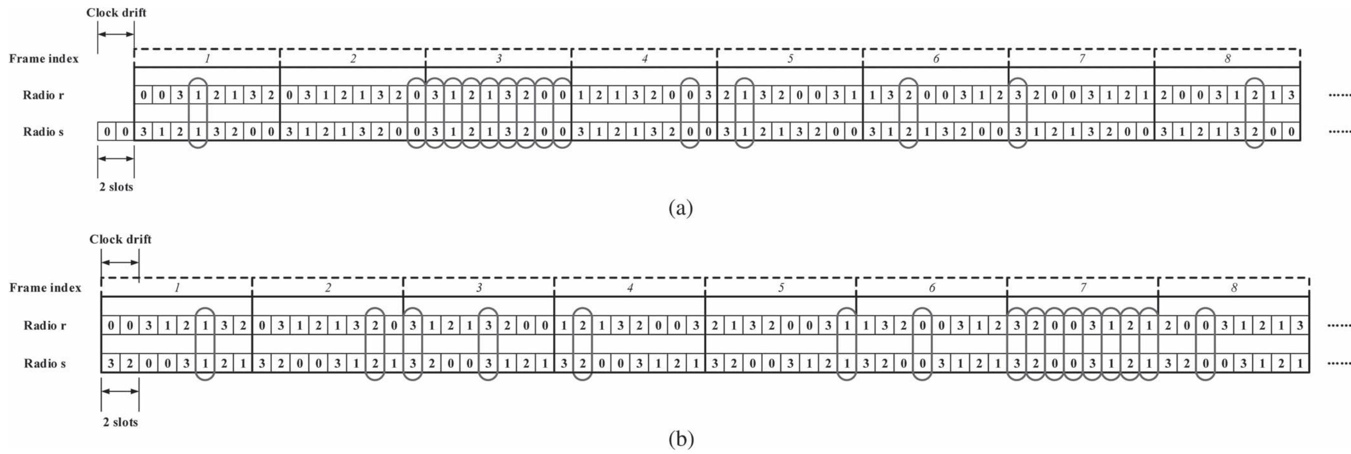

Fig. 3. Example illustrating the CH sequences of S-Broadcast when $N' = 4$. Radio r is the BS's broadcast radio, and there is only one user, i.e., radio s. We can observe that the CH sequences of the two radios can achieve full diversity within a period of $4N'^2 = 64$ time slots, given a clock drift of two slots, either forward or backward. The delivery latency in Fig. 3(a) and (b) is three and five time slots, respectively, both less than $2N' - 1 = 7$. Moreover, the delivery ratios in these two figures are both $(1/N') = (1/4)$.

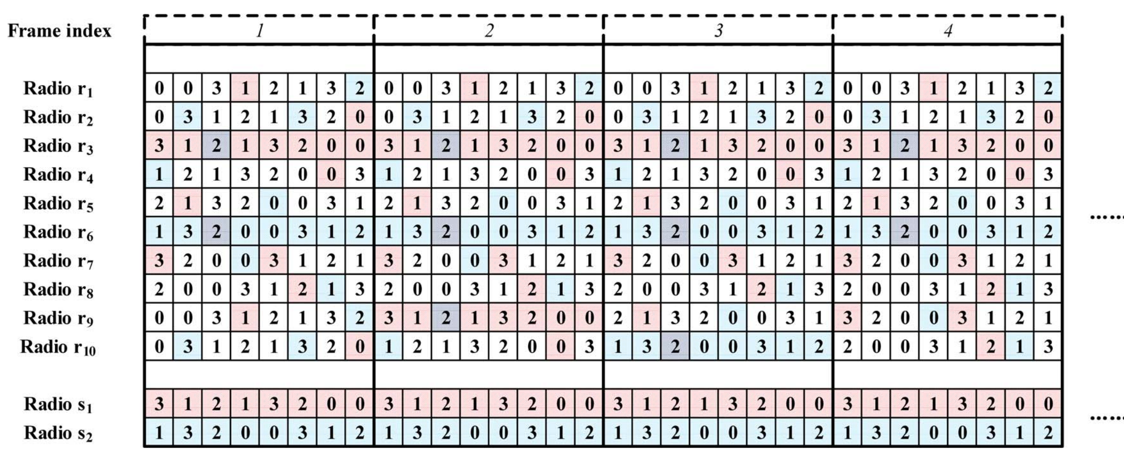

Fig. 4. Example illustrating the CH sequences of A-Broadcast when $N' = 4$. Radios $r_1, r_2, r_3, \ldots, r_{10}$ are the BS's broadcast radios, and there are two users, i.e., radio s_1 and radio s_2. The clock of radio s_1 is two time slots behind (or six time slots ahead of) that of the BS, whereas the clock of radio s_2 is three time slots ahead of (or five time slots behind) that of the BS. The blocks with the red/blue fill pattern represent the time slots in which radio s_1/s_2 receives successful broadcast delivery from the BS, respectively, whereas the blocks with the gray fill pattern represent the time slots in which both users (radios s_1 and s_2) receive successful broadcast delivery from the BS simultaneously. We can observe that both users can receive broadcast delivery from at least $2q = 2\lfloor 10/8 \rfloor = 2$ and, on average, $(R/N') = (10/4) = 2.5$ BS's broadcast radios every time slot and achieve full broadcast channel diversity.

down to zero, and guarantee successful broadcast delivery in every time slot.

We define the *balance sequences*

$$\psi_i^{n,k} \; (0 \leq i < \mathrm{lcm}(n,k)/k)$$

where $\psi_i^{n,k}$ is a sequence of k elements, and $\mathrm{lcm}(n,k)$ is the least common multiple of n and k. We denote the jth element in $\psi_i^{n,k}$ by $\psi_{i,j}^{n,k}$, where $0 \leq j < k$. Moreover, let

$$\psi_{i,j}^{n,k} \triangleq (ik + j) \mod n \in [0, \, n-1].$$

It is easy to see that $\{ik + j | 0 \leq i < \mathrm{lcm}(n,k)/k, 0 \leq j < k\} = \{0, 1, 2, 3, \ldots, \mathrm{lcm}(n,k) - 1\}$. Hence, $\forall n_0 \in [0, \, n-1]$, there exist exactly $\mathrm{lcm}(n,k)/n$ (i,j)-pairs such that $\psi_{i,j}^{n,k} = (ik+j) \bmod n = n_0$. As a result, we have the following lemma.

Lemma 8: $\forall n_0 \in [0, n-1]$, there exist exactly $\mathrm{lcm}(n,k)/n$ (i,j)-pairs such that $\psi_{i,j}^{n,k} = (ik+j) \bmod n = n_0$.

1) A-Broadcast When $R \geq 2N'$: If the BS has a large number of broadcast radios, i.e., $R \geq 2N'$, the AMB system can be designed to have guaranteed successful broadcast delivery in every time slot and full delivery channel diversity. Suppose $R = 2qN' + w$, where $q = \lfloor (R/2N') \rfloor \geq 1$, and $0 \leq w < 2N'$. We design a protocol, which is called A-Broadcast, as follows.

- $\forall\, 1 \leq i \leq 2qN'$, $r_i = \mathrm{rotate}(u, (i-1) (\bmod 2N'))$.
- $\forall\, 1 \leq i \leq w$, from time slot $\lfloor (t_B/2N') \rfloor$ to time slot $\lfloor (t_B/2N') \rfloor + (2N' - 1)$, radio $r_{2qN'+i}$ uses

$$\mathrm{rotate}\left(u, \psi^{2N',w}_{\lfloor \frac{t_B}{2N'} \rfloor \bmod \mathrm{lcm}(2N',w)/w, i-1}\right)$$

  as its CH sequence, where t_B is the BS's local clock time (i.e., according to the BS's local clock, it is the t_Bth time slot).
- $\forall\, 1 \leq j \leq U$, $s_j = u$.

An example illustrating the CH sequences of A-Broadcast when $N' = 4$ is shown in Fig. 4.

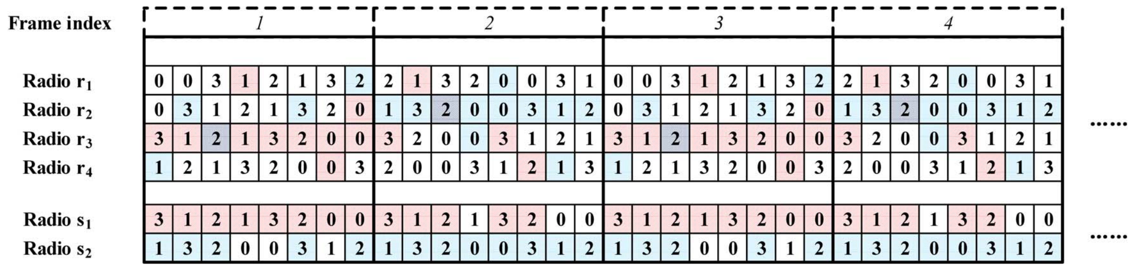

Fig. 5. Example illustrating the CH sequences of L-Broadcast when $N' = 4$. Radios r_1, r_2, r_3, and r_4 are the BS's broadcast radios, and there are two users, i.e., radio s_1 and radio s_2. The clock of radio s_1 is two time slots behind (or six time slots ahead of) that of the BS, whereas the clock of radio s_2 is three time slots ahead of (or five time slots behind) that of the BS. The blocks with the red/blue fill pattern represent the time slots in which radio s_1/s_2 receives successful broadcast delivery from the BS, respectively, whereas the blocks with the gray fill pattern represent the time slots in which both users (radios s_1 and s_2) receive successful broadcast delivery from the BS simultaneously. We can observe that both users can receive broadcast delivery from $R/N' = 4/4 = 1$ BS's broadcast radio, on average, and achieve full delivery channel diversity every $2N' \cdot \lceil 2N'/R \rceil = 8 \cdot (8/4) = 16$ time slots.

Theorem 9 evaluates the performance of A-Broadcast. As shown in Theorem 9, we have the following.

- A-Broadcast has zero broadcast latency.
- It achieves full delivery channel diversity. Moreover, the interval (i.e., $2N'$) is bounded, i.e., within every $2N'$ time slots, it achieves full delivery channel diversity.
- The delivery ratio is $1/N'$.
- In every time slot, every user can receive broadcast delivery from at least $2q$ BS radios and, on average, R/N' radios.

Theorem 9: $\mathcal{M} = (\mathcal{B},\mathcal{U})$ is an AMB system that implements A-Broadcast, where $|\mathcal{B}| \geq 2N'$. Then, we have the following.

1) The broadcast latency is 0.
2) Within every $2N'$ slots, it achieves full delivery channel diversity.
3) The delivery ratio $\rho = 1/N'$.
4) In every time slot, $\forall\, s \in \mathcal{U}$, s can have successful deliveries with at least $2q$ radios in $\mathcal{B}$.
5) In every time slot, $\forall\, s \in \mathcal{U}$, s can have successful deliveries with $R/N' = 2q + (w/N')$ radios in $\mathcal{B}$ on average.

2) L-Broadcast When $R < 2N'$: If the number of BS broadcast radios R is less than $2N'$, we can also use the balance sequence to achieve delivery channel diversity and maximize delivery ratio. We design an ELP-based protocol, i.e., L-Broadcast, as follows.

- $\forall\, 1 \leq i \leq R$, from time slot $\lfloor t_B/2N' \rfloor$ to time slot $\lfloor t_B/2N' \rfloor + (2N' - 1)$, radio r_i uses

$$\text{rotate}\left(u, \psi^{2N',R}_{\lfloor \frac{t_B}{2N'} \rfloor \bmod \text{lcm}(2N',R)/R, i-1}\right)$$

  as its CH sequence, where t_B is the BS's local clock time (i.e., according to the BS's local clock, it is the t_Bth time slot).
- $\forall\, 1 \leq j \leq U$, $s_j = u$.

An example illustrating the CH sequences of L-Broadcast when $N' = 4$ is shown in Fig. 5.

Theorem 10 analyzes the performance of an AMB system that implements L-Broadcast. Note that the aforementioned S-Broadcast is a special case of L-Broadcast. The following is shown in Theorem 10.

- The broadcast latency of the AMB system that implements L-Broadcast is, at most, $2N' - 1$.
- It achieves full delivery channel diversity. Moreover, the interval (i.e., $2N' \cdot \lceil 2N'/R \rceil$) is bounded, i.e., it achieves full delivery channel diversity every $2N' \cdot \lceil 2N'/R \rceil$ slots.
- In every time slot, every user can receive broadcast delivery from R/N' radios on average.

Theorem 10: $\mathcal{M} = (\mathcal{B},\mathcal{U})$ is an AMB system that implements L-Broadcast, where $|\mathcal{B}| < 2N'$. Then, we have the following.

1) $\forall\, s \in \mathcal{U}$, the broadcast latency is, at most, $2N' - 1$.
2) It achieves full delivery channel diversity every $2N' \cdot \lceil 2N'/R \rceil$ slots.
3) The delivery ratio $\rho = 1/N'$.
4) In every time slot, $\forall\, s \in \mathcal{U}$, s can have broadcast delivery with R/N' radios in $\mathcal{B}$ on average.

VI. Performance Evaluation

A. Simulation Settings

Here, we compare the performance of the proposed Mc-Broadcast protocol and other existing protocols, including the distributed broadcast protocol (or simply called "distributed") proposed in [19] and the random CH scheme, via simulation results. In each simulated network cell, the BS has R broadcast radios available; a number of U users are connected to the BS, and each user has a single radio interface; each radio can access N broadcast channels (i.e., the number of broadcast channels available to the network is N). The BS or its connected users generate their CH sequences using the agreed broadcast scheme (i.e., either Mc-Broadcast, the distributed protocol, or the random CH protocol) and perform CH in accordance with the sequences. Once two nodes hop onto the same channel that is free of PU signals, the broadcast delivery between them is successful.

Traffic model: We simulated a number of X primary transmitters operating on X channels independently, and these channels were randomly chosen in each simulation run. In most existing work, it is assumed that a PU transmitter follows a "busy/idle" transmission pattern on a licensed channel [6], [8], and we assume the same traffic pattern here. That is, the busy period has a fixed length of b time slots, and the idle period follows an exponential distribution with a mean of l time slots. A channel is considered "unavailable" when PU signals are present in it. The intensity of PU traffic can be characterized as $PU = (X/N) \cdot (b/l + b)$.

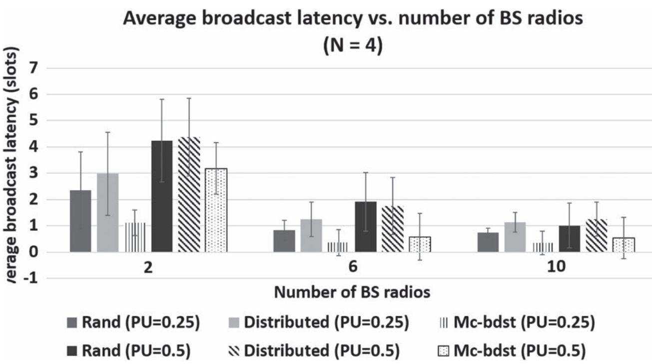

Fig. 6. Average broadcast latency versus the number of broadcast radios at the BS ($N = 4$), with a 95% confidence interval attached to each bar.

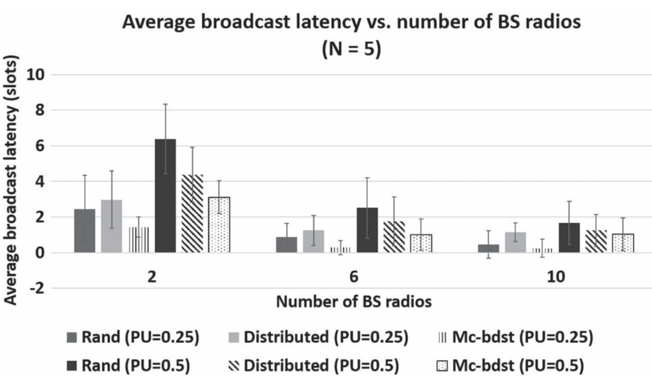

Fig. 7. Average broadcast latency versus the number of broadcast radios at the BS ($N = 5$), with a 95% confidence interval attached to each bar.

Random clock drift: In a CR network, the BS and the user may lose clock synchronization or even link connectivity at any time when they experience the broadcast failure problem due to PU affection. Hence, the clock of the BS and those of the users are not necessarily synchronized. In each simulation run, each secondary node (the BS and the users) determines its clock time independently of other nodes. Note that the radios of the BS are synchronized, and there is a random clock drift between the BS and any of its connected users.

B. Impact of Simulation Parameters

To show the impact of the number of broadcast radios R, we conduct a set of simulations by varying R (R can be 2, 6, and 8) under different scenarios in which the number of channels available to the network N can be 4, 5, and 8. In our simulations, each network has $U = 1000$ user nodes. PU traffic PU ranges from 0 (no PU activity) to 0.5, and its activity pattern is described in Section VI-A. Specifically, in the traffic model prescribed in Section VI-A, X is a randomly chosen integer in $[1, N]$, and l is randomly chosen from $[1, 2N]$; then, according to the desired PU traffic PU, we determine b such as $b = l/(1 - PU \cdot N/X) - l$. Our proposed Mc-Broadcast protocol and other existing CH protocols are compared. We show the impact of the number of broadcast radios in terms of the average broadcast latency (see Figs. 6–8) and the maximum broadcast latency (i.e., the worst-case performance; see Figs. 9–11). Additionally, we add 95% confidence intervals to these figures.

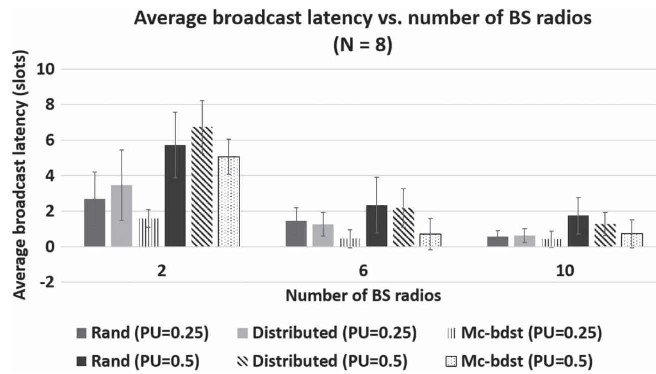

Fig. 8. Average broadcast latency versus the number of broadcast radios at the BS ($N = 8$), with a 95% confidence interval attached to each bar.

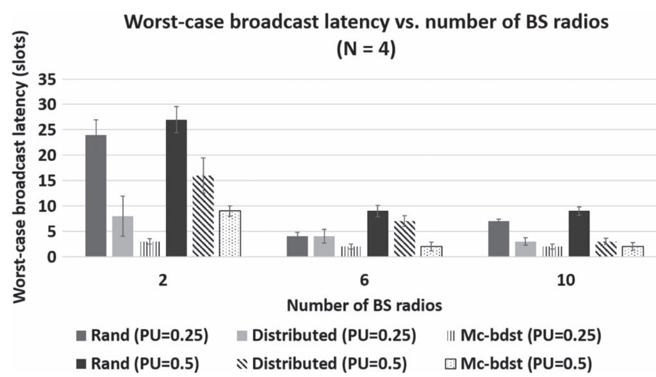

Fig. 9. Worst-case broadcast latency versus the number of broadcast radios at the BS ($N = 4$), with a 95% confidence interval attached to each bar.

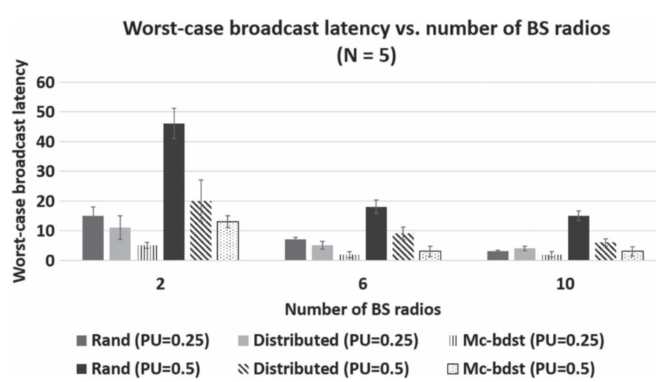

Fig. 10. Worst-case broadcast latency versus the number of broadcast radios at the BS ($N = 5$), with a 95% confidence interval attached to each bar.

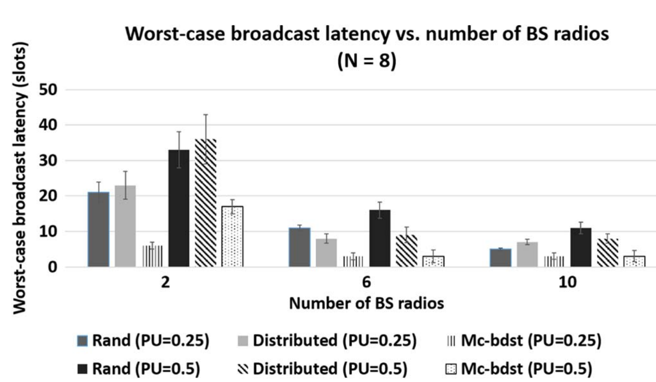

Fig. 11. Worst-case broadcast latency versus the number of broadcast radios at the BS ($N = 8$), with a 95% confidence interval attached to each bar.

TABLE I
AVERAGE DELIVERY RATIO

Network	$PU = 0$			$PU = 0.25$			$PU = 0.5$		
	Mc-Broadcast	Rand.	Dist.	Mc-Broadcast	Rand.	Dist.	Mc-Broadcast	Rand.	Dist.
1	0.125	0.125756	0.102156	0.09375	0.093801	0.088193	0.0625	0.057219	0.059482
2	0.125	0.125487	0.081807	0.09375	0.094214	0.096395	0.0625	0.064242	0.064952
3	0.125	0.124920	0.121225	0.09375	0.093950	0.086093	0.0625	0.062298	0.059874
4	0.125	0.124958	0.131584	0.09375	0.093069	0.091691	0.0625	0.063116	0.069454
5	0.125	0.125042	0.090916	0.09375	0.094237	0.095495	0.0625	0.063719	0.054845

TABLE II
MINIMUM DELIVERY RATIO

Network	$PU = 0$			$PU = 0.25$			$PU = 0.5$		
	Mc-Broadcast	Rand.	Dist.	Mc-Broadcast	Rand.	Dist.	Mc-Broadcast	Rand.	Dist.
1	0.125	0.092773	0.119845	0.09375	0.067383	0.0767121	0.0625	0.036133	0.0609353
2	0.125	0.099609	0.107964	0.09375	0.068359	0.0695130	0.0625	0.045573	0.0482546
3	0.125	0.105469	0.116589	0.09375	0.07666	0.0862363	0.0625	0.04541	0.0536983
4	0.125	0.109375	0.112300	0.09375	0.072266	0.0799273	0.0625	0.044531	0.0437375
5	0.125	0.103841	0.098455	0.09375	0.077474	0.0886291	0.0625	0.048828	0.0569861

1) Impact of the Number of Broadcast Radios: Average broadcast latency: Figs. 6–8 show the simulation results with respect to the average broadcast latency under the conditions $N = 4$, 5, and 8, respectively. It is illustrated that as the number of broadcast radios increases, it takes fewer time slots on average before the first successful delivery for both schemes under different PU traffic patterns. This implies that a greater number of broadcast radios is conducive to mitigating the average broadcast latency.

It is noteworthy that for different numbers of available channels and different PU traffic patterns, the average latency of Mc-Broadcast is smaller than those of other existing broadcast protocols.

Maximum broadcast latency: Figs. 9–11 show the simulation results with respect to the maximum (worst-case) broadcast latency, which characterizes the worst-case performance of broadcast schemes, under the conditions $N = 4$, 5, and 8, respectively. These results support the observation that as the number of broadcast radios increases, it takes fewer time slots in the worst case before the first successful delivery under different PU traffic patterns. Similarly, it is noteworthy that the proposed Mc-Broadcast scheme outperforms other existing protocols in the worst case regardless of different numbers of available channels and PU traffic patterns.

Limit for the number of broadcast radios: There is a limit for the number of broadcast numbers. There exists an optimal number of radios in terms of cost and latency reduction performance. The theoretical optimal number of radios is twice the channel number. Although generally a greater number of broadcast radios is conducive to mitigating broadcast latency, according to the simulation results, there is only marginal improvement in terms of cost and latency reduction performance if the BS is equipped with more than the theoretical optimal number of radios. For example, if a network has four broadcast channels and the BS has been equipped with eight radios, its performance almost remains the same when adding more radios to the BS.

2) Impact of PU Traffic Patterns: In Figs. 6–11, we can observe that the increase in PU traffic intensity causes a degradation in performance (in terms of both the average broadcast latency and the maximum broadcast latency) of both schemes. However, under the same PU traffic pattern and with the numbers of available channels and broadcast radios fixed, the proposed Mc-Broadcast scheme outperforms other existing broadcast protocols both on average and in the worst case.

C. Delivery Ratio

In this set of simulations, we measure the average delivery ratios (see Table I) and the minimum (worst-case) delivery ratios (see Table II) if a network uses the proposed Mc-Broadcast scheme, the random CH scheme, or the distributed broadcast protocol proposed in [19] under different PU traffic patterns. We simulate five networks (numbered from Network 1 to Network 5) in which the BS is equipped with eight broadcast radios, and there are $N = 8$ broadcast channels with PU traffic ranging from 0 to 0.5. In each simulated network, there are $U = 1000$ user nodes. The setting of a traffic model is described in Section VI-A. Specifically, in the traffic model prescribed in Section VI-A, X is a randomly chosen integer in $[1, N]$, and l is randomly selected from $[1, 2N]$; then, according to the desired PU traffic PU, we determine b such as $b = l/(1 - PU \cdot N/X) - l$.

The delivery ratio is the percentage of the number of slots in which successful broadcast delivery occurs in a period. The average delivery ratio of a network is the average of the delivery ratios between the BS and each user node in this network. The minimum (worst-case) delivery ratio of a network is the minimum of the delivery ratios between the BS and each user node.

We can observe from Table I that with the same PU traffic, the average delivery ratios of the three simulated broadcast protocols come very close. Despite that, Mc-Broadcast has the following advantages.

- Mc-Broadcast significantly outperforms the other two existing protocols in the worst-case performance (i.e., the minimum delivery ratio). Table II shows that Mc-Broadcast has an advantage of 20%–40% over the random CH protocol.

- Mc-Broadcast's performance is very stable. Tables I and II show that its worst-case performance coincides with its average performance regardless of different PU activities. Furthermore, the worst-case/average delivery ratio that Mc-Broadcast maintains with PU traffic being $PU \in [0,1]$ is exactly the theoretical optimal value $(1-PU)/N'$ proposed in Theorems 9 and 10. In contrast, the other existing protocols have obvious variation as to their worst-case performance.

VII. Conclusion

In this paper, we have proposed an efficient broadcast protocol that allows a network BS to broadcast over multiple channels via a CH process such that the broadcasts can be successfully delivered to SUs. The CH sequence is generated using a mathematical construct called LP. Our approach is novel in that the following statements hold.

- It significantly reduces the broadcast latency and achieves full broadcast channel diversity between the BS and the users while their clocks are asynchronous.
- It guarantees the upper-bounded interval to achieve full broadcast channel diversity.
- Its performance is very stable in view of the worst-case performance that coincides with the average performance.

Our analytical and simulation results show that the proposed method reduces the broadcast latency and achieves full broadcast channel diversity, and it is robust to the broadcast failure (broadcast link breakage) caused by PU activities under various network conditions.

Appendix A
Proof of Theorem 3

If $k \equiv l \pmod{2N'}$, then it is obvious that $\text{rotate}(u,k) = \text{rotate}(u,l)$

$$\mathcal{C}\left(\text{rotate}(u,k), \text{rotate}(u,l)\right) = \{0,1,2,\ldots,N'-1\}$$

and that $\mathcal{T}(\text{rotate}(u,k),\text{rotate}(u,l)) = \{0,1,2,3,\ldots,2N'-1\}$.

Now, we consider the case where $k-l \equiv g \not\equiv 0 \pmod{2N'}$, $|g| \leq N'$. We have $\mathcal{C}(\text{rotate}(u,k),\text{rotate}(u,l)) = \mathcal{C}(\text{rotate}(u,k-l),u) = \mathcal{C}(\text{rotate}(u,g),u)$. Moreover, $\forall\, 0 \leq j \leq T-1$, $\text{rotate}(u,g)_j = u_{j+g \bmod T}$, where $\text{rotate}(u,g)_j$ denotes the jth time slot of the CH sequence $\text{rotate}(u,g)$.

If $g > 0$, we have $1 \leq g \leq N'$ and $0 \leq g-1 \leq N'-1$. By the definition of ELP, there exists $i_0, j_0 \in [0, 2N'-1]$ such that $i_0 < j_0$ and $u_{i_0} = u_{j_0} = g-1$. Thus, we have $j_0 - i_0 = g$ and

$$\text{rotate}(u,g)_{i_0} = u_{i_0+g} = u_{j_0} = g-1.$$

Since $\text{rotate}(u,g)_{i_0} = g-1$ and we have proved that $u_{i_0} = g-1$, we thus obtain that

$$g-1 \in \mathcal{C}\left(\text{rotate}(u,g),u\right) = \mathcal{C}\left(\text{rotate}(u,k),\text{rotate}(u,l)\right).$$

Suppose $x \in \mathcal{C}(\text{rotate}(u,k),\text{rotate}(u,l)) = \mathcal{C}(\text{rotate}(u,g),u)$, then $\exists 0 \leq i_0 \leq 2N'-1$ s.t. $\text{rotate}(u,g)_{i_0} = u_{i_0+g} = u_{i_0} = x$, and by the definition of ELP, $x+1 = (i_0+g) - i_0 = g$, $x = g-1$. Therefore, $\mathcal{C}(\text{rotate}(u,k),\text{rotate}(u,l)) = \{|g|-1\}$.

If $g < 0$, we have $1 \leq -g \leq N'$, $0 \leq -g \leq N'-1$, and $\mathcal{C}(\text{rotate}(u,g),u) = \mathcal{C}(u,\text{rotate}(u,-g))$. Moreover, we reduce it to the case where $g > 0$. Hence, $\{|g|-1\} = \{(-g)-1\} = \mathcal{C}(u,\text{rotate}(u,-g)) = \mathcal{C}(\text{rotate}(u,g),\text{rotate}(u)) = \mathcal{C}(\text{rotate}(u,k), \text{rotate}(u,l))$. ■

Appendix B
Proof of Corollary 4

By Theorem 3, if $k-l \equiv 0 \pmod{2N'}$

$$|\mathcal{C}\left(\text{rotate}(u,k),\text{rotate}(u,l)\right)| \geq N' \geq 1$$

and if $k-l \not\equiv 0 \pmod{2N'}$

$$|\mathcal{C}\left(\text{rotate}(u,k),\text{rotate}(u,l)\right)| = 1.$$

Thus, we conclude that $\forall\, k,l \in \mathbb{Z}$

$$|\mathcal{C}\left(\text{rotate}(u,k),\text{rotate}(u,l)\right)| \geq 1.$$

■

Appendix C
Proof of Theorem 5

Proof: If $k \equiv l \pmod{2N'}$ [e.g., Frame 3 in Fig. 3(a)], then it is obvious that $\text{rotate}(u,k) = \text{rotate}(u,l)$ and that

$$\mathcal{T}\left(\text{rotate}(u,k),\text{rotate}(u,l)\right) = \{0,1,2,3,\ldots,2N'-1\}.$$

Now, we consider the case where $k-l \equiv g \not\equiv 0 \pmod{2N'}$, $|g| \leq N'$.

By Theorem 3, $|\mathcal{T}(\text{rotate}(u,k),\text{rotate}(u,l)| \geq 1$, and $\mathcal{C}(\text{rotate}(u,k),\text{rotate}(u,l)) = \{|g|-1\}$, i.e., the delivery channel $h = |g|-1$ and $\exists 0 \leq i_0 \leq 2N'-1$ s.t. $\text{rotate}(u,k)_{i_0} = \text{rotate}(u,l)_{i_0} = h$. There are only two h's in $\text{rotate}(u,k)$ and $\text{rotate}(u,l)$, respectively. Without loss of generality, suppose that the other h in $\text{rotate}(u,k)$ is $\text{rotate}(u,k)_{[i_0-(h+1)](\bmod 2N')}$, and the other h in $\text{rotate}(u,l)$ is $\text{rotate}(u,l)_{[i_0+(h+1)](\bmod 2N')}$.

If $i_0+(h+1) \equiv i_0-(h+1) \pmod{2N'}$, then we have $2(h+1) \equiv 2|g| \equiv 0 \pmod{2N'}$. Since $g \neq 0$, we obtain that $|g| = N'$. Therefore, if $|g| < N'$, we have $i_0+(h+1) \not\equiv i_0-(h+1) \pmod{2N'}$ and $|\mathcal{T}(\text{rotate}(u,k),\text{rotate}(u,l))| = 1$; if $|g| = N'$ [e.g., Frame 3 in Fig. 3(b)], then $i_0+(h+1) \equiv i_0-(h+1) \pmod{2N'}$, and we have $\text{rotate}(u,k)_{i_0} = \text{rotate}(u,l)_{i_0} = h$ and $\text{rotate}(u,k)_{[i_0-(h+1)](\bmod 2N')} = \text{rotate}(u,l)_{[i_0+(h+1)](\bmod 2N')} = h$.

Therefore, $|\mathcal{T}(\text{rotate}(u,k),\text{rotate}(u,l))| = 2$. ■

Appendix D
Proof of Theorem 6

If $\forall\, k \in \mathbb{Z}$, $\exists 1 \leq f \leq F$, s.t. $o_f - k \equiv 0 \pmod{2N'}$, then $\forall\, k,l \in \mathbb{Z}$, $\exists 1 \leq f \leq F$, s.t. $o_f - (l-k) \equiv 0 \pmod{2N'}$. We obtain that

$$\begin{aligned} N' &\geq |\mathcal{C}(\text{rotate}(r,k),\text{rotate}(s,l)| \\ &= |\mathcal{C}(\text{rotate}(r,k-l),s| \\ &\geq |\mathcal{C}\left(\text{rotate}\left(u,o_f-(l-k)\right),u\right)| \\ &= |\mathcal{C}(u,u)| = N'. \end{aligned}$$

Thus, $\text{DIV}(\mathcal{M}) = N'$.

If $\exists k_0 \in \mathbb{Z}$, s.t. $\forall\, 1 \leq f \leq F, o_f - k_0 \not\equiv 0 (\mathrm{mod}\, 2N')$, by Theorem 3, $\mathcal{C}(\mathrm{rotate}(u, o_f), \mathrm{rotate}(u, k)) = \{\delta(o_f - k_0)\}$. Thus, $|\mathcal{C}(r, \mathrm{rotate}(s, k_0))| = |\{\delta(o_f - k_0)\}_{1 \leq f \leq F}| \leq F$, and $\min_{k \in \mathbb{Z}} |\{\delta(o_f - k)\}_{1 \leq f \leq F}| \leq \mathrm{DIV}(\mathcal{M}) \leq |\{\delta(o_f - k_0)_{1 \leq f \leq F}\}| \leq F$. ■

Appendix E
Proof of Theorem 7

Proof: We group every $2N'$ slots of r into a frame, i.e., $\{r_i\}_{0 \leq i < 2N'}$ forms the first frame, $\{r_i\}_{2N' \leq i < 4N'}$ forms the second frame, etc., as shown in Fig. 3(a) and (b). The segment of r in the fth frame, which is denoted by $r[f]$, is rotate$(u, f - 1)$, and the segment of s in the fth frame, which is denoted by $s[f]$, is always rotate(u, k_0) for some $k_0 \in \mathbb{Z}$ for all frames. For example, in Fig. 3(b), $s[f] = \mathrm{rotate}(u, 2)$ for $\forall\, f \in \mathbb{N}$. By Corollary 4, in the first frame, broadcast delivery can occur between $r[1] = u$ and $s[1] = \mathrm{rotate}(u, k_0)$. Thus, the broadcast latency is, at most, the frame size $2N' - 1$. Since every period is homogeneous, with loss of generality, we consider the first period, i.e., from the first frame to the $2N'$th frame. For $1 \leq f \leq 2N'$, $\{[(f - 1) - k_0](\mathrm{mod}\, 2N')\}$ is a permutation of $\{0, 1, 2, 3, \ldots, 2N' - 1\}$. By Theorem 5, we have

$$\begin{aligned}
&|\mathcal{T}(r, s)| \\
&= \sum_{f=1}^{n} |\mathcal{T}(r[f], s[f])| \\
&= \sum_{f=1}^{n} |\mathcal{T}(\mathrm{rotate}(u, f - 1), \mathrm{rotate}(u, k_0))| \\
&= \sum_{g(f)=0} |\mathcal{T}(\mathrm{rotate}(u, f - 1), \mathrm{rotate}(u, k_0)| \\
&\quad + \sum_{g(f)=N'} |\mathcal{T}(\mathrm{rotate}(u, f - 1), \mathrm{rotate}(u, k_0)| \\
&\quad + \sum_{0<g(f)<N'} |\mathcal{T}(\mathrm{rotate}(u, f - 1), \mathrm{rotate}(u, k_0)| \\
&= 1 \cdot 2N' + 1 \cdot 2 + (2N' - 2) \cdot 1 = 4N'
\end{aligned}$$

where $g(f) \equiv (f - 1) - k_0 (\mathrm{mod}\, 2N')$, and $|g(f)| \leq N'$. Thus, $\rho(r, s) = |\mathcal{T}(r, s)|/T = 4N'/(2N' \cdot 2N') = 1/N'$. Since $\exists 1 \leq f_0 \leq F$ s.t. $(f_0 - 1) - k_0 \equiv 0\ (\mathrm{mod}\, 2N')$ [in Fig. 3(a), $f_0 = 3$], we have $\mathrm{rotate}(u, f_0 - 1) = \mathrm{rotate}(u, k_0)$ and $N' \geq |\mathcal{C}(r, s)| \geq |\mathcal{C}(\mathrm{rotate}(u, f_0 - 1), \mathrm{rotate}(u, k_0))| = N'$. Thus, $|\mathcal{C}(r, s)| = N'$. ■

Appendix F
Proof of Theorem 9

Given an arbitrary $s \in \mathcal{U}$, we call the first $2N'$ time slots the first frame. The segment of s in the first frame is rotate(u, k_0) for some $k_0 \in \mathbb{Z}$. The segments of radios $r_1, r_2, r_3, \ldots, r_{2N'}$ in the first frame are rotate(u, l_0), rotate$(u, l_0 + 1)$, rotate$(u, l_0 + 2), \ldots,$ rotate$(u, l_0 + 2N' - 1)$. $\exists r_0 \in \mathcal{B}$ s.t. the segment of r_0 in the first frame is exactly rotate(u, k_0). Thus, $\mathcal{B}$ and s can have successful broadcast delivery in the first frame, and therefore, the broadcast latency is 0.

For arbitrarily given $2N'$ consecutive slots, within which the segments of CH sequences $r_1, r_2, r_3, \ldots, r_{2N'}$ are rotate(u, m_0), rotate$(u, m_0 + 1)$, rotate$(u, m_0 + 2), \ldots,$ rotate$(u, m_0 + 2N' - 1)$ for some $m_0 \in \mathbb{Z}$, respectively, and the segment of s is rotate(u, n_0) for some $n_0 \in \mathbb{Z}$, and we have $\exists r_0 \in \mathcal{B}$ s.t. the segment of r_0 is exactly rotate(u, n_0). Thus, full delivery diversity is achieved.

The period T of $\mathcal{M}$ is $2N'\mathrm{lcm}(2N', w)/w$. Within a period, without loss of generality, suppose for $i \in [1, 2qN']$, $r_i = \mathrm{rotate}(u, (i - 1)(\mathrm{mod}\, 2N'))$, for $i \in [2qN' + 1, R]$, we have $r_i = \mathrm{rotate}(u, \psi^{2N', w}_{\lfloor t_B/2N' \rfloor \bmod \mathrm{lcm}(n,k)/k, i - sqN' - 1})$, and $s = \mathrm{rotate}(u, \Delta)$ due to the clock drift. By Theorem 5, we have

$$\begin{aligned}
&\sum_{i=1}^{2qN'} |\mathcal{T}(r_i, s)| \\
&\quad = q \sum_{i=1}^{2N'} |\mathcal{T}(r_i, s)| \\
&\quad = \frac{T}{2N'} \cdot q \sum_{i=1}^{2N'} |\mathcal{T}(\mathrm{rotate}(u, i - 1), \mathrm{rotate}(u, \Delta))| \\
&\quad = \frac{T}{2N'} \cdot q\, [1 \cdot 2N' + 1 \cdot 2 + (2N' - 2) \cdot 1] \\
&\quad = \frac{T}{2N'} \cdot q(4N') = 2qT
\end{aligned}$$

where $g(i) \equiv (i - 1) - \Delta \mod 2N'$, and $|g(i)| \leq N'$. By Theorem 5 and Lemma 8, we have

$$\begin{aligned}
&\sum_{i=1}^{w} |\mathcal{T}(r_{2qN'+i}, s)| \\
&\quad = \frac{\mathrm{lcm}(2N', w)}{2N'} \cdot \sum_{i=1}^{2N'} |\mathcal{T}(\mathrm{rotate}(u, i), \mathrm{rotate}(u, \Delta))| \\
&\quad = \frac{\mathrm{lcm}(2N', w)}{2N'} \cdot [1 \cdot 2N' + 1 \cdot 2 + (2N' - 2) \cdot 1] \\
&\quad = \frac{\mathrm{lcm}(2N', w)}{2N'} \cdot 4N' \\
&\quad = 2 \cdot \frac{wT}{2N'} \\
&\quad = \frac{wT}{N'}
\end{aligned}$$

where $g(i) \equiv (i - 1) - \Delta \mod 2N'$, and $|g(i)| \leq N'$. Thus, the delivery ratio

$$\begin{aligned}
\rho &= \frac{\sum_{i=1}^{R} |\mathcal{T}(r_i, s)|}{RT} \\
&= \frac{\sum_{i=1}^{2qN'} |\mathcal{T}(r_i, s)| + \sum_{i=1}^{w} |\mathcal{T}(r_{2qN'+i}, s)|}{RT} \\
&= \frac{1}{R} \cdot (2q + w/N') \\
&= \frac{1}{N'}.
\end{aligned}$$

For an arbitrarily given time slot $t \in [0, T-1]$, suppose the tth time slot of CH sequence s is $s_t = h \in [0, N'-1]$. Suppose $t = 2q'N' + r'$, where $0 \le r' < 2N'$, and suppose the segments of radios in $r_1, r_2, r_3, \ldots, r_{2N'}$ from time slot $2q'N'$ to time slot $2q'N' + (2N'-1)$ are rotate(u, m_0), rotate(u, m_0+1), rotate$(u, m_0+2), \ldots,$ and rotate$(u, m_0 + 2N'-1)$, respectively. By the definition of ELP, there exists $0 \le i < j < 2N'$ such that rotate$(u, m_0)_i =$ rotate$(u, m_0)_j = h$. Thus, for $k \in [0, 2N'-1]$, we have

$$\text{rotate}(u, m_0+k)_{(i-k) \bmod 2N'} = \text{rotate}(u, m_0)_i = h$$

$$\text{rotate}(u, m_0+k)_{(j-k) \bmod 2N'} = \text{rotate}(u, m_0)_j = h.$$

Let $k_1 \stackrel{\Delta}{=} (i - r') (\text{mod}\, 2N')$, $k_2 \stackrel{\Delta}{=} (j - r') (\text{mod}\, 2N')$, and we have rotate$(u, m_0 + k_1)_{r'} =$ rotate$(u, m_0 + i - r')_{r'} =$ rotate$(u, m_0)_{i-r'+r'} =$ rotate$(u, m_0)_i = h$, and similarly, rotate$(u, m_0 + k_2)_{r'} =$ rotate$(u, m_0 + j - r')_{r'} =$ rotate$(u, m_0)_{j-r'+r'} =$ rotate$(u, m_0)_j = h$. Since $i \ne j$, $k_1 \ne k_2$. We obtain that in time slot t, s can have broadcast delivery with radio r_{k_1+1} and radio r_{k_2+1}. Similarly, in time slot t, s can broadcast delivery with these $2q$ radios $r_{k_1+1+2pN'}, r_{k_2+1+2pN'}$, where $p \in [0, q-1]$. In an average time slot, s can have broadcast delivery with

$$\frac{\sum_{i=1}^{R} |\mathcal{T}(r_i, s)|}{T} = R\rho = \frac{R}{N'} = 2q + \frac{w}{N'}$$

radios in $\mathcal{B}$. ■

Appendix G
Proof of Theorem 10

Proof: The period T of $\mathcal{M}$ is $(2N' \cdot \text{lcm}(2N', R))/R$.

For an arbitrarily given $s \in \mathcal{U}$, we group every $2N'$ slots of the BS into a frame, i.e., the first $2N'$ slots according to the BS's clock form the first frame, the next $2N'$ slots form the second frame, etc., and thus, the segment of r_1 in the first frame is u, and the segment of s in the first frame is rotate(u, Δ) for some $\Delta \in \mathbb{Z}$. By Corollary 4, r_1 and s can have broadcast delivery within the first frame, and thus, the broadcast latency is, at most, $2N' - 1$.

Since every period is homogeneous, without loss of generality, we consider the first period, i.e., from the first frame to the Fth frame, where $F = T/2N' = \text{lcm}(2N', R)/R$.

Without loss generality, suppose

$$r_i[f] = \text{rotate}\left(u, \psi_{f-1,i-1}^{2N',R}\right) = [(f-1)R + (i-1)] \bmod 2N'$$

and $s[f] =$ rotate(u, Δ) for some $\Delta \in \mathbb{Z}$ due to the clock drift. Thus, if f is incremented from f_0 to $f_0 + (\lceil 2N'/R \rceil - 1)$ and i is incremented from 0 to $R-1$, $\psi_{f-1,i-1}^{2N',R}$ can take values of $0, 1, 2, 3, \ldots, 2N'-1$. There exists $\tilde{f} \in [f_0, f_0 + (\lceil 2N'/R \rceil - 1)]$ and $\tilde{i} \in [0, R-1]$ such that $r_{\tilde{i}}[\tilde{f}] =$ rotate$(u, \Delta) = s[f]$. Therefore, it achives full diversity every $\lceil 2N'/R \rceil$ frames, i.e., $2N' \cdot \lceil 2N'/R \rceil$ slots.

By Theorem 5 and Lemma 8, we have

$$\begin{aligned}
&\sum_{i=1}^{R} |\mathcal{T}(r_i, s)| \\
&= \frac{\text{lcm}(2N', R)}{2N'} \cdot \sum_{i=1}^{2N'} |\mathcal{T}(\text{rotate}(u, i), \text{rotate}(u, \Delta))| \\
&= \frac{\text{lcm}(2N', R)}{2N'} \cdot [1 \cdot 2N' + 1 \cdot 2 + (2N'-2) \cdot 1] \\
&= \frac{\text{lcm}(2N', R)}{2N'} \cdot 4N' \\
&= 2\text{lcm}(2N', R) \\
&= \frac{RT}{N'}
\end{aligned}$$

where $g(i) \equiv (i-1) - \Delta \bmod 2N'$, and $|g(i)| \le N'$. Therefore, the delivery density

$$\rho = \frac{\sum_{i=1}^{R} |\mathcal{T}(r_i, s)|}{RT} = \frac{RT}{RT \cdot N'} = \frac{1}{N'}.$$

In an average time slot, s can have broadcast delivery with

$$\frac{\sum_{i=1}^{R} |\mathcal{T}(r_i, s)|}{T} = R\rho = \frac{R}{N'}$$

radios in $\mathcal{B}$. ■

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