Improving Targets Localization Accuracy by Their Spatial-Temporal Relationships in Wireless Sensor Networks

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Abstract

Due to the low cost and capabilities of sensors, wireless sensor networks (WSNs) are promising for military and civilian surveillance of people and vehicles. One important aspect of surveillance is target localization. A location can be estimated by collecting and analyzing sensing data on signal strength, time of arrival, time difference of arrival, or angle of arrival. However, this data is subject to measurement noise and sensitive to environmental conditions, so its location estimates can be inaccurate. In this paper, we add a novel process to further improve localization accuracy after the initial location estimates are obtained from some existing algorithm. Our idea is to exploit the consistency of the spatial-temporal relationships of targets we track. Spatial relationships are the relative target locations in a group and temporal

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relationships are the locations of a target at different times. We first develop algorithms that improve location estimates using spatial and temporal relationships of targets separately, and then together. We prove mathematically that our methods improve localization accuracy. Furthermore, we relax the condition that targets should strictly keep their relative positions in the group and also show that perfect time synchronization is not required. Simulations were also conducted to test the algorithms. They used initial target location estimates from existing signal-strength and time-of-arrival algorithms and implemented our own algorithms. The results confirmed improved localization accuracy, especially in the combined algorithms. Since our algorithms use the features of targets and not the underlying WSNs, they can be built on any localization algorithm whose results are not satisfactory.
Keywords: localization, spatial-temporal, surveillance, tracking, wireless sensor networks

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1. Introduction

Wireless sensor networks (WSNs) are systems of small, low-powered networked sensing devices deployed over an area of interest to monitor interesting events and perform application-specific tasks in response to them. Typically they monitor people and vehicles. Due to their low cost and capabilities, nonimaging sensors can avoid occlusion and confusion in depth, can violate privacy less of those tracked, can be easier to conceal from adversary countermeasures, and can be distributed over large areas to provide uniform coverage [22]. Wireless sensor networks have good potential in persistent pervasive surveillance in military and civilian contexts. A good example is monitoring of a bridge for sabotage or explosive-device emplacement in Afghanistan today: explosive devices rarely can be sensed directly, but sensors can be used to distinguish normal activity from suspicious loitering by tracking the positions of people and vehicles.

A fundamental problem addressed in surveillance is target localization. When a moving target enters a sensor network, it affects sensor readings by its properties such as temperature, sound, light, magnetism and seismic vibration. Most of the existing localization algorithms for wireless sensor networks employ a centralized approach that requires all sensory data to be delivered to the central processor where the data are processed to locate a target by techniques such as signal strength [2, 12, 21], time of arrival or time difference of arrival [5, 28], and angle of arrival [18, 20], etc. However, the accuracy of these localization techniques is affected by measurement noise and environmental conditions [2, 6], and often there is still room to improve the localization accuracy.

In this paper, we add an extra process to improve localization accuracy after the initial location estimates are obtained from some existing localization algorithm. We tackle the problem from a new direction: Instead of taking more and better measurements and using more refinement processes on the side of WSNs, we exploit the *spatial-temporal relationships* of tracking targets themselves. The spatial relationship of targets is defined as the relative locations of the targets within their group while the temporal relationship is defined as a target's locations at different times. To attain their goals, malicious people like criminals and terrorists rarely act as individuals. They usually have a team of collaborators as in actions such as explosive-device emplacement. Acting in a group can also confuse both manual and automated surveillance, especially if they dress and behave similarly. In nature, fish swim in schools to avoid sharks, and birds fly in flocks to avoid hawks, to make it difficult for predators to track each individual. On the other hand, the co-location of the elements of group provides an extra condition to locate them better. Similarly, if we know a target's locations over time, we can better locate it. These observations motivate us to explore methods to improve localization accuracy using spatial-temporal relationships of targets.

The relationship between our algorithms and the existing ones is that our algorithms take the location estimates from the existing ones as inputs and apply the spatial-temporal relationships of targets to further improve localization accuracy. In other words, our algorithms are built on the underlying existing localization algorithms which we refer to as *baseline* algorithms. Our algorithms can be used if the location estimates from the baseline algorithms are not satisfactory. Thus, our algorithms do not incur extra communication cost except the computation cost. For the baseline localization algorithm that uses a centralized approach, the major cost comes from the communication between sensors and the central processor. To reduce the communication cost, a cluster-based hierarchical routing protocol [11, 16] can be used: sensors are organized into clusters and cluster heads are selected. Routing is conducted by ordinary sensors first sending the sensory data to their cluster heads and then the cluster heads sending the data to the central processor. To the best of our knowledge, the idea to improve localization accuracy using the spatial-temporal relationships of targets has not been discussed before besides our preliminary effort in [4]. A major advance in the current work is that we prove mathematically that our methods can further improve localization accuracy. Another advance is the relaxation of the requirement for strict consistency of group positions and the un-necessity of perfect time synchronization. Still another advance is the results of the first good testing of the performance of our algorithms in simulations, including now data exploiting time of arrival.

The main contributions of this paper are:

- We propose new methods to improve localization accuracy using spatial-temporal relationships of targets, both separately and together.
- We prove mathematically that our methods can improve localization accuracy.
- We relax the condition that group members should strictly keep their relative positions in the group and show that perfect time synchronization is not required.

- We validate the effectiveness of our methods with simulations.
- Because our methods use the features related to targets themselves, and not those of WSNs, they can be used as an extra process to further improve localization accuracy if the results from the existing algorithms are not satisfactory.

The remainder of this paper is organized as follows: Section 2 references the related work. Section 3 formulates the problem of using spatial-temporal relationships among targets to improve localization accuracy. Sections 4, 5, and 6 put forward methods to achieve that. Section 7 provides theoretical analysis. Section 8 shows simulation results of the proposed approaches. Section 9 draws conclusions.

2. Related Work

Before a target can be localized, the locations of the sensors in WSNs should be identified. In the literature, there are many sensor localization algorithms for WSNs [3, 7, 10, 17, 19, 25, 26], so we ignore this problem here.

A target can be localized using many types of signals it sends out: temperature, sound, light, magnetism and seismic vibration. [1] lists different signals sent from an unarmed person, a soldier and a vehicle. An unarmed person is likely to disrupt the environment thermally, seismically, acoustically, electrically, chemically, and optically. A soldier is more likely to be detected by magnetic sensor because of the presence of metal on his body, e.g. a weapon. And a vehicle is likely to disrupt the environment thermally, seismically, acoustically, electrically, magnetically, chemically, and optically. Therefore a sensing modality can be determined based on the types of targets to be tracked.

The common localization techniques for nonimaging sensors are:

- Signal Strength [2, 12, 21]: Locations are estimated by comparing signal strengths at different locations, using a theoretical or empirical model to translate signal strength into distance.

- Time of Arrival/Time Difference of Arrival [5, 28]: Locations are estimated by comparing times of arrival of the signal or time difference of arrival at different locations. The propagation time can be directly translated into distance, based on the known signal propagation speed.

- Angle of Arrival [18, 20]: Locations are estimated by comparing relative angles of the signal at different locations.

There are also some existing papers on target tracking by data fusion. If we can obtain relatively independent estimates of the probability of a target at a location, we can combine the probability distributions by Bayesian, Dempster-Shafer, fuzzy, particle-filter, or other methods. Representative examples of this approach are [9, 13, 27, 29]. Some works have considered the effects of groups of targets moving together [14, 23, 24]. But these target tracking methods take better measurements and use more refinement processes on the side of WSNs. None of them has considered the spatial-temporal relationships of tracking targets. Also these methods are used to predict target locations or evaluate targets correlation and not to improve localization accuracy from estimated values.

3. Problem Formulation

We assume that sensor nodes are deployed over a two-dimensional area. They are responsible for tracking moving targets which intrude the monitored region. We further assume that targets move together in a group. When targets move in a group, their relative locations create the *spatial* relationships among them. And when a target moves along a constant direction with a constant speed, its *footsteps* in the trajectory, the locations of the target at different times, are *temporally* related.

The problem is formulated as follows: given the preliminary estimated locations of targets by some algorithm that treats the target locations as independent of one another and over time, is it possible to improve localization accuracy further by exploiting through spatial relationships and temporal relationships of the targets? To avoid confusion, we denote a target's actual location as (x, y), the estimate of the position from some existing algorithm as (x^*, y^*), and the adjusted estimate after applying our methods as (x', y').

4. Improving Localization Accuracy by Spatial Relationships

In this section, we put forward an algorithm which we refer to as *LAS* to improve localization accuracy using the spatial relationships among targets.

Suppose there are N targets $u_1, u_2, \cdots, u_N$ moving across a sensor field with their relative locations and apparent angles unchanged. For example, as shown in Figure 1, we can imagine four targets A, B, C, and D stay at the four vertexes of an iron frame. They do not change their relative positions during movement. At a certain time t, the estimated locations of these targets, calculated by some localization algorithm, are: (x^*_{u1}, y^*_{u1}),

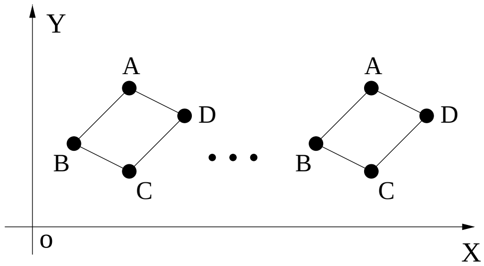


Figure 1: The four targets keep their configuration during movement

(x_{u2}^*, y_{u2}^*), $\cdots$, (x_{uN}^*, y_{uN}^*). Suppose initially the relative position of a target $u_i (1 \leq i \leq N)$ to the group's centroid o, which is known, is (x_i, y_i). When the group moves to a certain location in the sensor field, the relative location of target u_i to the group's centroid o' is (x_i', y_i'). If we translate o' to o (see Figure 2), the relationship between (x_i, y_i) and (x_i', y_i') can be expressed as:

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{1}$$

Here, α is a bearing after the group moves to a certain location, and r is an adjustment parameter.

From the estimated values: $(x_{u1}^*, y_{u1}^*), (x_{u2}^*, y_{u2}^*), \cdots, (x_{uN}^*, y_{uN}^*)$, we calculate the centroid o^* of the group and the relative position of each target (x_i^*, y_i^*) to their centroid o^* as follows:

$$\begin{cases} o_x^* = \dfrac{\sum\limits_{i=1}^{N} x_{ui}^*}{N}, \\ \\ o_y^* = \dfrac{\sum\limits_{i=1}^{N} y_{ui}^*}{N}. \end{cases}$$

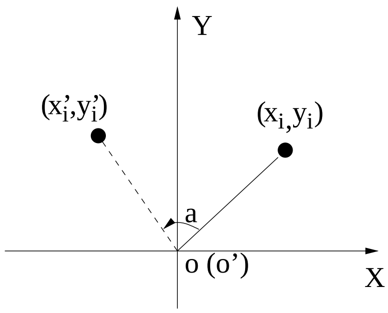


Figure 2: A target's location relative to the centroid of the group before and after the movement

$$\begin{cases} x_i^* = x_{ui}^* - o_x^*, \\ y_i^* = y_{ui}^* - o_y^*. \end{cases}$$

We translate o^* to o' and we want to minimize the error term between our adjusted relative locations of all targets $(x_i', y_i')(1 \leq i \leq N)$ and the estimated relative locations by some localization algorithm of all targets $(x_i^*, y_i^*)(1 \leq i \leq N)$. Thus, it is to minimize function:

$$f(\alpha, r) = \sum_{i=1}^{N} [(x_i' - x_i^*)^2 + (y_i' - y_i^*)^2] \quad (2)$$

To do that, apply Eq. (1),

$$f(\alpha, r) = \sum_{i=1}^{N} [(rx_i \cos\alpha - ry_i \sin\alpha - x_i^*)^2 + (rx_i \sin\alpha + ry_i \cos\alpha - y_i^*)^2]$$

Then the following partial derivatives should be equal to 0:

$$\begin{cases} \dfrac{\partial f(\alpha, r)}{\partial r} = 0, \\ \\ \dfrac{\partial f(\alpha, r)}{\partial \alpha} = 0. \end{cases}$$

After solving equations, α and r are as follows:

$$\begin{cases} \alpha = \arctan \dfrac{\sum\limits_{i=1}^{N}(x_i y_i^* - x_i^* y_i)}{\sum\limits_{i=1}^{N}(x_i x_i^* + y_i y_i^*)}, \\ \\ r = \dfrac{\sum\limits_{i=1}^{N}[(x_i x_i^* + y_i y_i^*)\cos\alpha + (x_i y_i^* - x_i^* y_i)\sin\alpha]}{\sum\limits_{i=1}^{N}(x_i^2 + y_i^2)}. \end{cases}$$

Knowing α and r, the adjusted location of each target u_i relative to centroid o^* can be calculated according to Eq. (1). Thus, the location of target u_i can be adjusted to:

$$\begin{aligned} \begin{bmatrix} x'_{ui} \\ y'_{ui} \end{bmatrix} &= \begin{bmatrix} o_x^* \\ o_y^* \end{bmatrix} + \begin{bmatrix} x'_i \\ y'_i \end{bmatrix} \\ &= \begin{bmatrix} o_x^* \\ o_y^* \end{bmatrix} + r \begin{bmatrix} x_i \cos\alpha - y_i \sin\alpha \\ x_i \sin\alpha + y_i \cos\alpha \end{bmatrix} \end{aligned} \tag{3}$$

From Eq. (3), the time complexity of improving localization accuracy by spatial relationships is $O(N)$.

5. Improving Localization Accuracy by Temporal Relationships

In this section, we improve localization accuracy by looking at the footsteps of a single target over time. Suppose we know that a target travels

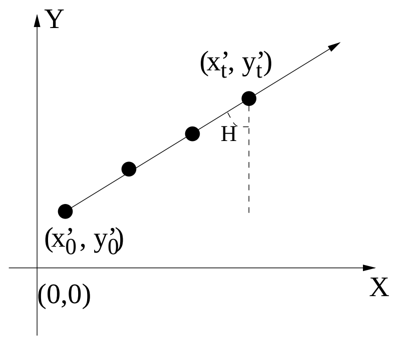


Figure 3: A target's trajectory over a sensor field

along a line with constant speed and direction, and the estimated locations of this target by using some localization method from time 1 to T are: $(x_1^*, y_1^*), (x_2^*, y_2^*), \cdots, (x_T^*, y_T^*)$. Here, time 1 may not be the time when the target starts moving. It can be any time during its movement that we start to observe. The problem is to adjust these estimated footsteps to make them closer to the actual locations. We explore methods in two conditions: (1) the speed and direction of the target are known, and (2) the speed and direction of the target are unknown. We refer to the resulting algorithms as *LAT1* and *LAT2*, respectively.

5.1. Speed and direction are known

Suppose the speed of the target is v and its bearing is H (see Figure 3), the starting point of this target, which is unknown, is (x'_0, y'_0), then the location of this target at time $t (1 \leq t \leq T)$ should be:

$$\begin{cases} x'_t = x'_0 + tv\sin(H) \\ y'_t = y'_0 + tv\cos(H) \end{cases} \tag{4}$$

Now our task is to reduce the error term between the adjusted locations and the estimated locations. So it is to minimize function:

$$f(x_0', y_0') = \sum_{t=1}^{T} [(x_t' - x_t^*)^2 + (y_t' - y_t^*)^2] \tag{5}$$

To do that, apply Eq. (4),

$$\begin{aligned} f(x_0', y_0') &= \sum_{t=1}^{T} [(x_0' + tv\sin(H) - x_t^*)^2 \\ &+ (y_0' + tv\cos(H) - y_t^*)^2] \end{aligned}$$

Then the following partial derivatives should be equal to zero:

$$\begin{cases} \dfrac{\partial f(x_0', y_0')}{\partial x_0'} = 0, \\ \\ \dfrac{\partial f(x_0', y_0')}{\partial y_0'} = 0. \end{cases}$$

Thus, x_0' and y_0' can be found as:

$$\begin{cases} x_0' = \frac{1}{T} \sum_{t=1}^{T} x_t^* - \dfrac{T+1}{2} v \sin(H), \\ \\ y_0' = \frac{1}{T} \sum_{t=1}^{T} y_t^* - \dfrac{T+1}{2} v \cos(H). \end{cases} \tag{6}$$

Next, each estimated location (x_t^*, y_t^*) can be adjusted to (x_t', y_t') as follows:

$$\begin{bmatrix} x_t' \\ y_t' \end{bmatrix} = \begin{bmatrix} x_0' + tv\sin(H) \\ y_0' + tv\cos(H) \end{bmatrix} \tag{7}$$

From Eq. (7), the time complexity of improving localization accuracy by temporal relationships when speed and direction are known is $O(T)$.

5.2. Speed and direction are unknown

Suppose the unknown speed of the target is v_x in x direction and v_y in y direction and the starting point of this target, which is also unknown, is (x'_0, y'_0), then the location of this target at time $t(1 \leq t \leq T)$ should be:

$$\begin{cases} x'_t = x'_0 + tv_x \\ y'_t = y'_0 + tv_y \end{cases} \tag{8}$$

Now our task is to reduce the error term between the adjusted locations and the estimated locations. So it is to minimize function:

$$f(v_x, v_y, x'_0, y'_0) = \sum_{t=1}^{T} [(x'_t - x^*_t)^2 + (y'_t - y^*_t)^2] \tag{9}$$

To do that, apply Eq. (8),

$$f(v_x, v_y, x'_0, y'_0) = \sum_{t=1}^{T} [(x'_0 + tv_x - x^*_t)^2 + (y'_0 + tv_y - y^*_t)^2]$$

Then the following partial derivatives should be equal to zero:

$$\begin{cases} \dfrac{\partial f(v_x, v_y, x'_0, y'_0)}{\partial v_x} = 0, \\ \\ \dfrac{\partial f(v_x, v_y, x'_0, y'_0)}{\partial v_y} = 0, \\ \\ \dfrac{\partial f(v_x, v_y, x'_0, y'_0)}{\partial x'_0} = 0, \\ \\ \dfrac{\partial f(v_x, v_y, x'_0, y'_0)}{\partial y'_0} = 0. \end{cases}$$

Thus, v_x, v_y, x'_0 and y'_0 can be found as:

$$\begin{cases} v_x = \dfrac{4\sum\limits_{t=1}^{T} t x_t^* - 2(T+1)\sum\limits_{t=1}^{T} x_t^*}{4\sum\limits_{t=1}^{T} t^2 - T(T+1)^2}, \\ \\ v_y = \dfrac{4\sum\limits_{t=1}^{T} t y_t^* - 2(T+1)\sum\limits_{t=1}^{T} y_t^*}{4\sum\limits_{t=1}^{T} t^2 - T(T+1)^2}, \\ \\ x'_0 = \dfrac{1}{T}\sum_{t=1}^{T} x_t^* - \dfrac{T+1}{2} v_x, \\ \\ y'_0 = \dfrac{1}{T}\sum_{t=1}^{T} y_t^* - \dfrac{T+1}{2} v_y. \end{cases} \tag{10}$$

Next, each estimated location (x_t^*, y_t^*) can be adjusted to (x'_t, y'_t) as follows:

$$\begin{bmatrix} x'_t \\ y'_t \end{bmatrix} = \begin{bmatrix} x'_0 + t v_x \\ y'_0 + t v_y \end{bmatrix} \tag{11}$$

From Eq. (11), the time complexity of improving localization accuracy by temporal relationships when speed and direction are unknown is $O(T)$.

6. Improving Localization Accuracy by Combining the Spatial and Temporal Relationships

In this section, we combine the spatial relationships with the temporal relationships. We do it in two orders: (1) Spatial first and then temporal, and (2) temporal first and then spatial. We refer to the resulting algorithms as $LAST$ and $LATS$, respectively.

6.1. Spatial first, temporal next

In the spatial first, temporal next combination, suppose we know the estimated locations of all group members over the past T time points, we first use Eq. (3) to adjust their estimated locations using the spatial relationships in the group. After that, we apply Eq. (7) or Eq. (11) to adjust each target according to its footsteps in the past T time points.

6.2. Temporal first, spatial next

In the temporal first, spatial next combination, suppose we know the estimated locations of all group members over the past T time points, we first use Eq. (7) or Eq. (11) to adjust each target according to its footsteps in the past T time points. After that, we apply Eq. (3) to adjust the locations of all the targets using the spatial relationships in the group.

7. Theoretical Analysis

In this section, we first prove mathematically that our methods of using spatial-temporal relationships of targets can improve localization accuracy, then relax the constraint that group members should strictly maintain their relative positions in the group and show that perfect time synchronization is not required.

7.1. Theorem proving

To prove our results, we define the *tracking error*$= \sum_{i=1}^{N}(|ab|^2)$. $|ab|^2$ is the distance square of a target's estimated location/adjusted location a to its actual location b. We prove that the tracking error of the estimated values is greater or equal to the adjusted values: $\sum_{i=1}^{N}[(x_i^* - x_i)^2 + (y_i^* - y_i)^2] \geq \sum_{i=1}^{N}[(x_i' - x_i)^2 + (y_i' - y_i)^2]$.

Theorem 1. *Suppose there are N targets $u_1, u_2, \cdots, u_N$ moving coherently in a group and the estimated locations of these targets, calculated by some localization algorithm, are: $(x_{u1}^*, y_{u1}^*), (x_{u2}^*, y_{u2}^*), \cdots, (x_{uN}^*, y_{uN}^*)$ and if the targets keep their relative positions in the group, adjust them by Eq. (3) improves localization accuracy.*

Proof. Since (x_i, y_i), (x_i^*, y_i^*) and (x_i', y_i') are targets' relative positions to their centroids in the actual values, estimated values and adjusted values and o, o' and o^* are translated to the same point, it is equivalent to prove that

$$\sum_{i=1}^{N}[(x_i^* - x_i)^2 + (y_i^* - y_i)^2] \geq \sum_{i=1}^{N}[(x_i' - x_i)^2 + (y_i' - y_i)^2] \tag{12}$$

From Eq. (2), $f(\alpha, r) = \sum_{i=1}^{N}[(x_i' - x_i^*)^2 + (y_i' - y_i^*)^2]$

From $\dfrac{\partial f(\alpha, r)}{\partial \alpha} = 0$ and $\dfrac{\partial f(\alpha, r)}{\partial r} = 0$,

$$\sum_{i=1}^{N}[-(x_i' - x_i^*)y_i' + (y_i' - y_i^*)x_i'] = 0 \tag{13}$$

$$\sum_{i=1}^{N}[(x'_i - x^*_i)x'_i + (y'_i - y^*_i)y'_i] = 0 \tag{14}$$

Eq. (13) $\times \sin\alpha-$ Eq. (14) $\times \cos\alpha$,

$$\sum_{i=1}^{N}[(x'_i - x^*_i)x_i + (y'_i - y^*_i)y_i] = 0 \tag{15}$$

$$\begin{aligned}
\textstyle\sum_{i=1}^{N} \quad & [(x^*_i - x_i)^2 + (y^*_i - y_i)^2] \\
= \quad & \sum_{i=1}^{N}[(x^*_i - x'_i)^2 + (x'_i - x_i)^2 + 2(x^*_i - x'_i)(x'_i - x_i) \\
+ \quad & (y^*_i - y'_i)^2 + (y'_i - y_i)^2 + 2(y^*_i - y'_i)(y'_i - y_i)]
\end{aligned}$$

Apply Eq. (14),

$$\begin{aligned}
= \quad & \sum_{i=1}^{N}[(x^*_i - x'_i)^2 + (x'_i - x_i)^2 - 2(x^*_i - x'_i)x_i \\
+ \quad & (y^*_i - y'_i)^2 + (y'_i - y_i)^2 - 2(y^*_i - y'_i)y_i] \\
= \quad & \sum_{i=1}^{N}[(x'_i - x_i)^2 + (x^*_i - x'_i)(x^*_i - x'_i - 2x_i) \\
+ \quad & (y'_i - y_i)^2 + (y^*_i - y'_i)(y^*_i - y'_i - 2y_i)]
\end{aligned}$$

Apply Eq. (14),

$$\begin{aligned}
= \quad & \sum_{i=1}^{N}[(x'_i - x_i)^2 + (x^*_i - x'_i)(x^*_i - 2x_i) \\
+ \quad & (y'_i - y_i)^2 + (y^*_i - y'_i)(y^*_i - 2y_i)]
\end{aligned}$$

Apply Eq. (15),

$$\begin{aligned}
= \quad & \sum_{i=1}^{N}[(x'_i - x_i)^2 + (x^*_i - x'_i)x^*_i \\
+ \quad & (y'_i - y_i)^2 + (y^*_i - y'_i)y^*_i]
\end{aligned}$$

Apply Eq. (14),

$$
\begin{aligned}
&= \sum_{i=1}^{N}[(x'_i - x_i)^2 + (x^*_i - x'_i)(x^*_i - x'_i) \\
&+ (y'_i - y_i)^2 + (y^*_i - y'_i)(y^*_i - y'_i)] \\
&= \sum_{i=1}^{N}[(x'_i - x_i)^2 + (x^*_i - x'_i)^2 \\
&+ (y'_i - y_i)^2 + (y^*_i - y'_i)^2] \\
&\geq \sum_{i=1}^{N}[(x'_i - x_i)^2 + (y'_i - y_i)^2]
\end{aligned}
$$

Thus proves the theorem. □

Theorem 2. *When the speed and the direction of a target are known, given the estimated locations of the target over T time points calculated by some localization algorithm: $(x^*_1, y^*_1), (x^*_2, y^*_2), \cdots, (x^*_T, y^*_T)$, adjust them by Eq. (7) improves localization accuracy.*

Proof. Proving this theorem is equivalent to prove that

$$\sum_{t=1}^{T}[(x^*_t - x_t)^2 + (y^*_t - y_t)^2] \geq \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \tag{16}$$

From Eq. (5), $f(x'_0, y'_0) = \sum_{t=1}^{T}[(x'_t - x^*_t)^2 + (y'_t - y^*_t)^2]$

From $\dfrac{\partial f(x'_0, y'_0)}{\partial x'_0} = 0$ and $\dfrac{\partial f(x'_0, y'_0)}{\partial y'_0} = 0$,

$$
\begin{cases}
2\sum_{t=1}^{T}(x'_t - x^*_t) = 0, \\
2\sum_{t=1}^{T}(y'_t - y^*_t) = 0.
\end{cases}
$$

So,

$$\begin{cases} \sum_{t=1}^{T} x'_t = \sum_{t=1}^{T} x^*_t, \\ \sum_{t=1}^{T} y'_t = \sum_{t=1}^{T} y^*_t. \end{cases} \tag{17}$$

If a target is moving with a certain speed v and a bearing H, the actual location of the target (x_t, y_t) at time t is:

$$\begin{cases} x_t = x_0 + tv\sin(H) \\ y_t = y_0 + tv\cos(H) \end{cases} \tag{18}$$

$$\begin{aligned} \sum_{t=1}^{T} & [(x^*_t - x_t)^2 + (y^*_t - y_t)^2] \\ = & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (x^*_t - x'_t)^2 + 2(x^*_t - x'_t)(x'_t - x_t) \\ + & (y'_t - y_t)^2 + (y^*_t - y'_t)^2 + 2(y^*_t - y'_t)(y'_t - y_t)] \end{aligned}$$

Apply Eq. (18),

$$\begin{aligned} = & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (x^*_t - x'_t)^2 + 2(x^*_t - x'_t)(x'_t - x_0 - tv\sin(H)) \\ + & (y'_t - y_t)^2 + (y^*_t - y'_t)^2 + 2(y^*_t - y'_t)(y'_t - y_0 - tv\cos(H))] \end{aligned}$$

Expand and apply Eq. (17),

$$\begin{aligned} = & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\ + & \sum_{t=1}^{T}(x^*_t)^2 - 2v\sin(H)\sum_{t=1}^{T} tx^*_t - \sum_{t=1}^{T}(x'_t)^2 + 2v\sin(H)\sum_{t=1}^{T} tx'_t \\ + & \sum_{t=1}^{T}(y^*_t)^2 - 2v\cos(H)\sum_{t=1}^{T} ty^*_t - \sum_{t=1}^{T}(y'_t)^2 + 2v\cos(H)\sum_{t=1}^{T} ty'_t \end{aligned}$$

Apply Eq. (4),

$$
\begin{aligned}
= & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ & \sum_{t=1}^{T}(x^*_t)^2 - 2v\sin(H)\sum_{t=1}^{T} t x^*_t - \sum_{t=1}^{T}(x'_0 + tv\sin(H))^2 + 2v\sin(H)\sum_{t=1}^{T} t(x'_0 + tv\sin(H)) \\
+ & \sum_{t=1}^{T}(y^*_t)^2 - 2v\cos(H)\sum_{t=1}^{T} t y^*_t - \sum_{t=1}^{T}(y'_0 + tv\cos(H))^2 + 2v\cos(H)\sum_{t=1}^{T} t(y'_0 + tv\cos(H))
\end{aligned}
$$

Apply Eq. (6) and expand $\sum_{t=1}^{T}(x^*_t)^2$,

$$
\begin{aligned}
= & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ & \frac{1}{T}[(T-1)\sum_{t=1}^{T}(x^*_t)^2 - 2Tv\sin(H)\sum_{t=1}^{T} t x^*_t - 2\sum_{t=1}^{T-1}\sum_{k=1}^{T-t} x^*_t x^*_{t+k} \\
+ & T(T+1)v\sin(H)\sum_{t=1}^{T} x^*_t + \frac{4T\sum_{t=1}^{T} t^2 - T^2(T+1)^2}{4} v^2\sin^2(H) \\
+ & (T-1)\sum_{t=1}^{T}(y^*_t)^2 - 2Tv\cos(H)\sum_{t=1}^{T} t y^*_t - 2\sum_{t=1}^{T-1}\sum_{k=1}^{T-t} y^*_i y^*_{t+k} \\
+ & T(T+1)v\cos(H)\sum_{t=1}^{T} y^*_t + \frac{4T\sum_{t=1}^{T} t^2 - T^2(T+1)^2}{4} v^2\cos^2(H)]
\end{aligned}
$$

$$
\begin{aligned}
= & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ & \frac{1}{T}\sum_{t=1}^{T-1}\sum_{k=1}^{T-t}[(x^*_t - x^*_{t+k} + kv\sin(H))^2 + (y^*_t - y^*_{t+k} + kv\cos(H))^2] \\
\geq & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2]
\end{aligned}
$$

Thus proves the theorem. $\square$

Theorem 3. *When the speed and the direction of a target are unknown, given the estimated locations of the target over T time points calculated by some localization algorithm: $(x_1^*, y_1^*), (x_2^*, y_2^*), \cdots, (x_T^*, y_T^*)$, adjust them by Eq. (11) improves localization accuracy.*

Proof. Proving this theorem is equivalent to prove that

$$\sum_{t=1}^{T}[(x_t^* - x_t)^2 + (y_t^* - y_t)^2] \geq \sum_{t=1}^{T}[(x_t' - x_t)^2 + (y_t' - y_t)^2] \tag{19}$$

From Eq. (9), $f(v_x, v_y, x_0', y_0') = \sum_{t=1}^{T}[(x_t' - x_t^*)^2 + (y_t' - y_t^*)^2]$

From $\dfrac{\partial f(v_x, v_y, x_0', y_0')}{\partial v_x} = 0$ and $\dfrac{\partial f(v_x, v_y, x_0', y_0')}{\partial v_y} = 0$,

$$\begin{cases} \sum_{t=1}^{T} x_t' = \sum_{t=1}^{T} x_t^*, \\ \sum_{t=1}^{T} y_t' = \sum_{t=1}^{T} y_t^*. \end{cases} \tag{20}$$

If a target is moving with a certain speed v and a bearing H, the actual location of the target (x_t, y_t) at time t is:

$$\begin{cases} x_t = x_0 + tv_x \\ y_t = y_0 + tv_y \end{cases} \tag{21}$$

From Eq. (8) and Eq. (21),

$$\begin{cases} x_t' - x_t = x_0' - x_0 \\ y_t' - y_t = y_0' - y_0 \end{cases} \tag{22}$$

$$
\begin{aligned}
&\sum_{t=1}^{T} \quad [(x_t^* - x_t)^2 + (y_t^* - y_t)^2] \\
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (x_t^* - x'_t)^2 + 2(x_t^* - x'_t)(x'_t - x_t) \\
+ \quad & (y'_t - y_t)^2 + (y_t^* - y'_t)^2 + 2(y_t^* - y'_t)(y'_t - y_t)] \\
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \sum_{t=1}^{T}[(x_t^*)^2 - 2x_t x_t^* - x'_t(x'_t - x_t - x_t) \\
+ \quad & (y_t^*)^2 - 2y_t y_t^* - y'_t(y'_t - y_t - y_t)]
\end{aligned}
$$

Apply Eq. (22),

$$
\begin{aligned}
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \sum_{t=1}^{T}[(x_t^*)^2 - 2x_t x_t^* - x'_t(x'_0 - x_0 - x_t) \\
+ \quad & (y_t^*)^2 - 2y_t y_t^* - y'_t(y'_0 - y_0 - y_t)]
\end{aligned}
$$

Apply Eq. (20),

$$
\begin{aligned}
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \sum_{t=1}^{T}(x_t^*)^2 - 2\sum_{t=1}^{T} x_t x_t^* - (x'_0 - x_0)\sum_{t=1}^{T} x_t^* + \sum_{t=1}^{T} x'_t x_t \\
+ \quad & \sum_{t=1}^{T}(y_t^*)^2 - 2\sum_{t=1}^{T} y_t y_t^* - (y'_0 - y_0)\sum_{t=1}^{T} y_t^* + \sum_{t=1}^{T} y'_t y_t
\end{aligned}
$$

Apply Eq. (21) and Eq. (8),

$$
\begin{aligned}
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \sum_{t=1}^{T}(x^*_t)^2 - 2\sum_{t=1}^{T} x_t x^*_t - x'_0 \sum_{t=1}^{T} x^*_t + 2x_0 \sum_{t=1}^{T} x^*_t + x'_0 v_x \sum_{t=1}^{T} t + v_x^2 \sum_{t=1}^{T} t^2 \\
+ \quad & \sum_{t=1}^{T}(y^*_t)^2 - 2\sum_{t=1}^{T} y_t y^*_t - y'_0 \sum_{t=1}^{T} y^*_t + 2y_0 \sum_{t=1}^{T} y^*_t + y'_0 v_y \sum_{t=1}^{T} i + v_y^2 \sum_{t=1}^{T} t^2
\end{aligned}
$$

Replace x'_0 and y'_0 using Eq. (10),

$$
\begin{aligned}
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \frac{1}{T}[(T-1)\sum_{t=1}^{T}(x^*_t)^2 - 2Tv_x \sum_{t=1}^{T} t x^*_t + T(T+1)v_x \sum_{t=1}^{T} x^*_t \\
- \quad & 2\sum_{t=1}^{T-1}\sum_{k=1}^{T-t} x^*_t x^*_{t+k} + \frac{4T\sum_{t=1}^{T} t^2 - T^2(T+1)^2}{4} v_x^2 \\
+ \quad & (T-1)\sum_{t=1}^{T}(y^*_t)^2 - 2Tv_y \sum_{t=1}^{T} t y^*_t + T(T+1)v_y \sum_{t=1}^{T} y^*_t \\
- \quad & 2\sum_{t=1}^{T-1}\sum_{k=1}^{T-t} y^*_t y^*_{t+k} + \frac{4T\sum_{t=1}^{T} t^2 - T^2(T+1)^2}{4} v_y^2] \\
= \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2] \\
+ \quad & \frac{1}{T}\sum_{t=1}^{T-1}\sum_{k=1}^{T-t}[(x^*_t - x^*_{t+k} + kv_x)^2 + (y^*_t - y^*_{t+k} + kv_y)^2] \\
\geq \quad & \sum_{t=1}^{T}[(x'_t - x_t)^2 + (y'_t - y_t)^2]
\end{aligned}
$$

Thus proves the theorem. $\square$

7.2. Relaxing the constraint in LAS

In our algorithm LAS that uses the spatial relationships to improve localization accuracy, we assumed that the group members strictly maintain their relative positions within the group over time. However, it may not be realistic in real-world situations. After we look at LAS and the proof of Theorem 1 closely, we find that this constraint can be relaxed. The inputs to LAS are the estimated locations of targets from the baseline algorithm (from which we can calculate the estimated relative positions to the centroid) and targets' actual relative positions. We can treat targets' actual relative positions as their initial relative positions before they enter the sensor field. Thus, the adjustment in Eq. (3) is based on targets' estimated positions, targets' calculated relative positions to the centroid, and targets' initial relative positions to the centroid. The actual deviation of targets from the group during movement is neither known nor used. Therefore, whether targets move closer towards or farther away from each other, the proof of Theorem 1 is still valid as long as they do not exchange positions in the group so that we can correctly identify them.

7.3. Time synchronization issue

In our algorithms LAT1 and LAT2 that use temporal relationships to improve accuracy, we have not discussed whether our algorithms still work if the sensor clocks are not synchronized. In real-world applications, it is hard to make sensor clocks perfectly synchronized due to inaccurate crystal quartz and ambient influence. But our algorithms do not require accurate time stamps. After the T estimated positions corresponding to T time points are obtained from the baseline algorithm, Eqs. (7) and (11) match the first position with time 1, the second position with time 2, etc. The accurate time is not a parameter in our algorithms. In other words, as long as the baseline localization algorithm is good enough to determine the correct order of the T estimated positions in the timeline, which is not a hard task, we can improve localization accuracy by LAT1 and LAT2 as proved in Theorems 2 and 3. The order of the positions is required because otherwise we cannot correctly identify positions (e.g. mistake position 1 for position 2).

8. Simulations

Since our approach is the first of its kind, which serves as an extra step after obtaining the location estimates from existing algorithms, there is no work in the literature to compare it with. Besides the theoretical proofs, we also conducted simulations to see if our algorithms can further improve localization accuracy using a simulator built in the Matlab language. We can start with location estimates from some localization algorithm that needs improvement in the literature. Here we chose two most investigated localization techniques in WSNs: Tracking by signal strength and by time of arrival, as our baseline algorithms. Signal-strength localization has errors due to ambient noise and to errors in matching signals between sensors, and time-of-arrival localization has errors due to path nonlinearity by refraction and to measurement of short times, so estimates from both often can be improved in real-world applications. Location estimation based on signal strength is usually less accurate than location estimation based on time of arrival, so our simulations provided an opportunity to see how our methods handled different degrees of initial accuracy. Our previous work [22] implemented these two baseline localization algorithms, so we build our simulation using these.

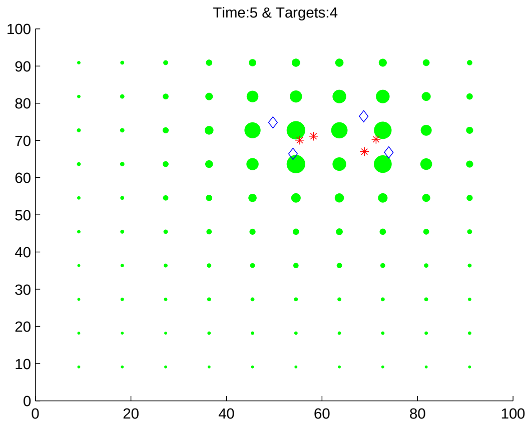
8.1. Simulation for tracking by signal strength

We call the baseline algorithm using signal strength in our previous work [22] our original method (ORG). We compared it with LAS, LAT1, LAT2, LAST and LATS.

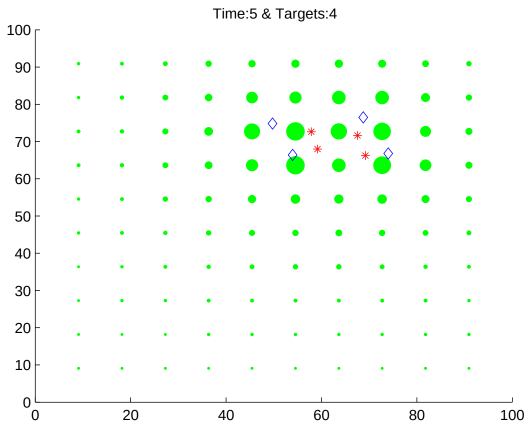
We built on the code for the simulation in [22]. It uses a grid of 100×100 sensors (the green dots in Figure 4). The size of a green dot shows the signal strength received by this sensor. The larger the size, the stronger the signal. Random targets (the blue diamonds in Figure 4) are created in the sensor space, and signals from these targets are received by the sensors in the field to make estimates of locations (the red stars in Figure 4). Following discussions of acoustic, seismic, and magnetic signals in the literature [15], we assumed that the inverse-square law is a good model to calculate signal strength for this simulation, where each signal strength is:

$$s_i = c + \frac{a}{(m+d)^2}, \quad 1 \leq i \leq N$$

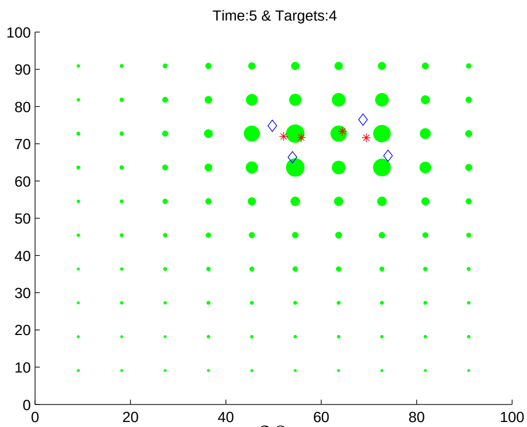
Here, s_i is the sensed signal strength in the ith sensor, c is a random factor, a is the intensity of the target, d is the distance from the target to the sensor, m is the minimum-distance factor from the target which is a feature of each sensor, and N is the total number of sensors in the network. Parameter c was set to 0 in these experiments to model the situation of no background noise. To avoid unstable behavior with very-near targets, m was set to 5 based on experiments in [22]. The total signal strength received by



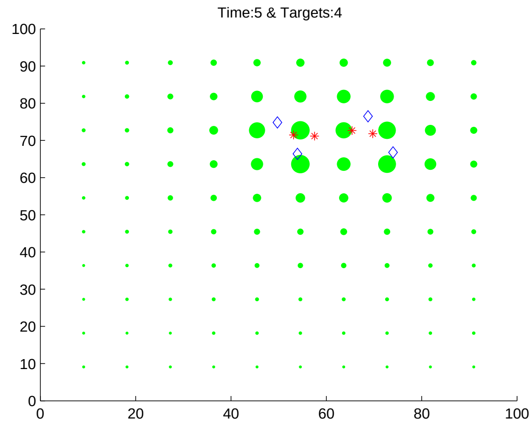
(a) Snapshot using ORG



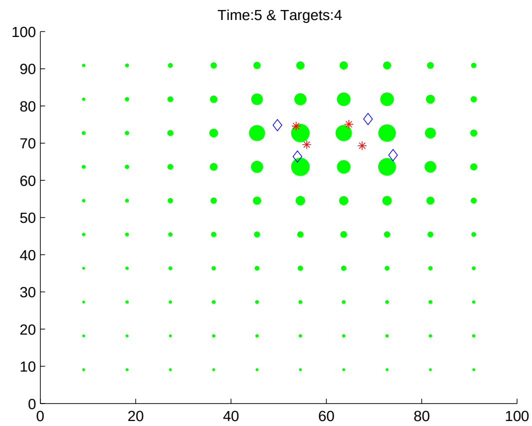
(b) Snapshot using LAS



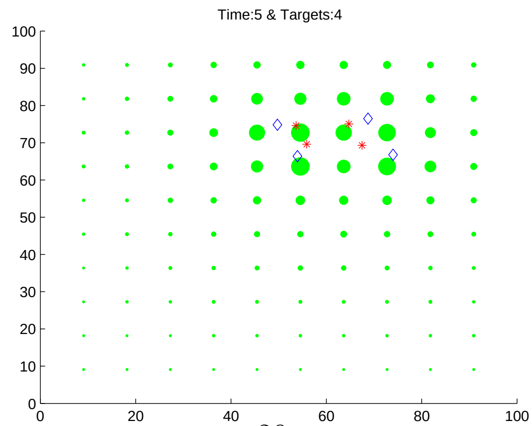
(c) Snapshot using LAT1



(d) Snapshot using LAT2



(e) Snapshot using LAST



(f) Snapshot using LATS

Figure 4: Snapshots of comparing algorithms: the blue diamonds show the actual locations of the targets, the green dots represent the sensors and the red stars are the estimated locations

each sensor is assumed to be additive from all these targets. For instance, if the signal is sound, the intensity from each target would be added at each sensor.

Paper [22] estimates the locations of targets in two steps. First, we assign crude estimates of target locations as the sensor locations that receive the maximum signal strength in their neighborhood (local maxima). In this simulation, the neighborhood of a sensor included all the sensors that are one grid space away from the current one. Second, we adjust locations based on the observed ratio of signal strengths, a variation on the approach of [15]. If initially the observed signal strength is assumed only due to each sensor's nearest target, then for any two sensors 1 and 2, the following holds true due to the assumed inverse square law:

$$s_1[(x - x_{1s})^2 + (y - y_{1s})^2 + m^2] = s_2[(x - x_{2s})^2 + (y - y_{2s})^2 + m^2]$$

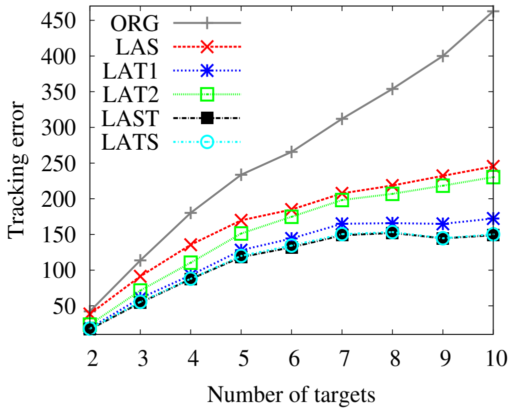
Here, s_1 and s_2 represent the signal strength received by the two sensors, m represents the minimum-distance factor as mentioned above, (x, y) is the position of the tracked target, and (x_{1s}, y_{1s}) and (x_{2s}, y_{2s}) are the coordinates of the two sensors. Rearranging this gives an equation of a circle for the locus of points on which the target could lie. The center and radius of this circle are:

$$x_c = \frac{s_1 x_1 - s_2 x_2}{s_1 - s_2}, \quad y_c = \frac{s_1 y_1 - s_2 y_2}{s_1 - s_2}$$
$$r = \sqrt{\frac{s_1 s_2[(x_{1s} - x_{2s})^2 + (y_{1s} - y_{2s})^2]}{(s_1 - s_2)^2} - m^2}$$

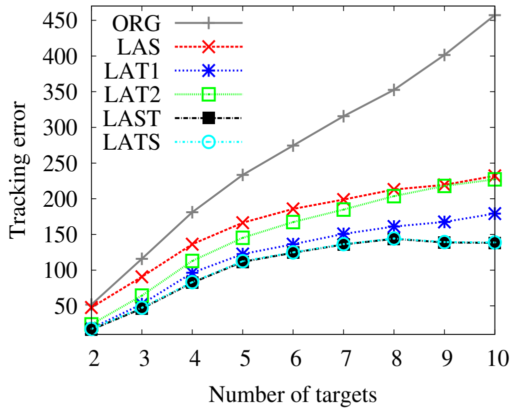
Next, use the idea of trilateration [8] to locate the targets. If we can get three sensors measuring signal strength of the same target, we can intersect their circles to reduce the locus of the target to two points. If there are more than three sensors measuring the same target, a *consensus center* can be obtained by finding the set of all intersection points and repeatedly removing the point furthest from the centroid of the set until there are only two points remaining. The centroid of these two points is the inferred target location.

For our simulation, we generated a group of N targets randomly in a 100×100 grid. It traveled in some direction with some speed across the grid. As it traveled, the signal strengths received by the sensors were calculated at evenly spaced times. The time starts from 1 and the interval between time points is one second. We infer target locations from the signal strength patterns using the ORG algorithm. Then, by considering spatial-temporal relationships of targets, we apply LAS, LAT1, LAT2, LAST, and LATS to adjust locations of targets. For example, Figures 4 (a)-(f) show the snapshots of the tracking simulator with a group of 4 targets at time 5.

We tested groups with 2, 3, $\cdots$, and 10 targets and with signal variances 1 and 3. Signal variance is the error in the signal strength perceived by the sensor. For each experiment, we did 100 runs. Tracking error is averaged over these runs to assess estimation performance. Figures 5(a) and (b) confirmed the performance of our methods. The proposed algorithms improve the localization accuracy of the ORG algorithm. With the increase of target number, the increase of the tracking error of our proposed algorithms slows down, especially the combined ones. The figures show that LAT1 performs better than LAT2. This is because if more parameters are known, the results



(a) Tracking error using signal strength with variance 1



(b) Tracking error using signal strength with variance 3

Figure 5: Tracking error with different group sizes and variances using signal strength

can be more accurate. Similarly, the combination of the spatial and temporal relationships is better than each one alone. However, there was not much difference between LAST and LATS. That means, whether the locations are adjusted first by the spatial relationships or by the temporal relationships does not matter much. Also, the signal variance has little effect on tracking error.

8.2. Simulation for tracking by time of arrival

In the second simulation, we localized using time of arrival [22], a method which we named ORG-T. All other methods follow the same naming scheme. Analogously to before, we created five more methods: LAS-T, LAT1-T, LAT2-T, LAST-T, and LATS-T.

The principle of time-of-arrival tracking is that the speed of transmission of nearly all signals is relatively constant over space, so differences in times at which a signal is received are near to proportional to differences in distances from the source of the sound. Thus in a two-dimensional plane, the locus of points of a target based on the differences in time of arrival of the same signal transmitted from that target to two sensors is a hyperbola. Three sensor readings from targets reduce the locus to (generally) two points, and four readings reduce it to one point. However, it is best to obtain as many readings as we can to compensate for inaccuracies, and then use a fitting method like least-squares to minimize the overall error.

Again suppose we have a set of N sensors at locations (x_i, y_i) for $i = 1$ to N. Assume that corresponding peaks arrive at each sensor at time t_i . We want to minimize:

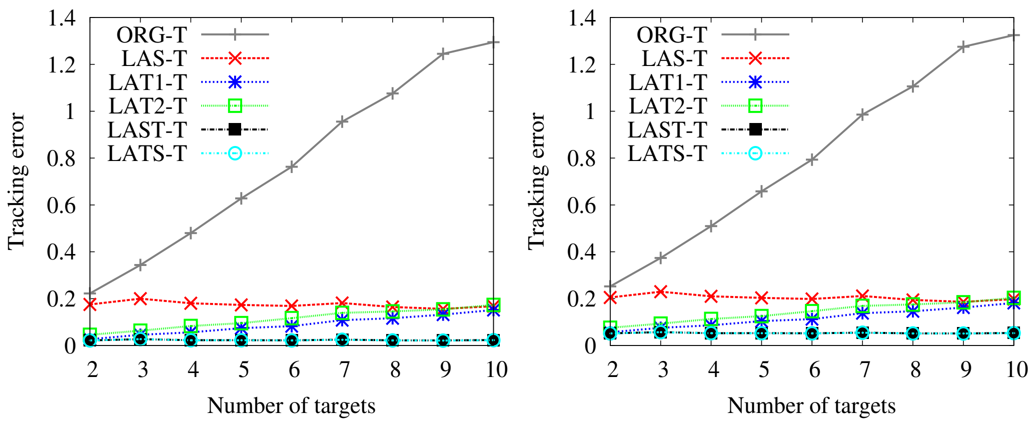
$$G_D = \sum_{i=1}^{N} \sum_{j=i+1}^{N} |E_D(i,j)|$$

where

$$\begin{aligned} E_D(i,j) &= \sqrt{(x-x_i)^2 + (y-y_i)^2} - \sqrt{(x-x_j)^2 + (y-y_j)^2} \\ &+ c(t_j - t_i) \end{aligned}$$

Here c is the average speed of the signal and (x, y) is the position of the tracked target as before. The derivatives of G are (where "sgn" is the sign of its argument):

$$\begin{aligned} \frac{\partial G_D}{\partial x} &= \sum_{i=1}^{N} \sum_{j=i+1}^{N} 2 * sgn(E_D) * [\frac{x - x_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2}} \\ &- \frac{x - x_j}{\sqrt{(x-x_j)^2 + (y-y_j)^2}}] \end{aligned}$$



(a) Tracking error using time of arrival with variance 1

(b) Tracking error using time of arrival with variance 3

Figure 6: Tracking error with different group sizes and variances using time of arrival

$$\frac{\partial G_D}{\partial y} = \sum_{i=1}^{N}\sum_{j=i+1}^{N} 2 * sgn(E_D) * [\frac{y-y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} - \frac{y-y_j}{\sqrt{(x-x_j)^2+(y-y_j)^2}}]$$

The two ratios are equivalent to the cosine (for x) and sine (for y) of the bearing angles from the estimated target location to the sensor. Thus we can optimize the location of the tracked target by moving its position by a weighted sum of the vectors towards or away from each of the sensors.

Figures 6(a) and (b) show the results based on time-of-arrival localization. We observe that the localization accuracy of tracking by time of arrival is much better than that of tracking by signal strength. Similar to the results in signal strength, the tracking error of ORG is much higher than that of the locations adjusted by our proposed algorithms. Also with an increase of number of targets, the increase of the tracking error of our algorithms slows down or stops. Other observations for the signal-strength experiments also hold in time of arrival experiments.

9. Conclusion

In this paper, we presented that using two additional constraints can improve localization accuracy of positions of targets estimated from sensors alone. The additional constraints were an assumption of consistency of relative locations within a group of targets and an assumption of consistency in velocity vector. We showed relatively simple mathematics by which they can be applied to sensor-based estimates, and proved that these methods never decreased the quality of the estimates. Furthermore, we relaxed the condition that targets should strictly keep their relative positions in the group and explained that perfect time synchronization is not required. We also displayed results of simulation on idealized sensor grid with random targets confirming that the methods did improve tracking accuracy considerably. In the future, we will consider fault-tolerance, connectivity and security issues in the localization algorithms.

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